Webpage-MTH111-Course Portfolio-Fall 2020

Ayman Badawi

Table of contents

1 Section : Course Syllabus

AUS | الجامعة الأمريكية في الشارقة American University of Sharjah

COURSE SYLLABUS

Warning: During this difficult time, "trust" relationship between students and instructor will definitely facilitate our work, to ensure that this "trust" is not violated, suspicious Respondus reports (after exams) will be sent to the Associate Dean.

A	**Course Title & Number**	MTH 111, Mathematics for Architects
B	**Pre/Co-requisite(s)**	Prerequisites: MTH 001 or MTH 003 or Architecture Math Placement Test or Engineering Math Placement Test or SAT II Math Level 1 test with score 600 and above
C	**Number of credits**	3-0-3
D	**Faculty Name**	Ayman Badawi
E	**Term/ Year**	Fall 2020
F	**Sections**	

CRN	Course	Days	Time	Location
	MTH111	TRU	11-11:50	ON LINE

G Instructor Information

Instructor	Office	Telephone	Email
Ayman Badawi	Nab 262 /Home		abadawi@aus.edu

Office Hours:

- TRU: 15—16
- Other office hours are available by appointment(just email me)

H Course Description from Catalog

Introduces the topics of geometry and calculus needed for architecture. Reviews conic sections, Areas and volumes of elementary geometric figures, and the analytic geometry of lines, planes and vectors in two and three dimensions. Covers differential and integral calculus, including applications on optimization problems, and areas and volumes by integration. Restricted to CAAD students.

I Course Learning Outcomes

Upon completion of the course, students will be able to:

1. Solve problems involving comic sections (Parabola, Ellipse, and Hyperbola). **Exam One, Final**
2. Find the derivative of a function and apply it to solve a variety of problems involving optimization and curve sketching. Exam 2, Final
3. Apply the Fundamental Theorem of Calculus to find the area under a curve and compute volumes of revolution. **Exam 2, Final**
4. Apply the analytic geometry of conic sections to solve word problems. **Exam one**
5. Express geometric quantities using vectors and their standard operations in 2 and 3 dimensions. **Exam one, Final**
6. Solve geometric problems involving lines and planes in 2 and 3 dimensions. **Exam one, Final**

J Textbook and other Instructional Material and Resources

Class notes (very crucial) , Materials posted on I-Learn , and my personal webpage (for old quizzes, exams, finals) : http://www.ayman-badawi.com/MTH%20%20111.html

K	**Teaching and Learning Methodologies**	This is a traditional lecture based course. Students are tested and given feedback throughout the semester via regular quizzes and exams.
L	**Grading Scale, Grading Distribution, and Due Dates**	**Grading Distribution:**

Assessment	Weight	Due Date
Quizzes	15%	TBA
Exam I	25%	Tuesday (@18:00) October 13
Exam II	25%	Tuesday (@18:00) November 24
Final Exam	35%	TBA
Total	100%	

Grading Scale

Letter	GPA	Percentage
A	4.0	92-100
A-	3.7	88-91.99
B+	3.3	84-87.99
B	3.0	80-83.99
B-	2.7	77-79.99
C+	2.3	74-76.99
C	2.0	67-73.99
C-	1.7	60-66.99
D	1.0	41-59.99
F	0	0-40.99

M Explanation of Assessments

- Quizzes: There will be in-class quizzes.
- Midterm Tests: There will be two midterm exams. The dates of the exams are given in this syllabus. .
- Final Exam: Final examination will be comprehensive. The date and time of the final exam is also given in this syllabus.

N Student Academic Integrity Code

All students are expected to abide by the Student Academic Integrity Code as articulated in the AUS Undergraduate Catalog.

Remarks and Rules:

- Quizzes will be pre-announced at least one lecture in advance.
- No make-up quizzes will be given. However the lowest quiz will not be counted toward your final grade.

SCHEDULE

CHAPTER	Week
Conic sections, ellipse, parabola, and hyperbola	One
Continue: Conic sections, ellipse, parabola, and hyperbola	• Two
Lines in 2D , Vectors in 2 D , and projection	• Three
Dot Product, Cross Product and applications	• Four
Line and planes in 3 dimensional space , and Parametric Equations	• Five
Continue: Line and planes in 3 dimensional space, and Parametric Equations	• Six
Definition of derivatives and apply derivative to polynomials, exponential function, and logarithms	• Seven
Tangent lines and normal lines, product formula, quotient formula, and chain rule	Eight
Applications of Derivatives: Maximaze and Minimize	Nine
Integration (anti-derivative), techniques and properties	• Ten
Integration by substitution and by simple fractions	Eleven
Calculating areas by definite integrals	Twelve
More techniques on Integration (Integral of a polynomial times exponential function)	Thirteen
Volume by definite integrals	Fourteen
Voume /Area and Reviews	Fifteen
Final Exam	

2 Academic Integrity Measures

Academic Integrity Measures in Online Exams

List the measures taken to ensure the academic integrity of the exam.

Quizzes 1-6, all students were in the lecture room (blackboard Ultra room). All students had 20-25 minutes. All questions are essay. Students submitted their solution in a folder that I created on I-learn.

Students used lockdown browser for exams one, two and final exam. All questions are essay. Students submitted their solution in a folder that I created on I-learn. The outcome (scores) was not significantly different from a normal in-class exams (see the scores of the students in the excel-sheet)

I am completely satisfied with the outcome of MTH111.

3 Section : Instructor Teaching Material-Handouts

3.1 Questions with Solutions on Ellipse from previous semesters

Krstin Raed

15/15 :)

Quiz I MTH 111, Spring 2019

Ayman Badawi

$CF^2 = \left(\frac{k}{2}\right)^2 - b^2$

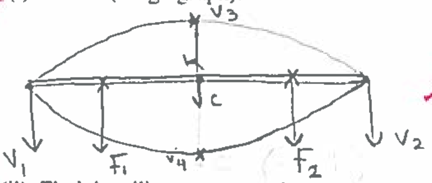
$CF^2 = 25 - 9$
$= 16$
$\boxed{CF = 4}$

QUESTION 1. Consider the ellipse $\frac{(x+2)^2}{25} + \frac{(y-1)^2}{9} = 1$

$25 = \left(\frac{k}{2}\right)^2$, $9 = b^2$

2 (i) Sketch (rough graph).

$c = (-2, 1)$



2 (ii) Find the ellipse-constant, k

$\left(\frac{k}{2}\right)^2 = 25$ $\frac{k}{2} = \sqrt{25}$ $\rightarrow$ $k = 5 \times 2$ $\rightarrow$ $\boxed{k = 10}$

$b^2 = 9$
$\boxed{b = 3}$

2 (iii) Find all 4 vertices

V_1 $(-2-5, 1)$ $(-7, 1)$
V_2 $(-2+5, 1)$ $(3, 1)$

V_3 $(-2, 4)$
V_4 $(-2, -2)$

2 (iv) Find the Foci

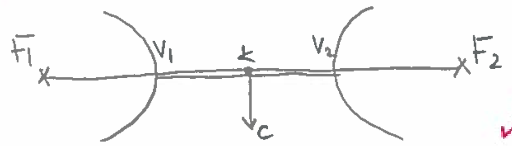
F_2 $(-2+4, 1)$
$(2, 1)$

F_1 $(-2-4, 1)$
$(-6, 1)$

QUESTION 2. Consider the hyperbola $\frac{(x-3)^2}{9} - \frac{(y+2)^2}{16} = 1.$

2 (i) Sketch (rough graph).

$c = (3, -2)$



1 (ii) Find the heperbola-constant, k

$\left(\frac{k}{2}\right)^2 = 9$ $\frac{k}{2} = \sqrt{9}$ $\boxed{k = 3 \times 2 = 6}$

2 (iii) Find all vertices

V_1 $(3-3, -2)$
$(0, -2)$

V_2 $(3+3, -2)$
$(6, -2)$

2 (iv) Find the Foci

$CF^2 = \left(\frac{k}{2}\right)^2 + b^2$
$CF^2 = 9 + 16$

$CF^2 = 25$
$\boxed{CF = 5}$

F_1 $(3-5, -2)$
$(-2, -2)$

F_2 $(3+5, -2)$
$(8, -2)$

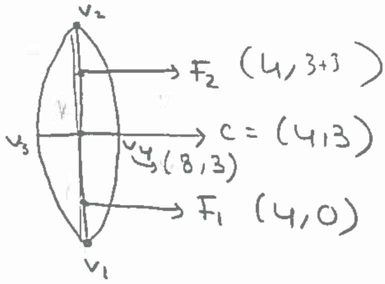
Faculty information

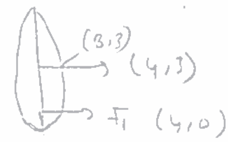
Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: abadawi@aus.edu, www.ayman-badawi.com

QUESTION 5. An ellipse is centered at $(4,3)$, $F_1 = (4,0)$ is one of the foci, and $(8,3)$ is one of the vertices.

(i) **(2 points)** Roughly, sketch such ellipse.

x does not change





$CF^2 = \left(\frac{k}{2}\right)^2 - b^2$

$3^2 = \left(\frac{k}{2}\right)^2 - 4^2$

$25 = \left(\frac{k}{2}\right)^2$

$\boxed{CF = 3}$

$\boxed{b = 4}$

(ii) **(3 points)** Find the ellipse-constant K.

$$CF^2 = \left(\frac{k}{2}\right)^2 - b^2$$

$$3^2 = \left(\frac{k}{2}\right)^2 - 4^2$$

$$\boxed{k = 10}$$

(iii) **(2 points)** Find the second foci of the ellipse.

$$F_2 = (4, 3+3)$$

$$(4, 6)$$

(iv) **(3 points)** Find the remaining three vertices of the ellipse

$V_1 = \left(4, 3 - \frac{10}{2}\right)$ $\boxed{(4, -2)}$ $V_3\ (0, 3)$

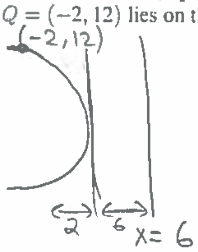
$V_2\ \left(4, 3 + \frac{10}{2}\right)$ $(4, 8)$

(v) **(3 points)** Find the equation of the ellipse.

$$\frac{(y-3)^2}{\left(\frac{10}{2}\right)^2} + \frac{(x-4)^2}{4^2} = 1$$

$$\frac{(y-3)^2}{25} + \frac{(x-4)^2}{16} = 1$$

QUESTION 11. (4 points) Given that $x = 6$ is the directrix line of a parabola that has F as its focus point. If the point $Q = (-2, 12)$ lies on the parabola. Find $|QF|$ (i.e., the distance between Q and F).



$|QF| = |QL| = 8$

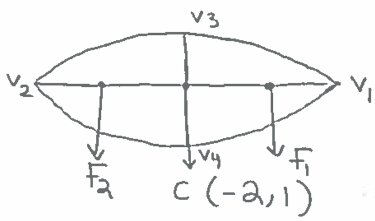
QUESTION 12. (6 points) Consider the ellipse $\dfrac{(y-1)^2}{9} + \dfrac{(x+2)^2}{25} = 1.$

$9 \rightarrow b^2$

$25 \rightarrow$ big # so its $\left(\frac{k}{2}\right)^2$

so the shape is

(i) Sketch (roughly)



(ii) Find the foci of the ellipse

$$CF^2 = \left(\frac{k}{2}\right)^2 - b^2 = 25 - 9 = 16$$

$CF^2 = 16$ so $\boxed{CF = 4}$

so $F_1\ (-2+4,\ 1)$ $(2, 1)$

$F_2\ (-2-4,\ 1)$ $(-6, 1)$

(iii) Find all four vertices of the ellipse.

$\left(\frac{k}{2}\right)^2 = 25$ $\boxed{\pm\frac{k}{2}}$

$\boxed{\frac{k}{2} = 5}$ $\boxed{\pm b}$

$b^2 = 9$ $\boxed{b=3}$

$V_1 = (-2+5,\ 1)$ $(3, 1)$

$V_2 = (-2-5,\ 1)$ $(-7, 1)$

$V_3 = (-2,\ 1+3)$ $(-2, 4)$

$V_4 = (-2,\ 1-3)$ $(-2, -2)$

QUESTION 13. (4 points) Given $Q = (1, 6, 4)$ is not on the line $L : x = t + 1, y = 2t + 4, z = -5t + 3$ $(t \in R)$. Find $|QL|$.

$$|QL| = \frac{|D \times IQ|}{|D|} = \frac{\sqrt{12^2+1^2+2^2}}{\sqrt{1^2+2^2+5^2}} = \frac{\sqrt{149}}{\sqrt{30}}$$

$D = \langle 1, 2, -5 \rangle$

$I = \langle 1, 4, 3 \rangle$

$\underset{Q-I}{IQ} = \langle 0, 2, 1 \rangle$

$$IQ \times D = \begin{vmatrix} i & j & k \\ 0 & 2 & 1 \\ 1 & 2 & -5 \end{vmatrix} = -12i + 1j - 2k$$

Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: `abadawi@aus.edu`, `www.ayman-badawi.com`

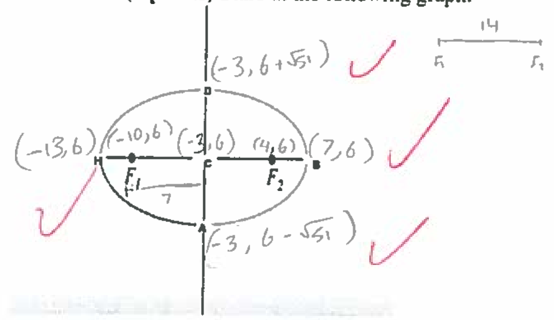
Name Haya Sujaa, ID 200082558

Final Exam, MTH 111, Spring 2019

Ayman Badawi

$$Score = \frac{75}{78}$$

QUESTION 1. (7 points) Stare at the following graph.



(i) Given $F1 = (-10, 6)$, $F2 = (4, 6)$ and the ellipse-constant is 20.

(ii) Find the center $c =$ $|CF_1| = 7$ $\therefore C = (-3, 6)$

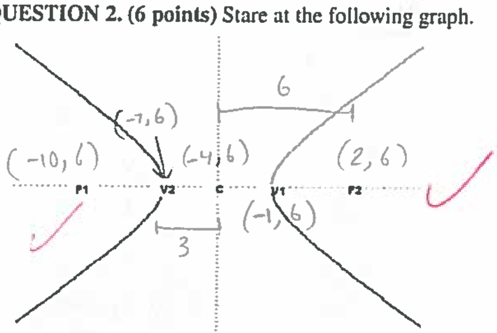
(iii) Find the vertices $A = (-3, -1.14)$, $D = (-3, 13.14)$, $H = (-13, 6)$, and $B = (7, 6)$

10, 7, b $\quad 100 - 49 = b^2$, $b = \sqrt{51}$

(iv) Find the equation of the ellipse.

$$\frac{(x+3)^2}{100} + \frac{(y-6)^2}{51} = 1$$

QUESTION 2. (6 points) Stare at the following graph.



Given $c = (-4, 6)$, $|cv2| = 3$, and $F2 = (2, 6)$.

(i) Find $v1 = (-1, 6)$ $F1 = (-10, 6)$, $v2 = (-7, 6)$, and the hyperbola-constant $k = 6$

$|CF_1| = \sqrt{(k/2)^2 + b^2} = 6$

$\sqrt{9 + b^2} = 6$

$9 + b^2 = 36$

$b^2 = 36 - 9$

$b^2 = 27$

(ii) Find the equation of the hyperbola

$$\frac{(x+4)^2}{9} - \frac{(y-6)^2}{27} = 1$$

alleem AlAli

Quiz I: MTH 111, Spring 2018

Ayman Badawi

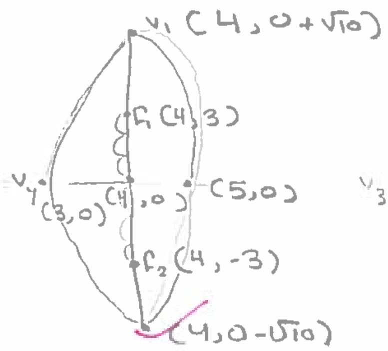
$\frac{(y-y_0)^2}{10} + (x-x_0)^2$ 15/15

QUESTION 1. Consider the ellipse given by $\frac{y^2}{10} + (x-4)^2 = 1$

$C = (4, 0)$

(i) Sketch, roughly.

$b^2 = 1 \quad b = 1$



(ii) Find the ellipse-constant K. $\sqrt{\left(\frac{K}{2}\right)^2} = \sqrt{10}$ $\boxed{K/2 = \sqrt{10}}$

$K = 2\sqrt{10}$

(iii) Find the foci. $|CF_1| = \sqrt{(K/2)^2 - b^2} = \sqrt{10-1} = 3$

3 $F_1\ (4, 3)$ $F_2 = (4, -3)$

(iv) Find all vertices, $V_1 = (4, 0+\sqrt{10})$ $V_3 = (4+1, 0)$

$V_2 = (4, 0-\sqrt{10})$ $V_4 = (4-1, 0)$

QUESTION 2. Consider the parabola $y = 3x^2 + 18x + 5$

(i) Sketch, roughly. Standard form

5

$y = 3[x^2 + 6x] + 5$

$y = 3[(x+3)^2 - 9] + 5$

$y = 3(x+3)^2 - 27 + 5$

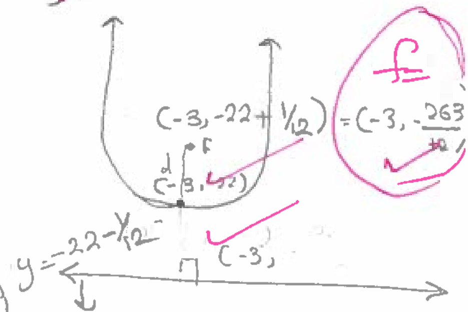
$y = 3(x+3)^2 - 22$

$y + 22 = 3(x+3)^2$

$\frac{1}{3}(y+22) = (x+3)^2$

vertex $= (-3, -22)$

$4d = \frac{1}{3}$ $d = \frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$



$= (-3, \frac{-263}{12})$

(ii) Find the focus.

write answers here!

$(-3,$

(iii) Find the directrix line.

$y = \frac{-265}{12}$

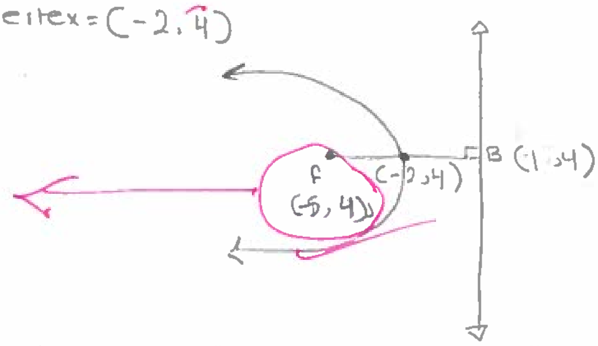
QUESTION 3. Consider the parabola $-12(x+2) = (y-4)^2$

vertex $= (-2, 4)$

(i) Sketch, roughly.

$4d = -12$

$\boxed{d = -3}$



(ii) Find the focus.

5 $(-5, 4)$

(iii) Find the directrix line. $x = 1$

Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: abadawi@aus.edu, www.ayman-badawi.com

$x = 1$

$(1, 4)$

$(-5, 4)$

QUESTION 4. Given $y = x^2 - 6x - 1$ is an equation of a parabola.

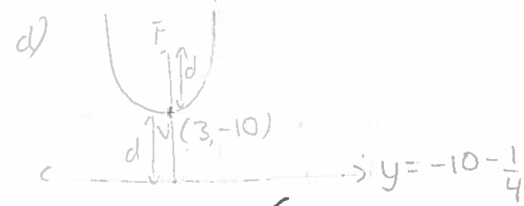
a)**(3 points)** Write the equation in the standard form.

$y = (x-3)^2 - 9 - 1$

$y = (x-3)^2 - 10$

$\boxed{(y+10) = (x-3)^2}$

$4d = 1 \Rightarrow d = 1/4$



b) **(2 points)** Find the equation of the directrix line.

$\boxed{y = -10 - \frac{1}{4} = -\frac{41}{4}}$

c)**(2 points)** Find the focus, say F

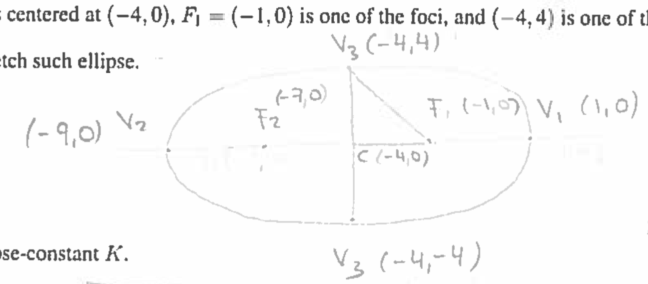
$\boxed{F(3, -10 + \frac{1}{4})} \to \boxed{F(3, -\frac{39}{4})}$

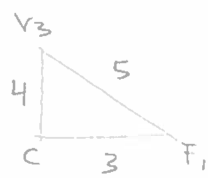
d)**(2 points)** Roughly, sketch the graph of such parabola.

(see picture)

QUESTION 5. An ellipse is centered at $(-4, 0)$, $F_1 = (-1, 0)$ is one of the foci, and $(-4, 4)$ is one of the vertices.

(i) **(2 points)** Roughly, sketch such ellipse.





(ii) **(3 points)** Find the ellipse-constant K.

$|V_3F_1| = \frac{K}{2} = 5 \Rightarrow \boxed{K = 10}$

(iii) **(2 points)** Find the second foci of the ellipse.

$\boxed{F_2(-7, 0)}$

(iv) **(3 points)** Find the remaining three vertices of the ellipse

$\boxed{V_3(-4,-4)}$

$\boxed{V_1(1,0)}$

$\boxed{V_2(-9,0)}$

(v) **(3 points)** Find the equation of the ellipse.

$$\boxed{\frac{(x+4)^2}{25} + \frac{y^2}{16} = 1}$$

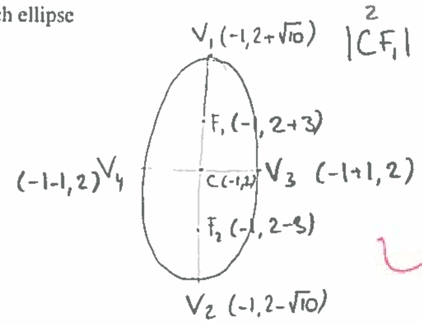
(x) Consider the ellipse $(x+1)^2 + \frac{(y-2)^2}{10} = 1$

$C(-1,2)$

$\frac{k}{2} = \sqrt{10}$

$|CF_1| = \sqrt{10-1} = 3$

a. (2 points) Roughly, draw such ellipse



b. (2 points) Find the foci

$F_1(-1, 5)$

$F_2(-1, -1)$

c. (2 points) Find the ellipse constant

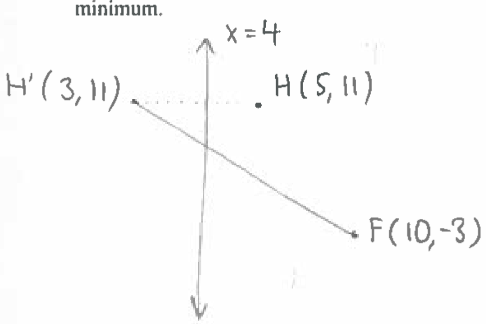
$k = 2\sqrt{10}$

d. (2 points) Find all four vertices

$V_1(-1, 2+\sqrt{10})$ $V_3(0,2)$

$V_2(-1, 2-\sqrt{10})$ $V_4(-2,2)$

(xi) (6 points) Let $H = (5, 11)$ and $F = (10, -3)$. Find a point Q on the vertical line $x = 4$ such that $|HQ| + |QF|$ is minimum.



$m = \frac{-3-11}{10-3} = -2$

$11 = -2(3) + b$

$b = 17$

$y = -2x + 17$

$y = -2(4) + 17 = 9$

$Q(4,9)$

Inaam Shamma

Quiz I: Math. for the Architects, MTH 111, Spring 2017

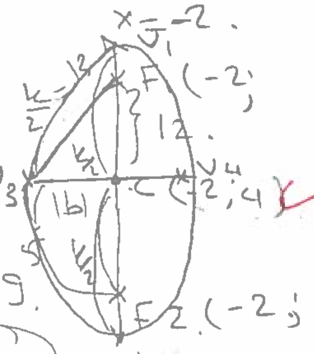
Ayman Badawi

15/15

QUESTION 1. Consider the Ellipse $\frac{(x+2)^2}{25} + \frac{(y-4)^2}{169} = 1$

(i) Sketch (rough sketch)

2



$\left(\frac{k}{2}\right)^2 = 169$

$\frac{k}{2} = 13$

$\boxed{k = 26}$

$C(-2;+4)$

$b^2 = 25$

$\boxed{b = 5}$

$hyp^2 = side^2 + side^2$

$169 = 25 + |F_1C|^2$

$|F_1C|^2 = 144$

$\boxed{F_1C = 12}$

(ii) Find the Foci

3

$F_1(-2;16)$

$F_2(-2;-8)$

(iii) Find the ellipse-contant k

3

$\boxed{k=26}$

(iv) Find all 4 vertices.

3

$V_3(-7;4)$ $V_4(3;4)$ $V_1(-2;17)$ $V_2(-2;-9)$

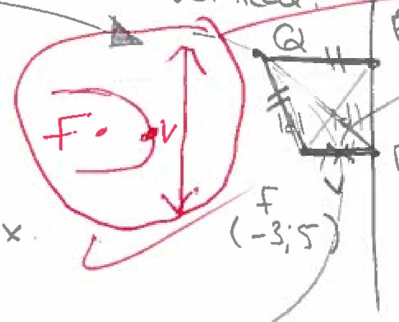
QUESTION 2. Given $(-3, 5)$ is the focus of a parabola with directrix line $x = 9$.

vertical

(i) Sketch (rough sketch)

2

graph sketched on the back



(ii) Find the equation of the Parabola.

eq: $4d(x-x_1) = (y-y_1)^2$.

3

vertix.

midpt of $|FB|$ is the vertix.

$x_V = \frac{x_F + x_B}{2} = \frac{-3+9}{2} = 3$.

$|FV| = |VB| = |d| = |\Delta x| = |-3-3| = |-6| = 6$.

since on the left side $d < 0$.

$\boxed{d = -6}$

$4(-6)(x-3) = (y-5)^2$

$\boxed{-24(x-3) = (y-5)^2}$

(iii) ~~If Q is a point on the curve of the parabola. What is the distance between Q and the directrix~~?

2

$|QF| = |QL|$ QL we draw $\perp$ to L.

intersect at point E

$E(9;?)$

Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: abadawi@aus.edu, www.ayman-badawi.com

iii) find the distance between vertix and directrix.

$|VB| = \sqrt{\Delta x^2} = |\Delta x| = |9-3| = 6$.

Haya Alshamsi

Exam I: MTH 111, Fall 2017

Ayman Badawi

Excellent

Points = $\frac{70}{70}$

QUESTION 1. **(6 points)** Given $y = 11$ is the directrix of of a parabola that has the point $(6,5)$ as its vertex point.

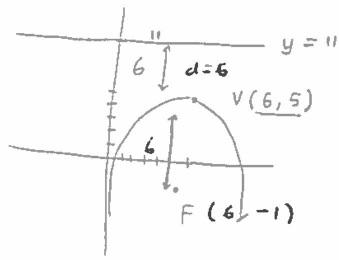
a) Find the equation of the parabola

$\boxed{d = 6}$

$$-4d\,(y - y_1) = (x - x_1)^2$$

$$-4(6)(y-5) = (x-6)^2$$

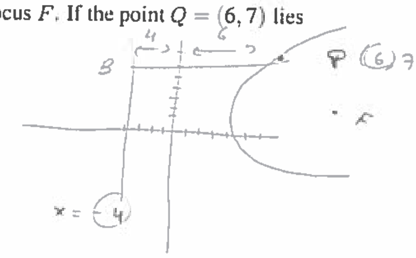
$$-24(y-5) = (x-6)^2$$



b) Find the focus of the parabola.

$F\ (6, -1)$

QUESTION 2. **(3 points)** Given that $x = -4$ is the directrix of a parabola that has focus F. If the point $Q = (6, 7)$ lies on the curve of the parabola, find $|QF|$ (i.e., find the distance between F and Q).



$|QL| = |QF|$

$|QB| = |QF|$

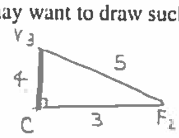
$|QF| = 10$ units

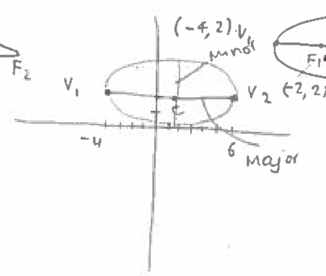
QUESTION 3. **(8 points)** Given $(-4, 2), (6, 2)$ are the vertices of the major axis of an ellipse (recall major axis is the longer axis) and $(4, 2)$ is one of the foci.

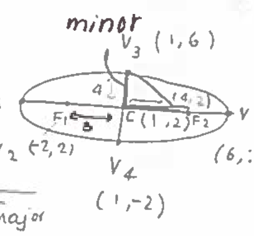
(i) Find the vertices of the minor axis (shorter axis). (you may want to draw such ellipse (roughly)).

$V_3\ (1, 6)$

$V_4\ (1, -2)$







(ii) Find the ellipse-constant K. $\quad C\ (1, 2)\ ;\ V_2\ (6, 2)$

$\frac{K}{2} = 5 \quad \Rightarrow \boxed{K = 10}$

(iii) Find the second foci of the ellipse.

$F_1\ (-2, 2)$

(iv) Find the equation of the ellipse. horizontal ellipse $;\ K = 10\ ;\ \cancel{(b = 3)}\ b = 4$

$$\frac{(x-1)^2}{25} + \frac{(y-2)^2}{16} = 1$$

Final Exam: MTH 111, Fall 2017

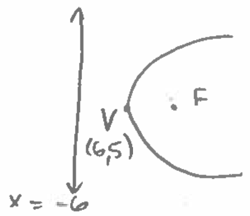
Ayman Badawi

Katia

Points = $\frac{81}{82}$

QUESTION 1. (6 points) Given $x = -6$ is the directrix of of a parabola that has the point $(6, 5)$ as its vertex point.

a) Find the equation of the parabola



$|VL| = |-6-6| = |-12| = 12$

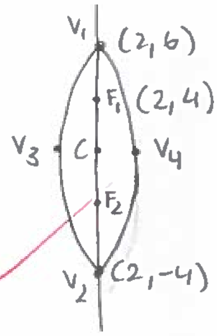
$4(12)(x-6) = (y-5)^2 \Rightarrow 48(x-6) = (y-5)^2$

b) Find the focus of the parabola.

$|VF| = 12 \to F(18, 5)$

QUESTION 2. (8 points) Given $(2, -4), (2, 6)$ are the vertices of the major axis of an ellipse (recall major axis is the longer axis) and $(2, 4)$ is one of the foci.

(i) Find the vertices of the minor axis (shorter axis). (you may want to draw such ellipse (roughly)).



$|V_1V_2| = K = |6+4| = 10 \to \frac{K}{2} = 5 = |V_1C|$

$C = (2,1) \to |F_1C| = |4-1| = 3 \to b^2 = (\frac{K}{2})^2 - |F_1C|^2$

$b^2 = 5^2 - 3^2 = 16 \to V_3(18,1), V_4(-14,1)$

(ii) Find the ellipse-constant K.

$K = 10$

(iii) Find the second foci of the ellipse.

$F_2(2, -2)$

(iv) Find the equation of the ellipse.

$$\frac{(x-2)^2}{16} + \frac{(y-1)^2}{25} = 1$$

QUESTION 3. (5 points) Given $y = 3x^2 + 12x + 9$ is an equation of a parabola. Write the equation of the parabola in the standard form and find the equation of its directrix.

$y = 3x^2 + 12x + 9 \to y = 3(x^2 + 4x + 3) \to y = 3[(x+2)^2 - 4 + 3]$

$y = 3(x+2)^2 - 1(3) \to \frac{1}{3}(y+3) = (x+2)^2$

$4d = \frac{1}{3} \to d = \frac{1}{12}$

$V = (-2, -3) \to$ directrix $\to x = -2 - \frac{1}{12} \Rightarrow \boxed{\frac{-25}{12} = x}$

$y = -3 - \frac{1}{12}$

3.2 Questions with Solutions on parabola from previous semesters

Krstin Raed
g00078656

Quiz II MTH 111, Spring 2019

Ayman Badawi

15/15 :)

QUESTION 1. Consider the parabola $y = 3x^2 - 6x + 2$

3 (i) Write the equation above in the standard form.

$y = 3x^2 - 6x + 2$

$y = (3(x^2 - 2x)) + 2$

$3((x-1)^2 - 1^2) + 2$

$y = 3(x-1)^2 - 3 + 2$

$y = 3(x-1)^2 - 1$

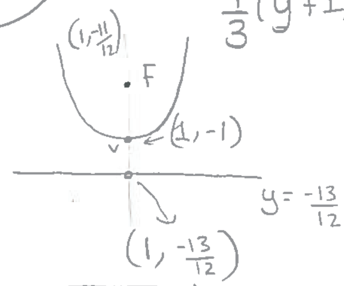
$\frac{(y+1)}{3} = \frac{3(x-1)^2}{3}$

$\frac{1}{3}(y+1) = (x-1)^2$

2 (ii) Sketch the graph (roughly)

$y=$ so up or down

$4d = \frac{1}{3}$ $d = \frac{1}{12}$ so up



1 (iii) Find the vertex.

$\boxed{(1, -1)}$

1 (iv) Find the FOCUS.

$F = \left(1, -1 + \frac{1}{12}\right)$ $\boxed{\left(1, \frac{-11}{12}\right)}$

1 (v) Find the equation of the directrix line

$\boxed{y = \frac{-13}{12}}$

QUESTION 2. Consider the parabola $-12(x+4) = (y-2)^2$.

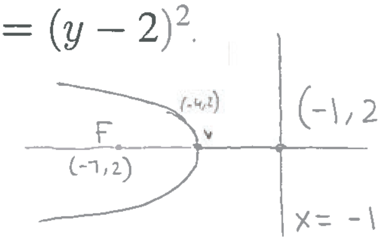
4 (i) Sketch (rough graph).

$4d = -12$

$\boxed{d = -3}$

(x so its right or left)

negative so left



1 (ii) Find the focus

$(-4-(3), 2)$ $\boxed{(-7, 2)}$

1 (iii) Find the vertex

$\boxed{(-4, 2)}$

1 (iv) Find the equation of the directrix line.

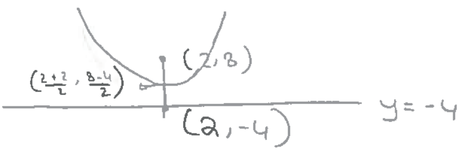
$\boxed{x = -1}$

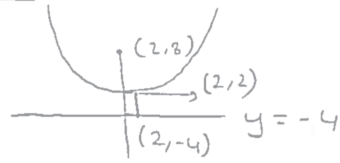
Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: abadawi@aus.edu, www.ayman-badawi.com

QUESTION 3. Given $y = -4$ is the directrix of a parabola that has the point $F = (2, 8)$ as its focus point.

a) **(2 points)** Roughly, sketch such parabola.





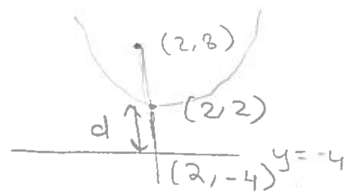
d = 6 & its up

4

b)**(4 points)** Find the equation of the parabola

$$4d(y-2) = (x-2)^2$$
$$4(6)(y-2) = (x-2)^2$$
$$24(y-2) = (x-2)^2$$

$\boxed{d=6}$



$d = (-4-2$
-6

c) **(2 points)** Find the vertex of the parabola, say V.

$$V = (2,2)$$

QUESTION 4. Given $y = 4x^2 + 24x - 3$ is an equation of a parabola.

a)**(3 points)** Write the equation in the standard form.

$$y = 4x^2 + 24x - 3$$
$$y = 4(x^2 + 6x) - 3$$
$$y = 4((x+3)^2 - 9) - 3$$
$$y = 4(x+3)^2 - 36 - 3$$
$$y = 4(x+3)^2 - 39$$
$$\frac{1(y+39)}{4} = \frac{4(x+3)^2}{4}$$
$$\frac{1}{4}(y+39) = (x+3)^2$$

$$4d = \frac{1}{4}$$
$$d = \frac{1}{4 \times 4}$$
$$\boxed{d = \frac{1}{16}}$$

so +

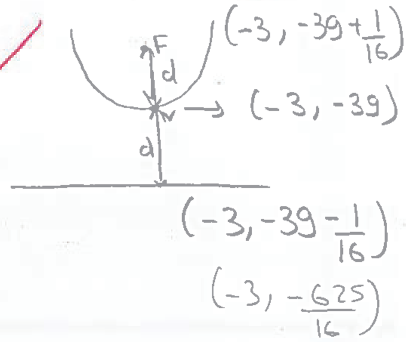
b) **(2 points)** Find the equation of the directrix line.

$$y = -\frac{625}{16}$$

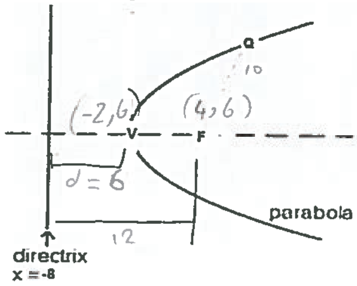
c)**(2 points)** Find the focus, say F

$$F = (-3, -39 + \tfrac{1}{16}) = (-3, -\tfrac{623}{16})$$

d)**(2 points)** Roughly, sketch the graph of such parabola.



QUESTION 3. **(4 points)** Stare at the following graph.



Given $F = (4, 6)$, the directrix line, L is $x = -8$, and $|QF| = 10$.

(i) Find $|QL| = |QF| = 10$

(ii) Find $v = (-2, 6)$

(iii) Find the equation of the parabola

$$24(x+2) = (y-6)^2$$

alleem AlAli

Quiz I: MTH 111, Spring 2018

Ayman Badawi

$\frac{(y-y_0)^2}{10} + (x-x_0)^2$

15/15

5

QUESTION 2. Consider the parabola $y = 3x^2 + 18x + 5$

(i) Sketch, roughly.

Standard form

$y = 3[x^2 + 6x] + 5$

$y = 3[(x+3)^2 - 9] + 5$

$y = 3(x+3)^2 - 27 + 5$

$y = 3(x+3)^2 - 22$

$y + 22 = 3(x+3)^2$

$\frac{1}{3}(y+22) = (x+3)^2$

vertex $= (-3, -22)$

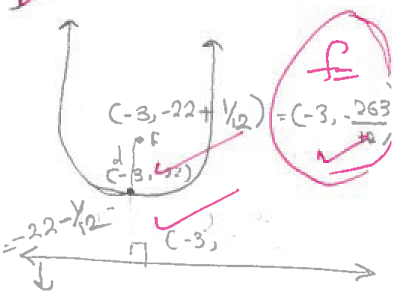
$4d = \frac{1}{3}$ $d = \frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$

5

(ii) Find the focus.

write answers here!

f: $(-3, -22 + 1/12) = (-3, -\frac{263}{12})$



(iii) Find the directrix line.

$y = -\frac{265}{12}$

QUESTION 3. Consider the parabola $-12(x+2) = (y-4)^2$

vertex $= (-2, 4)$

(i) Sketch, roughly.

$4d = -12$

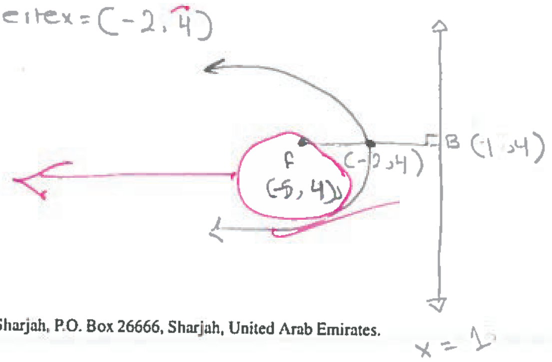
$\boxed{d = -3}$

(ii) Find the focus.

5

$(-5, 4)$

(iii) Find the directrix line. $x = 1$



(1, 4)

(-5, 4)

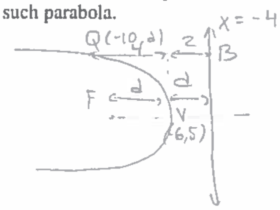
Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: abadawi@aus.edu, www.ayman-badawi.com

[redacted]

QUESTION 3. Given $x = -4$ is the directrix of of a parabola that has the point $(-6, 5)$ as its vertex point.

a) **(2 points)** Roughly, sketch such parabola.



$|d| = 2$

b) **(4 points)** Find the equation of the parabola

$4d(x - x_0) = (y - y_0)^2$

$-4(2)(x+6) = (y-5)^2$

$\boxed{-8(x+6) = (y-5)^2}$ ✓

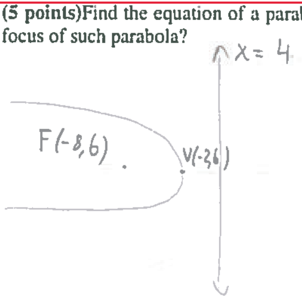
c) **(2 points)** Find the focus of the parabola, say F.

$\boxed{F(-8, 5)}$ ✓

d) **(2 points)** Given $Q = (-10, b)$ is a point on the curve of the parabola. Find $|QF|$ (HINT: You should know how to do this QUICKLY!, you do not need the value of b)

$|QL| = |QB| = |QF| = \boxed{6}$ ✓

(vii) **(5 points)** Find the equation of a parabola that has x = 4 as its directrix line and (-2, 6) as its vertex. What is the focus of such parabola?



$d = |-2-4| = 6$

$-4d\,(x-x_0) = (y-y_0)^2$

$\boxed{-24(x+2) = (y-6)^2}$

$\boxed{F(-8,6)}$

Inaam Shamma

Quiz I: Math. for the Architects, MTH 111, Spring 2017

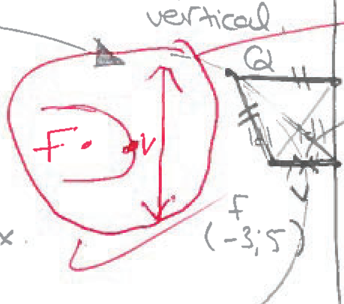
QUESTION 2. Given $(-3, 5)$ is the focus of a parabola with directrix line $x = 9$.

$x = 9$ vertical

(i) Sketch (rough sketch)

3

graph sketched on the back



(ii) Find the equation of the Parabola.

eq: $4d\,(x - x_1) = (y - y_1)^2$ vertex

3

midpt of $|FB|$ is the vertix.

$x_V = \frac{x_F + x_B}{2} = \frac{-3+9}{2} = 3.$

$|FV| = |VB| = |d| = |\Delta x| = |-3-3| = |-6| = 6.$

since on the left side $d < 0$. $\boxed{d = -6}$

$4(-6)(x-3) = (y-5)^2$

$\boxed{-24\,(x-3) = (y-5)^2}$

~~(iii) If Q is a point on the curve of the parabola. What is the distance between Q and the directrix?~~

2

$|QF| = |QL|$ QL we draw $\perp$ to L. intersect at point E $E(9; ?)$

Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: abadawi@aus.edu, www.ayman-badawi.com

iii) find the distance between vertix and directrix.

$|VB| = \sqrt{\Delta x^2} = |\Delta x| = |9 - 3| = 6.$

Haya Alshamsi

Exam I: MTH 111, Fall 2017

Ayman Badawi

Excellent

Points = $\frac{70}{70}$

QUESTION 1. (6 points) Given $y = 11$ is the directrix of of a parabola that has the point $(6, 5)$ as its vertex point.

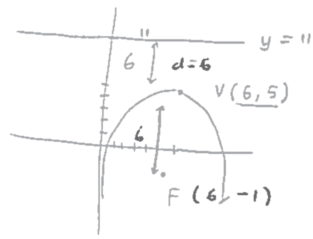
a) Find the equation of the parabola

$\boxed{d = 6}$

$$-4d\,(y - y_1) = (x - x_1)^2$$

$$-4(6)(y-5) = (x-6)^2$$

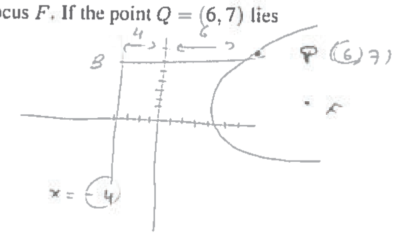
$$-24(y-5) = (x-6)^2$$



b) Find the focus of the parabola.

F $(6, -1)$

QUESTION 2. (3 points) Given that $x = -4$ is the directrix of a parabola that has focus F. If the point $Q = (6, 7)$ lies on the curve of the parabola, find $|QF|$ (i.e., find the distance between F and Q).



$|QL| = |QF|$

$|QB| = |QF|$

$|QF| = 10$ units

Exam I MTH 111, Fall 2016

Ayman Badawi

$\frac{93}{95}$ Excellent

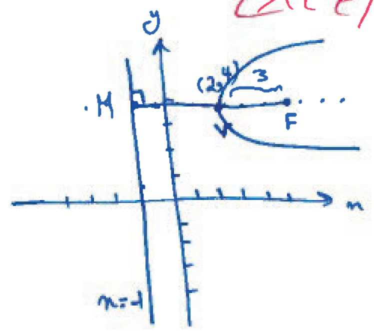
QUESTION 1. Given $12(x-2) = (y-4)^2$.

(i) Roughly, Sketch the graph of the given parabola.

$V = (2,4)$

$4d = 12 \rightarrow d = 3$

$M = (-1,4)$



(ii) What is the directrix line?

directrix: $\boxed{x = -1}$

(iii) What is the focus?

$\boxed{F = (5,4)}$

QUESTION 2. Given $y = x^2 - 6x + 4$

(i) Roughly, Sketch the graph of the given parabola.

$y - 4 = (x-3)^2 - 9 \rightarrow (y+5) = (x-3)^2$

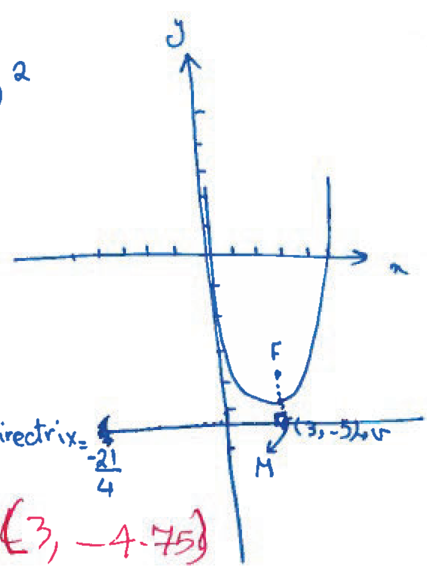
$\boxed{V = (3,-5)}$

$\boxed{4d = 1 \rightarrow d = \frac{1}{4}}$

(ii) What is the directrix line?

$M = (3, -5-\frac{1}{4})$

directrix: $y = -5 - \frac{1}{4} = \boxed{-5.25}$



∴ directrix $= -\frac{21}{4}$

(iii) What is the focus?

$\longrightarrow (3, -5+\frac{1}{4}) = (3, -4.75)$

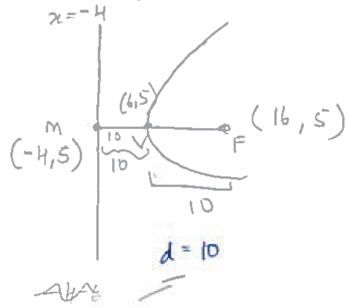
QUESTION 8. (6 points) Given $x = -4$ is the directrix of of a parabola that has the point $(6, 5)$ as its vertex point.

a) Find the equation of the parabola

$4(10)(x-6) = (y-5)^2$

$= 40(x-6) = (y-5)^2$

6/6



b) Find the focus of the parabola.

$F(16, 5)$

QUESTION 9. (6 points) Consider the parabola $x = -0.25(y + 3)^2 + 4$ [hint: first write it in the standard form].

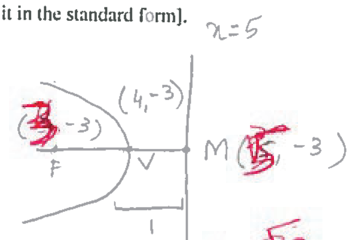
$x = -0.25(y+3)^2 + 4$

$(x-4) = -0.25(y+3)^2$

$-4(x-4) = (y+3)^2$

$4d = -4$

$d = -1$



a) Find the focus.

~~F(5,-3)~~ $F = (3, -3)$

b) Find the equation of the directrix

$x = 5$

6/6

$x = 5$

c) Draw the parabola

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: abadawi@aus.edu, www.ayman-badawi.com

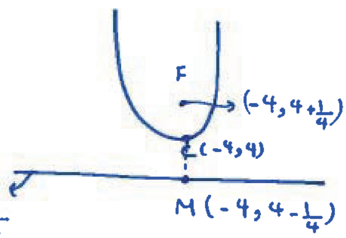
QUESTION 7. (8 points). Given $y = x^2 + 8x + 20$

(i) Roughly, Sketch the graph of the given parabola.

$y = (x+4)^2 - 16 + 20 \Rightarrow y = (x+4)^2 + 4$

$(y-4) = (x+4)^2$

$4d(y - y_0) = (x - x_0)^2$



$y = 4 - \frac{1}{4}$

(ii) What is the directrix line?

$4d = 1 \Rightarrow \boxed{d = \frac{1}{4}}$

$\boxed{C = (-4, 4)}$ $\rightsquigarrow$ directrix line: $\boxed{y = 4 - \frac{1}{4} = \frac{15}{4}}$

(iii) What is the focus?

$\boxed{F = (-4, 4+\frac{1}{4})}$

3.3 Questions with Solutions on hyperbola from previous semesters

Name , ID

Krstin Raed

15/15 :)

Quiz I MTH 111, Spring 2019

Ayman Badawi

$CF^2 = \left(\frac{k}{2}\right)^2 - b^2$

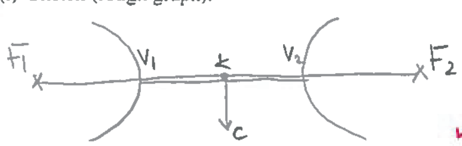
$CF^2 = 25 - 9$

QUESTION 1. Consider the elli... $\frac{(x+2)^2}{} + \frac{(y-1)^2}{} = 1$

$(2, 1)$

QUESTION 2. Consider the hyperbola $\frac{(x-3)^2}{9} - \frac{(y+2)^2}{16} = 1$.

2 (i) Sketch (rough graph).



$c = (3, -2)$

1 (ii) Find the heperbola-constant, k

$\left(\frac{k}{2}\right)^2 = 9$ $\quad \frac{k}{2} = \sqrt{9}$ $\quad \boxed{k = 3 \times 2 = 6}$

2 (iii) Find all vertices

$V_1 \ (3-3, -2)$
$(0, -2)$

$V_2 \ (3+3, -2)$
$(6, -2)$

2 (iv) Find the Foci

$CF^2 = \left(\frac{k}{2}\right)^2 + b^2$

$CF^2 = 9 + 16$

$CF^2 = 25$

$\boxed{CF = 5}$

$F_1 \ (3-5, -2)$
$(-2, -2)$

$F_2 \ (3+5, -2)$
$(8, -2)$

Faculty information

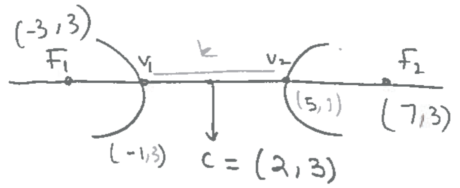
Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: abadawi@aus.edu, www.ayman-badawi.com

QUESTION 6. Consider the hyperbola $\frac{(x-2)^2}{9} - \frac{(y-3)^2}{16} = 1$.

$\left(\frac{k}{2}\right)^2$ b^2

a) **(2 points)** Draw the hyperbola, roughly.

under x so right left



b) **(2 points)** Find the hyperbola-constant K.

$\left(\frac{k}{2}\right)^2 = 9$

$\frac{k}{2} = \sqrt{9}$

$k = 3 \times 2$

$\boxed{k=6}$

c) **(3 points)** Find the two vertices of the hyperbola.

$V_2 = (2+3, 3)$
$(5,3)$

$V_1 = (2-3, 3)$
$(-1,3)$

d) **(3 points)** Find the foci of the hyperbola.

$F_1 = (2-5, 3)$ $(-3,3)$

$F_2 = (2+5, 3)$ $(7,3)$

$CF^2 = \left(\frac{k}{2}\right)^2 + b^2$

$CF^2 = 9 + 16$

$= 25$

$\boxed{CF = 5}$

QUESTION 7. (4 points) Given two lines $L_1 : x = t+1, y = 2t+4, z = -5t+3$ $(t \in R)$ and $L_2 : x = 2w ...$

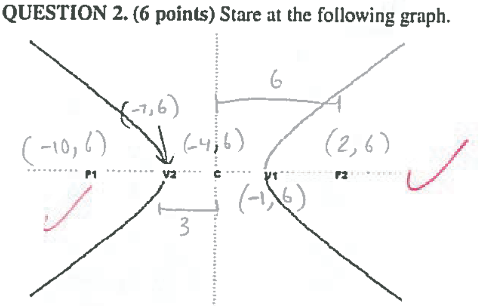
Name Haya Sujaa, ID g00082558

Final Exam, MTH 111, Spring 2019

Ayman Badawi

$$Score = \frac{75}{78}$$

QUESTION 2. (6 points) Stare at the following graph.



Given $c = (-4, 6)$, $|cv2| = 3$, and $F2 = (2, 6)$.

(i) Find $v1 = (-1, 6)$ $F1 = (-10, 6)$, $v2 = (-7, 6)$, and the hyperbola-constant $k = 6$

$|CF_1| = \sqrt{(\frac{k}{2})^2 + b^2} = 6$

$\sqrt{9 + b^2} = 6$

$9 + b^2 = 36$

$b^2 = 36 - 9$

$b^2 = 27$

(ii) Find the equation of the hyperbola

$$\frac{(x+4)^2}{9} - \frac{(y-6)^2}{27} = 1$$

Quiz II: MTH 111, Spring 2018

Ayman Badawi

$$\frac{(y-y_0)^2}{(\frac{k}{2})^2} - \frac{(x-x_0)^2}{b^2} = 1$$

$C(-1,2)$

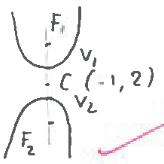
$(\frac{k}{2})^2 = 9 \Rightarrow \frac{k}{2} = 3 \Rightarrow k = 6$

$|CF_1| = \sqrt{16+9} = 5$

15/15

QUESTION 1. Consider the hyperbola given by $\frac{(y-2)^2}{9} - \frac{(x+1)^2}{16} = 1$

(i) Sketch, roughly.



(ii) Find the ellipse-constant K.

$\boxed{k=6}$

(iii) Find the foci.

$F_1(-1, 2+5) \Rightarrow \boxed{F_1(-1,7)}$ $F_2(-1, 2-5) \Rightarrow \boxed{F_2(-1,-3)}$

(iv) Find all vertices.

$V_1(-1, 2+3) \Rightarrow \boxed{V_1(-1,5)}$

$V_2(-1, 2-3) \Rightarrow \boxed{V_2(-1,-1)}$

QUESTION 2. Given a ~~parabola~~ hyperbola centered at $(-2,3)$ such that one of the vertices is $(0,3)$ and one of the foci is $(-6,3)$.

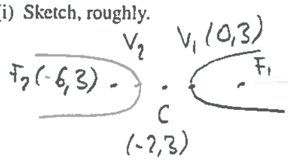
$$\frac{(x-x_0)^2}{(\frac{k}{2})^2} - \frac{(y-y_0)^2}{b^2} = 1$$

$|CF_1| = \sqrt{(\frac{k}{2})^2 + b^2}$ $|CF_1| = 4$

$b^2 = |CF_1|^2 - (\frac{k}{2})^2$

$b^2 = 4^2 - 4 \Rightarrow b^2 = 12$

(i) Sketch, roughly.



(ii) Find the constant K.

$\frac{k}{2} = |C_1V_2| = 2 \Rightarrow \boxed{k=4}$

(iii) Find the second focus and the second vertex.

$\boxed{V_2(-4,3)}$

$\boxed{F_1(2,3)}$

(iv) Write down the equation of the hyperbola.

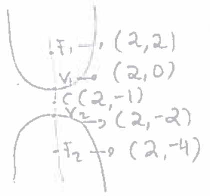
$$\frac{(x+2)^2}{4} - \frac{(y-3)^2}{12} = 1$$

Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: abadawi@aus.edu, www.ayman-badawi.com

QUESTION 6. Consider the hyperbola $(y+1)^2 - \frac{(x-2)^2}{8} = 1$.

a) **(2 points)** Draw the hyperbola, roughly



$|CF_1| = \sqrt{1+8} = 3$

b) **(2 points)** Find the hyperbola-constant K.

$\left(\frac{K}{2}\right)^2 = 1$

$\frac{K}{2} = 1 \Rightarrow \boxed{K = 2}$ ✓

c) **(3 points)** Find the two vertices of the hyperbola.

$\boxed{V_1\ (2,0)}$

$\boxed{V_2\ (2,-2)}$ ✓

d) **(3 points)** Find the foci of the hyperbola.

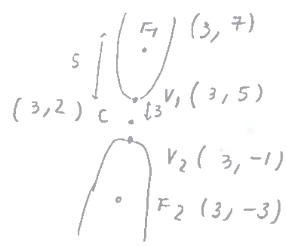
$\boxed{F_1\ (2,2)}$

$\boxed{F_2\ (2,-4)}$ ✓

QUESTION 4. (8 points)

Draw roughly the hyperbola $\dfrac{(y-2)^2}{9} - \dfrac{(x-3)^2}{16} = 1$. Then find

positive $y \Rightarrow$



a) The hyperbola-constant K.

$\left(\frac{k}{2}\right)^2 = 9 \rightarrow \frac{k}{2} = 3$

$\boxed{k = 6}$

b) The two vertices of the hyperbola.

V_1 (3, 5)

V_2 (3, −1)

c) The foci of the hyperbola.

$|CF_1| = \sqrt{9 + 16} = 5$

F_1 (3, 7)

F_2 (3, −3)

$4 \times 3 = 12$ $\qquad 2\sqrt{3}$

QUESTION 7. (8 points) First draw the hyperbola $\frac{y^2}{4} - \frac{(x-1)^2}{12} = 1$. Then find

a) The hyperbola-constant K.

$(\frac{K}{2})^2 = 4 \qquad \frac{K}{2} = 2 \qquad \underline{\underline{K = 4}}$

b) The two vertices of the hyperbola.

$V_1(1, 2)$ //

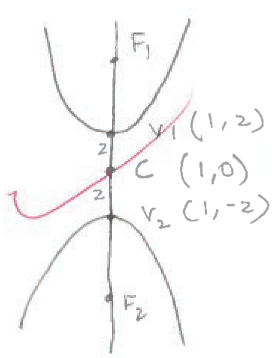
$V_2(1, -2)$ //

$8/8$

$b^2 = 12$

$b = \sqrt{12}$

$= 2\sqrt{3}$



c) The foci of the hyperbola.

$F_1C = \sqrt{b^2 + (4/2)^2} = \sqrt{\cancel{16+144}} = \cancel{12 \cdot 64}\ \sqrt{12+4} = \sqrt{16}$

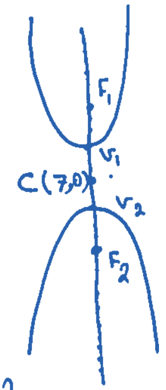
$= 4$

$F_1(1, 4)$ //

$F_2(1, -4)$ //

QUESTION 3. Given the hyperbola $\frac{y^2}{4} - \frac{(x-7)^2}{5} = 1$

(i) Roughly, Sketch the graph of the given hyperbola.



(ii) Find the two vertices, V_1 and V_2

$\left(\frac{K}{2}\right)^2 = 4 \to \frac{K}{2} = 2 \to \boxed{K=4} \to |V_1V_2| \to |CV_1| = |CV_2| = 2$

$\boxed{V_1 = (7,2)}$ / $\boxed{V_2 = (7,-2)}$

(iii) Find the two Foci: F_1, F_2

$|CF_1| = |CF_2| = \sqrt{\left(\frac{K}{2}\right)^2 + b^2} = \sqrt{5+4} = \boxed{\sqrt{9} = 3}$

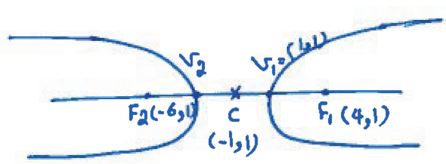
$\boxed{F_1 = (7,3)}$ / $\boxed{F_2 = (7,-3)}$

QUESTION 4. Given $F_1 = (4,1)$, $F_2 = (-6,1)$ are the foci of a hyperbola and $V_1 = (1,1)$ is one of the vertices.

(i) Find the hyperbola-constant K.

$C = \left(\frac{-6+4}{2}, 1\right) = (-1,1)$

$\frac{K}{2} = 2 \to \boxed{K=4}$



(ii) Find the second vertex of the hyperbola.

$|CV_2| = K/2 \to \boxed{V_2 = (-3,1)}$

(iii) Find the equation of the hyperbola.

$|CF_1| = |CF_2| = 5 = \sqrt{\left(\frac{K}{2}\right)^2 + b^2} \to 5 = \sqrt{4+b^2} \to 25 = 4 + b^2 \to \boxed{b^2 = 21}$

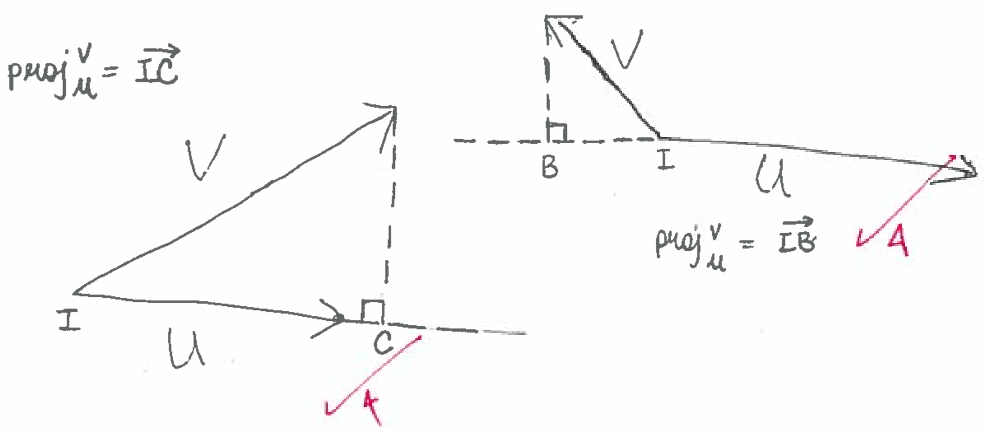
equation: $\boxed{\frac{(x+1)^2}{4} - \frac{(y-1)^2}{21} = 1}$

3.4 Questions with Solutions on Vector-Projections-Lines-in-3D from previous semesters

Name KAMYA KANSRA, ID 81881

Draw the projection V over U

15/15 ☺



Let $U = \langle 2, 2 \rangle$, $V = \langle -3, 4 \rangle$

$u \cdot v = -6 + 8 = 2$; $|u| = \sqrt{(2)^2 + (2)^2} = \sqrt{8} = 2\sqrt{2}$

Find $\text{proj}_u V = \left(\frac{u \cdot v}{|u|^2}\right) u = \frac{\cancel{2}}{\cancel{8}\,4} \langle 2, 2 \rangle = \frac{1}{4} \langle 2, 2 \rangle = \langle \frac{1}{2}, \frac{1}{2} \rangle$

4

Find $\left| \text{proj}_u^v \right| = \frac{|u \cdot v|}{|u|} = \frac{2}{\sqrt{8}} = \frac{\cancel{2}}{\cancel{2}\sqrt{2}} = \frac{1}{\sqrt{2}}$

3

Name Krstin Raed , ID 78656

MTH 111 Math for Architects Spring 2019, 1–6

Exam I: MTH 111, Spring 2019

$F = V \times W$

Ayman Badawi

$$\text{Points} = \frac{87}{87}$$

QUESTION 1. b) **(4 points)** Given $A = (6, 10)$, $B = (-7, 3)$, and $C = (-4, -2)$ are the vertices of a triangle. Find the area of the triangle ABC.

Area of the triangle ABC $= \frac{1}{2} |AB \times AC|$

$AB = \langle -13, -7 \rangle$ (B-A)

$AC = \langle -10, -12 \rangle$ (C-A)

$$AB \times AC = \begin{vmatrix} i & j & k \\ -13 & -7 & 0 \\ -10 & -12 & 0 \end{vmatrix} = 0i - 0j + \underline{86k} = \underline{86}$$

Area of $\Delta ABC = \frac{1}{2} 86 = \boxed{43 \text{ units}^2}$

c) **(3 points)** Find a vector F that is perpendicular to both vectors $V =< 2, 6, -3 >$ and $W =< 5, -4, 1 >$ such that $|F| = 111$.

$$F = V \times W = \begin{vmatrix} i & j & k \\ 2 & 6 & -3 \\ 5 & -4 & 1 \end{vmatrix} = -6i - 17j - 38k$$

$|F| = 111 \quad = \frac{111}{|F|} F$

$$= \boxed{\frac{111}{42} \langle -6, -17, -38 \rangle}$$

QUESTION 2. a) **(4 points)** The line $L_1 : x = -2t - 3, y = -3t + 3, z = 4t - 2$ $(t \in R)$ intersects the line $L_2 : x = 2w - 13, y = 4w - 15, z = 4w - 6$ $(w \in R)$ in a point Q. Find Q.

$L_1 : x = -2t - 3$
$y = -3t + 3$
$z = 4t - 2$

$L_2 : x = 2w - 13$
$y = 4w - 15$
$z = 4w - 6$

use substitution method

find pt of intersection: $-2t - 3 = 2w - 13$

$\frac{-2t}{-2} = \frac{2w - 13 + 3}{-2}$

$\boxed{t = -w + 5}$

Sub in second equ

$-3(-w+5)+3 = 4w-15$

$3w - 15 + 3 = 4w - 15$

$4w - 3w = -15 + 15 + 3$

$\boxed{1w = 3}$

$t = -3 + 5$

$\boxed{t = 2}$

• now sub in each line to get intersection pt

$-2(2) - 3 = 2(3) - 13$

$\boxed{-7 = -7}$

$-3(2) + 3 = 4(3) - 15$

$\boxed{-3 = -3}$

$4(2) - 2 = 4(3) - 6$

$\boxed{6 = 6}$

$\boxed{\text{Intersection pt} = Q = (-7, -3, 6)}$

b) **(2 points)** Are the lines in (a) perpendicular? Explain

$D_1 = \langle -2, -3, 4 \rangle$

$D_2 = \langle 2, 4, 4 \rangle$

$D_1 \cdot D_2 = (-2 \times 2) + (-3 \times 4) + (4 \times 4)$
$= 0$

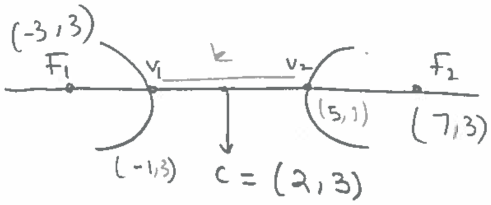
So they are perpendicular because their dot product is zero & they intersect

QUESTION 6. Consider the hyperbola $\frac{(x-2)^2}{9} - \frac{(y-3)^2}{16} = 1$.

$9 = \left(\frac{k}{2}\right)^2$, $16 = b^2$

a) **(2 points)** Draw the hyperbola, roughly.

under x so right left



b) **(2 points)** Find the hyperbola-constant K.

$$\left(\frac{k}{2}\right)^2 = 9$$

$$\frac{k}{2} = \sqrt{9}$$

$$k = 3 \times 2$$

$$\boxed{k = 6}$$

c) **(3 points)** Find the two vertices of the hyperbola.

$V_2 = (2+3, 3)$
$(5,3)$

$V_1 = (2-3, 3)$
$(-1,3)$

d) **(3 points)** Find the foci of the hyperbola.

$F_1 = (2-5, 3)$ $(-3,3)$

$F_2 = (2+5, 3)$ $(7,3)$

$CF^2 = \left(\frac{k}{2}\right)^2 + b^2$

$CF^2 = 9 + 16$
$= 25$

$\boxed{CF = 5}$

QUESTION 7. (4 points) Given two lines $L_1 : x = t+1, y = 2t+4, z = -5t+3$ $(t \in R)$ and $L_2 : x = 2w-1, y = 4w+1, z = -10w+13$ $(w \in R)$. Is L_1 parallel to L_2? Explain (show the work)

• 2 lines are // if they have cst & they do not intersect

L_1 : $x = t+1$
$y = 2t+4$
$z = -5t+3$

L_2 : $x = 2w-1$
$y = 4w+1$
$z = -10w+13$

$D_1 \langle 1, 2, -5 \rangle$ $D_2 \langle 2, 4, -10 \rangle$

$1 = c2$ $c = \frac{1}{2}$
$2 = c4$ $c = \frac{1}{2}$
$-5 = c(-10)$ $c = \frac{1}{2}$

they have a cst

take $t = 0$

$1 = 2w - 1$
$4 = 4w + 1$
$3 = -10w + 13$

$2w = 2$, $w = 1$
$4w = 4-1$, $w = \frac{3}{4}$
$10w = 13-3$, $10w = 10$, $w = 1$

$2w = 2$
$\boxed{w = 1}$
$4 - 1 = 4w$
$3 = 4w$
$\boxed{w = \frac{3}{4}}$
$3 - 13 = -10w$
$-10 = -10w$

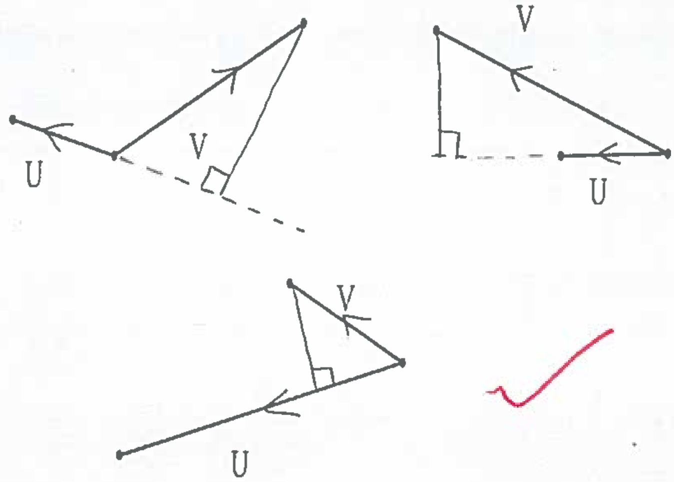
they do not intersect

$L_1 // L_2$

QUESTION 8. **(6 points)**

proj_U^V

Stare at the below. Then find Projection of V over U



QUESTION 9. **(4 points)** Find the equation of the plane that contains the points $Q_1 = (4,4,0)$. $Q_2 = (0,2,6)$ and $Q_3 = (4,0,8)$.

$N = \overrightarrow{Q_1Q_2} \times \overrightarrow{Q_1Q_3}$

$\langle -4,-2,6\rangle \times \langle 0,-4,8\rangle$

$$\begin{vmatrix} i & j & k \\ -4 & -2 & 6 \\ 0 & -4 & 8 \end{vmatrix} = 8i + 32j + 16k$$

$\langle 8, 32, 16\rangle$

choose a pt

$Q_1 = (4,4,0)$

$8(x-4) + 32(y-4) + 16(z-0) = 0$

$\boxed{8(x-4) + 32(y-4) + 16z = 0}$

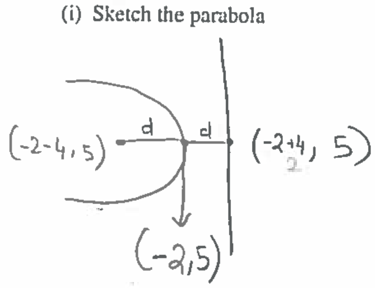
QUESTION 10. **(6 points)** Consider the parabola $-16(x+2) = (y-5)^2$.

(i) Sketch the parabola

$4d = -16$

$d = -4$

& before x so its left



(ii) Find the equation of the directrix line

$X = -2+4$

$\boxed{X = 2}$

(iii) Find the focus point.

Focus $= (-2-4, 5)$

$(-6, 5)$

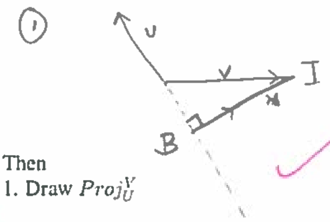
Quiz III: MTH 111, Spring 2018

Ayman Badawi

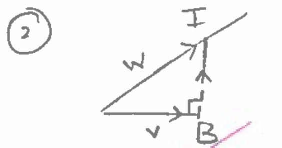
Archa Alukka
75223

$\frac{14.5}{15}$

QUESTION 1. Stare at the following vectors.



Projections are IB



Then
1. Draw $Proj_U^V$

3

2. Draw $Proj_V^W$

QUESTION 2. Given $(1,2,4)$ and $(7,-4,3)$ lie on a line L.

a) Find a parametric equations of L.

$D = (7-1,\ -4-2,\ 3-4) = (6,-6,-1)$

$(1,2,4)$ and $(6,-6,-1)$

$(1+6L,\ 2-6L,\ 4-L)$

$X = 1+6L \quad Y = 2-6L \quad Z = 4-L$

$6L = 2-Y$

b) Find a symmetric equations of L.

3

$L: \dfrac{X-1}{6} = \dfrac{2-Y}{6} = \dfrac{4-Z}{1}$

c) Does the point $(1,4,8)$ lie on the line L.

- $\dfrac{X-1}{6} = \dfrac{1-1}{6} = 0$
- $\dfrac{4-8}{6} = \dfrac{-2}{6} = -\dfrac{1}{3}$
- $4-8 = -4$

1

∴ It doesn't lie on the line L because the values varies when substituted.

QUESTION 3. Let $V = <1,1,2>$ and $W = <-2,2,-1>$. Find $Proj_V^W$. Will it be in the direction of V?

3.5

$Proj_V W = \dfrac{U \cdot W}{|V|^2} \times V \qquad |V| = \sqrt{1^2+1^2+2^2} = \sqrt{1+1+4} = \sqrt{6}$

$|V|^2 = 6$

$U \cdot W = 1(-2) + 1(2) + 2(-1)$
$= -2 + 2 - 2 = -2$

$Proj_V W = \dfrac{-2}{6} \times (1,1,2) = \left(\dfrac{-2}{6}, \dfrac{-2}{6}, \dfrac{-4}{6}\right)$

Yes, it will be in the direction of V

No

opposite

Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: abadawi@aus.edu, www.ayman-badawi.com

QUESTION 2. a) **(4 points)** Does the line $L_1 : x = 5t - 20, y = -t + 3, z = 3t - 27$ ($t \in R$) intersect the line $L_2 : x = -2w + 20, y = -4w - 5, z = 2w - 3$ ($w \in R$)? If yes find the intersection point Q.

$$L_1 : \begin{cases} x = 5t - 20 \\ y = -t + 3 \\ z = 3t - 27 \end{cases} \qquad L_2 : \begin{cases} x = -2w + 20 \\ y = -4w - 5 \\ z = 2w - 3 \end{cases}$$

$$5t - 20 = -2w + 20 \Rightarrow 5t + 2w = 40$$
$$-t + 3 = -4w - 5 \Rightarrow -t + 4w = -8$$
$$t = 8 \quad w = 0$$

check for z:

$$\left.\begin{array}{l} z = 3t - 27 = 3(8) - 27 = -3 \\ z = 2w - 3 = 2(0) - 3 = -3 \end{array}\right\} \text{they are equal} \Rightarrow L_1 \text{ and } L_2 \text{ intersect}$$

The point of intersection

$$x = 2w + 20 = 2(0) + 20 = 20$$
$$y = -4w - 5 = -4(0) - 5 = -5$$
$$z = 2w - 3 = 2(0) - 3 = -3$$

point of intersection is (20, -5, -3)

b) **(2 points)** Are the lines in (a) perpendicular? Explain Yes.

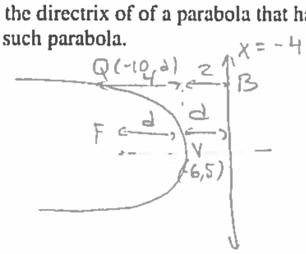
$D_1 = <5, -1, 3> \quad D_2 = <-2, -4, 2>$

$D_1 \cdot D_2 = 5(-2) - 1(-4) + 3(2) = 0$

dot product = 0 $\Rightarrow$ They are perpendicular.

QUESTION 3. Given $x = -4$ is the directrix of of a parabola that has the point $(-6, 5)$ as its vertex point.

a) **(2 points)** Roughly, sketch such parabola.



$|d| = 2$

b) **(4 points)** Find the equation of the parabola

$$4d(x - x_0) = (y - y_0)^2$$
$$-4(2)(x + 6) = (y - 5)^2$$
$$\boxed{-8(x + 6) = (y - 5)^2}$$

c) **(2 points)** Find the focus of the parabola, say F.

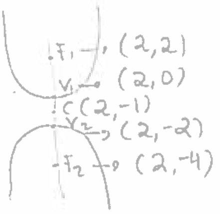
$\boxed{F(-8, 5)}$

d) **(2 points)** Given $Q = (-10, b)$ is a point on the curve of the parabola. Find $|QF|$ (HINT: You should know how to do this QUICKLY!, you do not need the value of b)

$|QL| = |QB| = |QF| = \boxed{6}$

QUESTION 6. Consider the hyperbola $(y+1)^2 - \frac{(x-2)^2}{8} = 1$.

a) **(2 points)** Draw the hyperbola, roughly



$|CF_1| = \sqrt{1+8} = 3$

b) **(2 points)** Find the hyperbola-constant K.

$(\frac{K}{2})^2 = 1$

$\frac{K}{2} = 1 \Rightarrow \boxed{K = 2}$ ✓

c) **(3 points)** Find the two vertices of the hyperbola.

$\boxed{V_1\ (2,0)}$

$\boxed{V_2\ (2,-2)}$ ✓

d) **(3 points)** Find the foci of the hyperbola.

$\boxed{F_1\ (2,2)}$

$\boxed{F_2\ (2,-4)}$ ✓

QUESTION 7. Given two lines $L_1 : x = t+1, y = 2t+4, z = -5t+3$ and $L_2 : x = 2w+7, y = 4w+16, z = -10w-27$.

(i) **(3 points)** Find the symmetric equation of L_1.

$\boxed{x-1 = \frac{y-4}{2} = \frac{-z+3}{5}}$ ✓

(ii) **(3 points)** Is D_1 parallel to D_2? (note that D_1 is the directional vector of L_1 and D_2 is the directional vector of L_2) Show the work

$D_1 < 1,2,-5 >$

$D_2 < 2,4,-10 >$

$D_1 = c D_2$

$<1,2,-5> = c <2,4,-10>$

$c = \frac{1}{2}$ ✓

$D_1 = \frac{1}{2} D_2 \Rightarrow$ They are parallel

(iii) **(2 points)** Is L_1 parallel to L_2? Explain (show the work)

Take $t=0 \rightarrow (1,4,3)$

check if $(1,4,3) \in L_2$

$1 = 2w+7 \Rightarrow w = -3$

$4 = 4w+16 \Rightarrow w = -3$

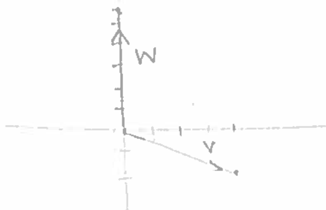
$3 = -10w-27 \Rightarrow w = -3$

$\Rightarrow$ it $\in$ to L_2. ✓

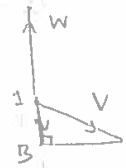
$\Rightarrow$ L_1 and L_2 intersect and they are NOT parallel. They are collinear (same line/ on top of each other)

QUESTION 8. Let (0, 0) be the initial point of the two vectors $V = < 4, -2 >$, and $w = < 0, 6 >$.

a) **(2 points)** Draw V and W in the xy-plane.



b)



$\text{proj}_W^V = \overrightarrow{1B}$

c)

$\text{proj}_V^W = \overrightarrow{1B}$

b) **(2 points)** Use the picture that you draw in (a) in order to draw $Proj_W^V$

c)**(2 points)** Use the picture that you draw in (a) in order to draw $Proj_v^w$

d) **(4 points)** Find $Proj_w^v$ and find its length.

$$\text{proj}_W^V = \frac{V \cdot W}{|W|^2} \cdot W = \frac{-12}{36} \cdot W = -\frac{1}{3} < 0, 6 > = \boxed{< 0, -2 >}$$

$$\boxed{|\text{proj}_W^V| = \sqrt{2^2} = 2.}$$

e)**(3 points)** Find the angle between V and W

$$\cos\theta = \frac{V \cdot W}{|V||W|} = \frac{-12}{(6)(2\sqrt{5})} = -\frac{\sqrt{5}}{5}$$

$$\theta = \cos^{-1}\left(-\frac{\sqrt{5}}{5}\right) = \boxed{116.565°}$$

Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: abadawi@aus.edu, www.ayman-badawi.com

Name [redacted], ID [redacted]
00073656

Quiz Four: MTH 111, Fall 2017

Ayman Badawi

15/15

QUESTION 1. Find a parametric equations of the line that passes through (1, 6, 9) and (0, 4, −1)

① $L_D : \langle 0-1, 4-6, -1-9 \rangle$

$= \langle -1, -2, -10 \rangle$

② $L : (1,6,9) + t\langle -1,-2,-10 \rangle$

$= (1+-t,\ 6+-2t,\ 9+-10t)$

$$\begin{bmatrix} x = 1-t \\ y = 6-2t \\ z = 9-10t \end{bmatrix} \quad t = \mathbb{R}$$

5

QUESTION 2. Find a parametric equations of the line that has directional vector $D = \langle 3, -4, 8 \rangle$ and it passes through (2, −6, 7)

$L : (2,-6,7) + t\langle 3,-4,8 \rangle$

$= (2+3t,\ -6+-4t,\ 7+8t)$

$$\begin{bmatrix} x = 2+3t \\ y = -6-4t \\ z = 7+8t \end{bmatrix} \quad t = \mathbb{R}$$

5

QUESTION 3. Does $L_1 : x = 2t+1, y = -4t+6, z = 3t+2$ ($t \in R$) intersect $L_2 : x = 4w+1, y = w-12, z = 4w+6$ ($w \in R$)? If yes, then find the intersection point.

L_1:
$$\begin{bmatrix} x = 2t+1 \\ y = -4t+6 \\ z = 3t+2 \end{bmatrix} \quad t = \mathbb{R}$$

L_2:
$$\begin{bmatrix} x = 4w+1 \\ y = w-12 \\ z = 4w+6 \end{bmatrix} \quad w = \mathbb{R}$$

① FIND T & W

$2t+1 = 4w+1$
$-4t+6 = w-12$

↓

$2t-4w = 1-1$
$-4t-w = -12-6$

↓

$2t-4w = 0$
$-4t-w = -18$

$$t = \frac{\begin{vmatrix} 0 & -4 \\ -18 & -1 \end{vmatrix}}{\begin{vmatrix} 2 & -4 \\ -4 & -1 \end{vmatrix}} \qquad w = \frac{\begin{vmatrix} 2 & 0 \\ -4 & -18 \end{vmatrix}}{\begin{vmatrix} 2 & -4 \\ -4 & -1 \end{vmatrix}}$$

$t = \dfrac{0-(72)}{-2-(16)}$ $\qquad w = \dfrac{-36-0}{-2-16}$

$t = \dfrac{-72}{-18}$ $\qquad w = \dfrac{-36}{-18}$

$t = 4$ $\qquad w = 2$

CHECK: z of $L_1 \stackrel{?}{=}$ z of L_2

$3t+2 \stackrel{?}{=} 4w+6$
$(3)(4)+2 \stackrel{?}{=} 4(2)+6$ → $14 = 14$

Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: abadawi@aus.edu, www.ayman-badawi.com

② L_1 INTERSECT L_2

YES!

③ FIND INTERSECTION POINT → (9, −10, 14)

$x = 2(4)+1 = 9$

$y = -4(4)+6 = -10$

$z = 3(4)+2 = 14$

5

QUESTION 10. (12 points)

a) Convince me that $q_1 = (0,4,2)$, $q_2 = (2,1,-1)$, and $q_3 = (2,3,5)$ are not co-linear

$\overrightarrow{Q_1Q_2} = \langle 2,-3,-3 \rangle$

$\overrightarrow{Q_1Q_3} = \langle 2,-1,3 \rangle$

$$\overrightarrow{Q_1Q_2} \times \overrightarrow{Q_1Q_3} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & -3 \\ 2 & -1 & 3 \end{vmatrix}$$

$= -12\hat{i} - 12\hat{j} + 4\hat{k}$

The cross-product is not a zero-vector $\Rightarrow$ The points are not collinear

b) Find the area of the triangle with vertices q_1, q_2, q_3. (q_1, q_2, q_3 as in (a))

$A\Delta = \frac{1}{2} | \overrightarrow{Q_1Q_2} \times \overrightarrow{Q_1Q_3} |$

$A\Delta = \frac{1}{2} \sqrt{144+144+16} = \frac{1}{2}(4\sqrt{19}) = 2\sqrt{19}$ units^2

c) Find a vector F that is perpendicular to both vectors $\overrightarrow{q_1q_2}$ and $\overrightarrow{q_1q_3}$. (q_1, q_2, q_3 as in (a))

$F = | \overrightarrow{Q_1Q_2} \times \overrightarrow{Q_1Q_3} | = -12\hat{i} - 12\hat{j} + 4\hat{k} = \langle -12, -12, 4 \rangle.$

d) Convince me that the line $L_1 : x = 2t+1, y = -t+3, z = 4t+1$ $(t \in R)$ is perpendicular to the line $L_2 : x = -2w+5, y = 4w-5, z = 2w-3$ $(w \in R)$.

$L_1 : \begin{cases} x = 2t+1 \\ y = -t+3 \\ z = 4t+1 \end{cases}$; $t \in \mathbb{R}$ $\quad D_1 : \langle 2,-1,4 \rangle$

$L_2 : \begin{cases} x = -2w+5 \\ y = 4w-5 \\ z = 2w-3 \end{cases}$; $w \in \mathbb{R}$ $\quad D_2 : \langle -2,4,2 \rangle$

$D_1 \cdot D_2 = 2(-2) - 4 + 8 = -4 - 4 + 8 = -8 + 8 = 0.$

$\rightarrow$ check if they intersect :

$2t + 1 = -2w + 5 \longrightarrow 2t + 2w = 4 \longrightarrow t + w = 2 \rightarrow \boxed{w = 2-t}$

$-t + 3 = 4w - 5 \longrightarrow -t - 4w = -8 \longrightarrow -t - 4w = -8$

$-t - 4(2-t) = -8$

$-t - 8 + 4t = -8$

$3t - 8 = -8$

$3t = 0$

$\boxed{t = 0}$

$\boxed{w = 2}$

$z = 4(0) + 1$, $\boxed{z = 1}$ $\qquad$ $z = 2(2) - 3$, $\boxed{z = 4 - 3 = 1}$

Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: abadawi@aus.edu, www.ayman-badawi.com

The two lines intersect at $(1,3,1)$ $\Rightarrow$ The two lines are perpendicular.

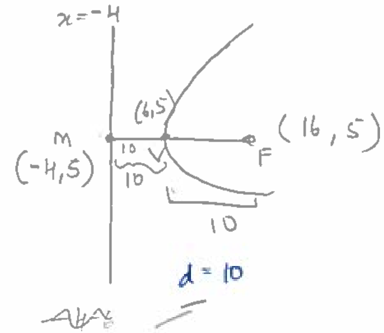
QUESTION 8. (6 points) Given $x=-4$ is the directrix of of a parabola that has the point $(6,5)$ as its vertex point.

a) Find the equation of the parabola

$4(10)(x-6)=(y-5)^2$

$=40(x-6)=(y-5)^2$

6/6



b) Find the focus of the parabola.

F (16, 5)

QUESTION 9. (6 points) Consider the parabola $x=-0.25(y+3)^2+4$ [hint: first write it in the standard form].

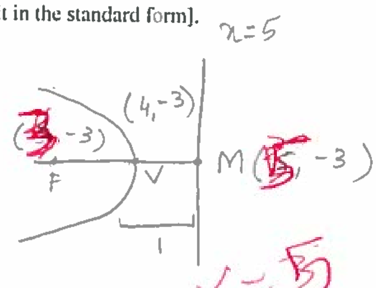
$x=-0.25(y+3)^2+4$

$(x-4)=-0.25(y+3)^2$

$-4(x-4)=(y+3)^2$

$4d=-4$

$d=-1$



a) Find the focus.

~~F (5, -3)~~ F = (3, -3) 4

b) Find the equation of the directrix

$x = 5$

6/6

c) Draw the parabola

x = 5

QUESTION 10. (6 points) Given two lines $L_1: x=t, y=1+t, z=3-2t$, $L_2: x=2+w, y=3-w, z=-1+2w$. If L_1 intersects L_2, find the intersection point.

L_1: $x=t$
$y=1+t$
$z=3-2t$

L_2: $x=2+w$
$y=3-w$
$z=-1+2w$

L_1: $x=2$
$y=3$
$z=-1$

L_2: $x=2$
$y=3$
$z=-1$

$t=2+w$

$t-w=2$ — (1) ×2

$1+t=3-w$

$t+w=3-1$

$t+w=2$ — (2) ~~×2~~

$3-2t=2w-1$

$2w+2t=4$ — (3)

~~2w+2t=4~~

~~2w~~

~~2w+2t=4~~

~~-2w+2t~~ =

$t-w=2$

$+\ t+w=2$

$2t=4$

$t=2$ $w=0$

∴ The point of intersection is (2, 3, -1)

QUESTION 11. Bonus: (4 points) Imagine this: You are staring at 4 tables; table one has 3 legs; table 2 has 4 legs; table 3 has 6 legs; table 4 has 8 legs. Which one of the tables is more stable? explain CLEARLY and briefly in order to get the full mark (NO PARTIAL CREDIT, i.e., 0 or 4)

The table 4 with 8 legs is more stable. Since there are 8 legs ~~each~~ the weight on the table will be equally distributed among more number of legs. ∴ ~~each leg~~ will have to support less weight as compared to table 1 when each leg will have to support more weight.

Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: abadawi@aus.edu, www.ayman-badawi.com

(iv) Find the fourth vertex of the ellipse.

$b = |CV_3| = 2 \rightsquigarrow \boxed{V_4 = (-1, -2)}$

(v) Find the two Foci: F_1, F_2 of the ellipse.

$|CF_1| = |CF_2| = \sqrt{(\frac{k}{2})^2 - b^2} = \sqrt{49 - 4} = \sqrt{45} \rightsquigarrow \boxed{F_1 = (1, -2+\sqrt{45})}$ / $\boxed{F_2 = (1, -2-\sqrt{45})}$

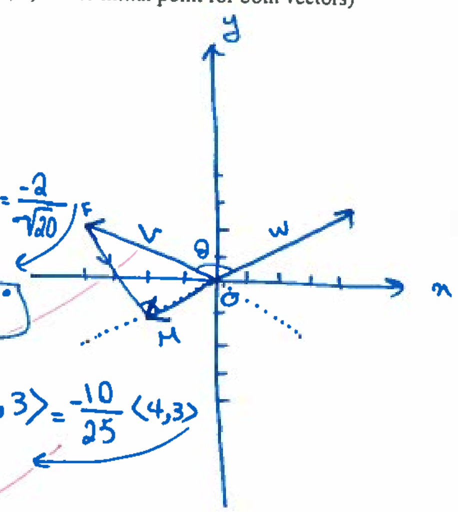
(vi) Find the equation of the ellipse.

$$\boxed{\frac{(y+2)^2}{49} + \frac{(x-1)^2}{4} = 1}$$

see back

QUESTION 7. Given $V = \langle -4, 2 \rangle$, $W = \langle 4, 3 \rangle$ (you may consider (0, 0) as the initial point for both vectors)

(i) Sketch both vectors in the xy–plane



(ii) Find the angle between V, W (to the nearest 2 decimals)

$\cos\theta = \frac{V \cdot W}{|V||W|} = \frac{-16+6}{(\sqrt{16+4})(\sqrt{16+9})} = \frac{-10}{5\sqrt{20}} = \frac{-2}{\sqrt{20}} = \frac{-1}{\sqrt{5}} = -\frac{\sqrt{5}}{5} \rightarrow \theta = \cos^{-1}\frac{-\sqrt{5}}{5} = \boxed{116.56°}$

(iii) Find $Proj_W^V$

$Proj_W^V = OM = \frac{W \cdot V}{|W|^2} W = \frac{(-10)}{(\sqrt{16+9})^2}\langle 4, 3\rangle = \frac{-10}{25}\langle 4, 3\rangle$

$\langle \frac{-2 \times 4}{5}, \frac{-2 \times 3}{5} \rangle = \boxed{\langle \frac{-8}{5}, \frac{-6}{5} \rangle}$

(iv) Find $|Proj_W^V|$

$|Proj_W^V| = \frac{|W \cdot V|}{|W|} = \frac{10}{\sqrt{16+9}} = \frac{10}{5} = \boxed{2}$

(v) Find Inj_W^V

$Inj_W^V = FM = V - Proj_W^V = \langle -4, 2 \rangle - \langle -\frac{8}{5}, -\frac{6}{5} \rangle = \langle -4 + \frac{8}{5}, 2 + \frac{6}{5} \rangle$

$\langle \frac{-12}{5}, \frac{16}{5} \rangle$

$|Inj_W^V| = \sqrt{(\frac{-12}{5})^2 + (\frac{16}{5})^2}$

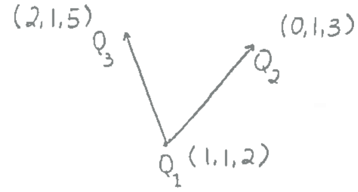
3.5 Questions with Solutions on Planes in 3 D from previous semesters

Quiz V MTH 111, Spring 2019

15/15 :)

Ayman Badawi

7/7 **QUESTION 1.** Let $Q_1 = (1, 1, 2), Q_2 = (0, 1, 3), Q_3 = (2, 1, 5)$. Find the equation of the plane that passes through $Q_1, Q_2, Q3$.



$N \perp$ Plane

$N = \overrightarrow{Q_1Q_2} \times \overrightarrow{Q_1Q_3}$

$\overrightarrow{Q_1Q_2} = \langle 0-1, 1-1, 3-2 \rangle = \langle -1, 0, 1 \rangle$ ✓1

$\overrightarrow{Q_1Q_3} = \langle 2-1, 1-1, 5-2 \rangle = \langle 1, 0, 3 \rangle$ 1

$\overrightarrow{Q_1Q_2} \times \overrightarrow{Q_1Q_3} = \begin{vmatrix} i & j & k \\ -1 & 0 & 1 \\ 1 & 0 & 3 \end{vmatrix} = (\ 0\ )i - (-3-1)j + (\ 0\ )k = 4j = \langle 0, 4, 0 \rangle$ ✓3

choose Q_1 & a random point

$w = (x, y, z)$

$Q_1 = (1, 1, 2)$

$\overrightarrow{Q_1w} = (x-1, y-1, z-2)$

$N \cdot \overrightarrow{Q_1w} = \langle 0, 4, 0 \rangle \cdot \langle x-1, y-1, z-2 \rangle = 0$

$0(x-1) + 4(y-1) + 0(z-2) = 0$

$4(y-1) = 0$ ✓2

 Haya

QUESTION 2. (i) **(6 points)** Does the line $L: x = 2t+1, y = 5t-1, z = -2t+3$ lie entirely inside the plane $x+2y+z = 23$? If not, does it intersect the plane? If yes, then find the intersection point.

it doesnt lie intirely on the plane but intersects it at (5.2, 9.5, -1.2)

$L: \left.\begin{array}{l} x = 2t+1 \\ y = 5t-1 \\ z = -2t+3 \end{array}\right\} t \in \mathbb{R}$

$P \Rightarrow x + 2y + z = 23$

$P(L) \Rightarrow (2t+1) + 2(5t-1) + (-2t+3) = 23 \leftarrow 1$

$\cancel{2t} + 1 + 10t - 2 - \cancel{2t} + 3 = 23$

$10t + 2 = 23$

$10t = 21$

$t = 21/10$

$= 2.1$

$x = 2(2.1) + 1 = 5.2$

2) → $y = 5(2.1) - 1 = 9.5$

$z = -2(2.1) + 3 = -1.2$

$Q = (5.2, 9.5, -1.2)$

the point of intersection is $(5.2, 9.5, -1.2) \leftarrow 3$

(ii) **(4 points)** Given $N = < -2, 3, 2 >$ is perpendicular to the plane P and the point $(-1, 4, 2)$ lies inside the plane P. Find the equation of the plane P.

$N_x(x - P_x) + N_y(y - P_y) + N_z(z - P_z) = 0$

$-2(x+1) + 3(y-4) + 2(z-2) = 0 \Leftrightarrow$ plane.

1

Scanned with CamScanner

QUESTION 9. (5 points). Can we draw the entire line $L: x = 2t, y = -3t + 1, z = 11t + 4$ inside the plane $2x - 6y - 2z = 20$? EXPLAIN

$N_{plane} \cdot D_{line}$ must $= 0$

$N = \langle 2, -6, -2 \rangle$

$D = \langle 2, -3, 11 \rangle$

$N \cdot D = 4 + 18 - 22$

$= 0$ ✓

take a point on L and check if the point lies in the plane or not

NO

yes the line can be entirely drawn on the plane because the dot product of the normal and directional vector is 0

3

Name ______, ID ______

Quiz 4 ~~HW 9~~: MTH 111, Spring 2018

Ayman Badawi

$\frac{15}{15}$

QUESTION 1. a) Find the equation of the plane that contains the points $Q_1 = (0, 1, 1), Q_2 = (0, 2, 3), Q_3 = (1, 3, 2)$.

$\overrightarrow{Q_1Q_2}$: $(0,2,3) - (0,1,1) \rightarrow \langle 0,1,2 \rangle$

$\overrightarrow{Q_1Q_3}$: $(1,3,2) - (0,1,1) \rightarrow \langle 1,2,1 \rangle$

$\overrightarrow{QQ_2} \cdot \overrightarrow{Q_1Q_3} = \langle N \rangle = i[(1\times1)-(2\cdot2)] - j[(0\cdot1)-(2\cdot1)] + K[(0\cdot2)-(1\cdot1)]$

$$\begin{array}{ccc} i & j & K \\ 0 & 1 & 2 \\ 1 & 2 & 1 \end{array}$$

$-3i + 2j - K$

$\langle N \rangle \quad \langle -3, 2, -1 \rangle$

5

equation: $\boxed{-3x + 2(y-2) - 1(z-1) = 0}$

c) Given a plane $P : 5x - 7y + z = 21$ Can we draw the vector $V = \langle -4, -3, -1 \rangle$ inside the plane P? explain

3

$N \langle 5, -7, 1 \rangle$ $\quad N \cdot V = 0 = \perp \longrightarrow$ so inside plane

$V \langle -4, -3, -1 \rangle$ $\quad (5\cdot -4) + (-7\cdot -3) + (1\cdot -1) = -20 + 21 - 1$

$= 0$

d) Find the distance between the point (0, 10, 5) and the plane ...

Rania Hegab

Exam II: MTH 111, Spring 2018

Ayman Badawi

Points = 55

QUESTION 2. (i) **(3 points)** What can you say about the line $L: x = 2t + 1, y = t - 1, z = -2t + 3$ and the plane $x + 2y + z = 16$? (i.e., Doe L lie inside the plane? Does L intersect the plane exactly in one point? or neither?

L: $x = 2t + 1$
$y = t - 1$
$z = -2t + 3$

P: $x + 2y + z = 16$

$(2t + 1) + 2(t - 1) - 2t + 3 = 16$

$2t + 1 + 2t - 2 - 2t + 3 = 16$

$2t = 14 \Rightarrow t = 14/2 \Rightarrow \boxed{t = 7}$

x: $2(7) + 1 = 15$
y: $7 - 1 = 6$
z: $-2(7) + 3 = -11$

$\boxed{\text{Q: intersection point} : (15, 6, -11)}$

(ii) **(4 points)** Given $N = \langle -2, 3, 2 \rangle$ is perpendicular to the plane P and the point $(-1, 4, 2)$ lies inside the plane P. Find the equation of the plane P.

Find eqn → Directional vector
→ point Q

$N = \langle -2, 3, 2 \rangle \perp P$ at $Q(-1, 4, 2)$

P: $-2(x + 1) + 3(y - 4) + 2(z - 2) = 0$

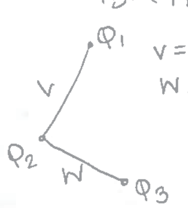
P: $-2x - 2 + 3y - 12 + 2z - 4 = 0$

$\boxed{P: -2x + 3y + 2z = 18}$

(iii) **(6 points)** Find the equation of the plane that contains the points $Q_1 = (4, 4, 0)$, $Q_2 = (0, 2, 6)$ and $Q_3 = (4, 0, 8)$.

Eqn of plane → directional vector and point Q_1

$Q_1: (4, 4, 0)$
$Q_2: (0, 2, 6)$
$Q_3: (4, 0, 8)$



$V = Q_1Q_2 = \langle 4, 2, -6 \rangle$

$W = Q_3Q_2 = \langle 4, -2, 2 \rangle$

$$V \times W = \begin{vmatrix} i & j & k \\ 4 & 2 & -6 \\ 4 & -2 & 2 \end{vmatrix} = \begin{vmatrix} 2 & -6 \\ -2 & 2 \end{vmatrix}, -\begin{vmatrix} 4 & -6 \\ 4 & 2 \end{vmatrix}, \begin{vmatrix} 4 & 2 \\ 4 & -2 \end{vmatrix}$$

$= \langle 4 - 12, -(8 + 24), -8 - 8 \rangle$

$= \langle -8, -32, -16 \rangle$

P: $-8(x - 4) - 32(y - 4) - 16(z + 0) = 0$

P: $-8x + 32 - 32y + 128 - 16z = 0$

$\boxed{P: -8x - 32y - 16z = -160}$

Exam II: MTH 111, Fall 2017

Ayman Badawi

Points = $\frac{47}{47}$

Haya Alshamsi

QUESTION 2. (i) **(3 points)** Can we draw the vector $v = <3, -5, 2>$ inside the plane $x - 4y - 11z = 7$? explain

NO. The two vectors are not perpendicular, hence v can't be drawn inside the plane.

$V = <3, -5, 2>$

$N = <1, -4, -11>$

$N \cdot V = 3(1) - 5(-4) + 2(-11)$

$N \cdot V = 3 + 20 - 22 = 1 \neq 0$

(ii) **(4 points)** Given $N = <4, 6, 2>$ is perpendicular to the plane P and the point $(4, 1, 1)$ lies inside the plane P. Find the equation of the plane P.

$N = <4, 6, 2>$
$<a, b, c>$

$Q(4, 1, 1)$

$Q(x_0, y_0, z_0)$

$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$

$4(x - 4) + 6(y - 1) + 2(z - 1) = 0$

$4x - 16 + 6y - 6 + 2z - 2 = 0$

$\boxed{4x + 6y + 2z = 24}$

(iii) **(6 points)** Find the equation of the plane that contains the points $Q_1 = (1, 1, 4), Q_2 = (2, 3, 6)$ and $Q_3 = (1, 1, 8)$.

$Q_1 (1, 1, 4)$

$Q_2 (2, 3, 6)$

$Q_3 (1, 1, 8)$

$\overrightarrow{Q_1Q_2} = <1, 2, 2>$

$\overrightarrow{Q_1Q_3} = <0, 0, 4>$

$$\vec{N} = \overrightarrow{Q_1Q_2} \times \overrightarrow{Q_1Q_3} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 2 \\ 0 & 0 & 4 \end{vmatrix}$$

$\vec{N} = 8\hat{i} - 4\hat{j} + 0\hat{k}$

$\boxed{\vec{N} = <8, -4, 0>}$

$\cancel{8(x-1) - 4(y+4) + 0(z)}$

$8(x - 1) - 4(y - 1) + 0(z - 4) = 0$

$8x - 8 - 4y + 4 = 0$

$\boxed{8x - 4y = 4}$

$\boxed{2x - y = 1}$

QUESTION 3. (i) (4 points) The line $L: x = 2w, y = -w + 1, z = 3$ intersects the plane $4x + 7y + z = 12$ in a point Q. Find Q.

$$L: \begin{cases} x = 2w \\ y = -w + 1 \quad ; w \in \mathbb{R} \\ z = 3 \end{cases}$$

$P: 4x + 7y + z = 12$

$4(2w) + 7(-w+1) + 3 = 12$

$8w - 7w + 7 + 3 = 12$

$w + 10 = 12$

$\boxed{w = 2}$ → The plane and the line intersect when $w = 2$

$\Rightarrow \boxed{Q(4, -1, 3)}$

Name ______________, ID ____________ $t\langle 1,-3,2\rangle+(2,0,1)$

Exam I: MTH 111, Spring 2017

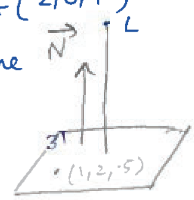
Ayman Badawi

$$\text{Points} = \frac{\cancel{58}\ 58}{58}$$

QUESTION 1. (4 points) Given that the line $L = 2+t, y = -3t, z = 1+2t$ is perpendicular to a plane, say P. If the point $(1,2,-5)$ lies in the plane P, find the equation of the plane P.

The parametric eqn can be written as $L: t\langle 1,-3,2\rangle + (2,0,1)$

since $L \perp$ to plane & pt $(1,2,-5)$ lies on the plane



$1(x-1) + -3(y-2) + 2(z+5) = 0$

$x - 1 - 3y + 6 + 2z + 10 = 0$

$x - 3y + 2z + 15 = 0$

4

(iii) Let $Q_1 = (1,1,0)$, $Q_2 = (0,-1,2)$ and $Q_3 = (2,2,2)$.

a. **(5 points)** Find the equation of the plane that contains Q_1, Q_2, Q_3.

$\overrightarrow{Q_1Q_2} \langle -1,-2,2\rangle \qquad \overrightarrow{Q_1Q_3} \langle 1,1,2\rangle$

$N = |Q_1Q_2 \times Q_1Q_3| = \begin{vmatrix} i & j & k \\ -1 & -2 & 2 \\ 1 & 1 & 2 \end{vmatrix} = \langle -6,4,1\rangle$

$P: -6(x-2)+4(y-2)+1(z-2)=0$

b. **(2 points)** Find the area of the triangle that has Q_1, Q_2, Q_3 as vertices.

$A = \frac{1}{2}\left|\overrightarrow{Q_1Q_2} \times \overrightarrow{Q_1Q_3}\right| = \frac{\sqrt{6^2+4^2+1^2}}{2} = \frac{\sqrt{53}}{2}$ units^2

(iv) **(4 points)** Given $L: x = t+1, y = 8, z = 4t+1$ lies entirely inside the plane $P: ax+2y+z = b$ Find the values of a, b. $D\langle 1,0,4\rangle \qquad N\langle a,2,1\rangle$

$N \cdot D = 0$

$a+4=0$

$\boxed{a=-4}$

$-4(t+1)+2(8)+4t+1 = b$

$-4t-4+16+4t+1 = b$

$\boxed{b=13}$

3.6 Questions with Solutions on Intersection of Planes in 3 D from previous semesters

Name____________________, ID __________

$t\langle 1,-3,2\rangle+\langle 2,0,1\rangle$

Exam I: MTH 111, Spring 2017

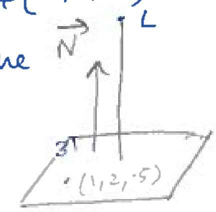
Ayman Badawi

Points = $\frac{\cancel{58}\ 58}{58}$

QUESTION 1. (4 points) Given that the line $L = 2+t, y = -3t, z = 1+2t$ is perpendicular to a plane, say P. If the point $(1,2,-5)$ lies in the plane P, find the equation of the plane P.

The parametric eqn can be written as $L: t\langle 1,-3,2\rangle + (2,0,1)$

since $L \perp$ to plane & pt $(1,2,-5)$ lies on the plane



$1(x-1) + -3(y-2) + 2(z+5) = 0$

$x - 1 - 3y + 6 + 2z + 10 = 0$

$x - 3y + 2z + 15 = 0$

4/4

QUESTION 2. (5 points) The two planes $P_1 : 2x - y + z = 6$ and $P_2 : -x + y + 4z = 4$ intersect in a line L. Find a parametric equations of L.

$P_1 : 2x - y + z = 6 \quad \langle 2,-1,1\rangle \to \vec{N_1}$

$P_2 : -x + y + 4z = 4 \quad \langle -1,1,4\rangle \to \vec{N_2}$

$\vec{N_1}\times\vec{N_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ -1 & 1 & 4 \end{vmatrix} = \hat{i}(-4-1) - \hat{j}(8+1) + \hat{k}(2-1)$

$= -5\hat{i} - 9\hat{j} + \hat{k} \to \langle -5,-9,1\rangle$

Assume $z = 0$

$$\begin{aligned} &2x - y = 6 \\ + \ &-x + y = 4 \\ \hline &x = 10 \end{aligned}$$

$-10 + y = 4$

$y = 14$

pt $(10,14,0)$

$L: t\langle -5,-9,1\rangle + (10,14,0)$

$: \langle -5t, -9t, t\rangle + (10,14,0)$

$x = -5t + 10$; $y = -9t + 14$; $z = t$

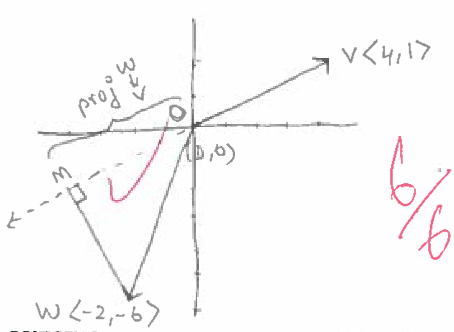
5/5

QUESTION 3. (6 points) From the origin (i.e, (0, 0)) draw the two vectors $V = \langle 4,1\rangle$, $W = \langle -2,-6\rangle$. First draw $Proj_V^W$. Then find $Proj_V^W$ and its length.

$\text{proj}_V^W = \frac{V\cdot W}{|V|^2}\cdot V = \frac{-8-6}{17}\langle 4,1\rangle$

$= \frac{-14}{17}\langle 4,1\rangle$

$= \left\langle \frac{-56}{17}, \frac{-14}{17}\right\rangle$



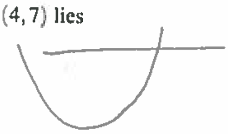
6/6

$|\text{proj}_V^W| = \sqrt{\left(\frac{-56}{17}\right)^2 + \left(\frac{-14}{17}\right)^2}$

$= \sqrt{11.52} = 3.39$

3136
+ 196
3332
289
11.52

QUESTION 4. (3 points) Given that $y = -2$ is the directrix of a parabola that has focus F. If the point $Q = (4,7)$ lies on the curve of the parabola, find $|QF|$ (i.e., find the distance between F and Q).



$|QL| = 9$

since $|QF| = |QL|$

$\therefore |QF| = 9$ units

3/3

(diagram on next page)

Quiz 5: MTH 111, Spring 2018

Ayman Badawi

$\frac{15}{15}$

QUESTION 1. a) The Plane $P: 2x + y - z = 16$ intersects the line $L: x = 3t, y = -2t + 4, z = -t - 2$ at a point Q find Q.

$2(3t) - 2t + 4 + t + 2 = 16$

$6t - 2t + 4 + t + 2 = 16$

$t = 2$ 1/1

$\boxed{Q(6,0,-4)}$ 1/1

$x = 3(2) = 6$

$y = -2(2) + 4 = 0$

$z = -2 - 2 = -4$

4/4

c) The two planes $P_1: 2x + y - z = 6$ and $P_2: 4x - y + z = 12$ intersect in a line L. Find a parametric equations of L.

$N_1: <2,1,-1> \quad N_2 <4,-1,1>$

$D = N_1 \times N_2 = \begin{vmatrix} i & j & k \\ 2 & 1 & -1 \\ 4 & -1 & 1 \end{vmatrix} = <0,-6,-6>$ 1/1

$$\boxed{L: \begin{cases} x = 3 \\ y = -6t \\ z = -6t \end{cases} ; t \in \mathbb{R}}$$

take $z = 0$

$2x + y = 6$

$4x - y = 12$

$x = 3 \quad y = 0$

$\Rightarrow Q(3,0,0)$ 1/1

4/4

QUESTION 2. Find $f'(x)$ and do not simplify

a) $f(x) = 3x^2(x+2)^2 + 2018x - 2017$

$f'(x) = \underbrace{6x(x+2)^2 + 6x^2(x+2)}_{\text{Product formula}} + 2018$ 2/2

or $f(x) = 3x^2(x^2 + 4x + 4) + 2018x - 2017$

$= 3x^4 + 12x^3 + 12x^2 + 2018x - 2017$

so $f'(x) = 12x^3 + 36x^2 + 24x + 2018$

b) $f(x) = 8\sqrt{x} + \frac{6}{x^3} + 2x^2$

$f'(x) = \frac{4}{\sqrt{x}} - \frac{18}{x^4} + 4x$ 1/1

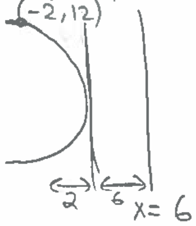
c) If $f(x) = 18\sqrt{x} + 7x + 1$. find $f'(9)$

$f'(x) = \frac{9}{\sqrt{x}} + 7 \Big|_{x=9} = 10$ 1/1

. **Faculty Information**

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: abadawi@aus.edu, www.ayman-badawi.com

QUESTION 11. (4 points) Given that $x = 6$ is the directrix line of a parabola that has F as its focus point. If the point $Q = (-2, 12)$ lies on the parabola. Find $|QF|$ (i.e., the distance between Q and F).



$|QF| = |QL| = 8$

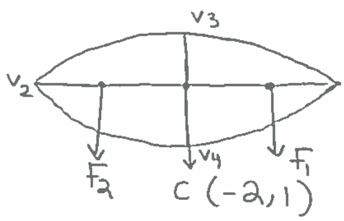
QUESTION 12. (6 points) Consider the ellipse $\frac{(y-1)^2}{9} + \frac{(x+2)^2}{25} = 1.$

$9 \to b^2$

$25 \to$ big # so its $\left(\frac{k}{2}\right)^2$

so the shape is

(i) Sketch (roughly)



(ii) Find the foci of the ellipse

$CF^2 = \left(\frac{k}{2}\right)^2 - b^2$
$= 25 - 9$
$= 16$

$CF^2 = 16$
so $\boxed{CF = 4}$

so $F_1\ (-2+4,\ 1)$
$(2, 1)$

$F_2\ (-2-4,\ 1)$
$(-6, 1)$

(iii) Find all four vertices of the ellipse.

$\left(\frac{k}{2}\right)^2 = 25$ $\boxed{\pm\frac{k}{2}}$

$\boxed{\frac{k}{2} = 5}$ $\boxed{\pm b}$

$b^2 = 9$ $\boxed{b = 3}$

$V_1 = (-2+5,\ 1)$
$(3, 1)$

$V_2 = (-2-5,\ 1)$
$(-7, 1)$

$V_3 = (-2,\ 1+3)$ $(-2, 4)$

$V_4 = (-2,\ 1-3)$ $(-2, -2)$

QUESTION 13. (4 points) Given $Q = (1, 6, 4)$ is not on the line $L : x = t + 1, y = 2t + 4, z = -5t + 3$ $(t \in R)$. Find $|QL|$.

$D = \langle 1, 2, -5 \rangle$

$I = \langle 1, 4, 3 \rangle$

$IQ = Q - I = \langle 0, 2, 1 \rangle$

$IQ \times D = \begin{vmatrix} i & j & k \\ 0 & 2 & 1 \\ 1 & 2 & -5 \end{vmatrix}$

$= -12i + 1j - 2k$

$|QL| = \frac{|D \times IQ|}{|D|} = \frac{\sqrt{12^2+1^2+2^2}}{\sqrt{1^2+2^2+5^2}}$

$= \frac{\sqrt{149}}{\sqrt{30}}$

Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: abadawi@aus.edu, www.ayman-badawi.com

 Haya

QUESTION 2. (i) **(6 points)** Does the line $L: x = 2t + 1, y = 5t - 1, z = -2t + 3$ lie entirely inside the plane $x + 2y + z = 23$? If not, does it intersect the plane? If yes, then find the intersection point.

$L: \left.\begin{array}{l} x = 2t+1 \\ y = 5t-1 \\ z = -2t+3 \end{array}\right\} t \in R$

$P \Rightarrow x + 2y + z = 23$

it doesn't lie intierly on the plane but intersects it at $(5.2, 9.5, -1.2)$

$P(L) \Rightarrow (2t+1) + 2(5t-1) + (-2t+3) = 23 \leftarrow 1$

$\cancel{2t} + 1 + 10t - 2 - \cancel{2t} + 3 = 23$

$10t + 2 = 23$

$10t = 21$

$t = 21/10$

$= 2.1$

$x = 2(2.1) + 1 = 5.2$

$2) \rightarrow y = 5(2.1) - 1 = 9.5$

$z = -2(2.1) + 3 = -1.2$

$Q = (5.2, 9.5, -1.2)$

the point of intersection is $(5.2, 9.5, -1.2) \leftarrow 3$

(ii) **(4 points)** Given $N = < -2, 3, 2 >$ is perpendicular to the plane P and the point $(-1, 4, 2)$ lies inside the plane P. Find the equation of the plane P.

$N_x(x - P_x) + N_y(y - P_y) + N_z(z - P_z) = 0$

$-2(x+1) + 3(y-4) + 2(z-2) = 0 \Leftrightarrow$ plane.

(iii) **(4 points)** Find the distance between $Q = (10, 10, 33)$ and the plane $P: -2x + 2y - 5z = 21$.

$D = \dfrac{|P(Q)|}{|N|} \quad \dfrac{|-2(10) + 2(10) - 5(33) - 21|}{\sqrt{4+4+25}} = \dfrac{186}{\sqrt{33}}$ units.

(iv) **(6 points)** The two planes $P_1: x + 4y + z = 10$ and $P_2: -x + 2y - z = 8$ intersects in a line L. Find a parametric equations of L.

$N_1 \times N_2 = D$

$N_1 = \langle 1, 4, 1 \rangle$

$N_2 = \langle -1, 2, -1 \rangle$

$\begin{array}{ccc} i & j & k \\ 1 & 4 & 1 \\ -1 & 2 & -1 \end{array}$

$(-4-2)i - (-1+1)j + (2+4)k$

$1 \rightarrow D = \langle -6, 0, 6 \rangle$

let $z = 0$

$-x + 2y = 8$

$x + 4y = 10$

$x = -2$

$y = 3$

$z = 0$

$2 \rightarrow Q = (-2, 3, 0)$

$D = \langle -6, 0, 6 \rangle$

$3 \rightarrow L: \left.\begin{array}{l} x = -6t - 2 \\ y = 3 \\ z = 6t \end{array}\right\} t \in R$

QUESTION 6. (5 points). Let $H = (4,6)$, $F = (6,34)$. Find a point Q on the line $x = -2$ such that $|HQ| + |FQ|$ is minimum.

$y = mx + b$

$m = \frac{6-34}{4+10} = -2$

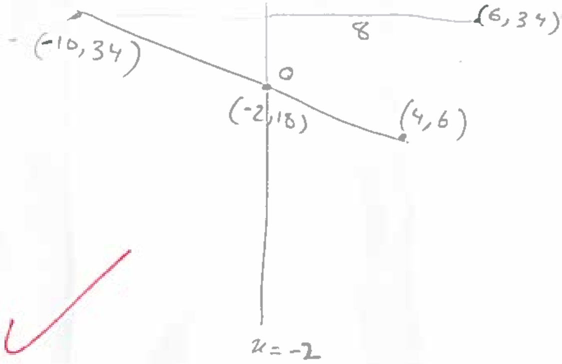
$6 = -2(4) + b$

$b = 14$

$y = -2x + 14$

$y = -2(-2) + 14$
$= 18$

$Q = (-2, 18)$



QUESTION 7. (4 points). For what values of x does the tangent line to the curve $y = ln(4x+1) + 7x + 2$ have slope equal 8?

$y' = 8$

$y' = \frac{4}{4x+1} + 7 = 8$

$\frac{4}{4x+1} = 1$

$4 = 4x + 1$

$4x = 4 - 1$

$x = 3/4$

check $\frac{4}{4(\frac{3}{4})+1} + 7 =$

$1 + 7 = 8$ //

the T line has slope 8 at $x = \frac{3}{4}$

QUESTION 8. (6 points). The plane $P_1 : x + 2y - 3z = 2$ intersects the plane $P_2 : -x + 5y + z = 19$ in a line L. Find a parametric equations of L.

① → $N_1 \times N_2 = D$

$N_1 = \langle 1, 2, -3 \rangle$

$N_2 = \langle -1, 5, 1 \rangle$

$D = (2+15)i - (1-3)j + (5+2)k$
$= \langle 17, 2, 7 \rangle$

② → $z = 0$
$x + 2y = 2$
$-x + 5y = 19$

$x = \frac{\begin{vmatrix} 2 & 2 \\ 19 & 5 \end{vmatrix}}{\begin{vmatrix} 1 & 2 \\ -1 & 5 \end{vmatrix}}$ $\quad y = \frac{\begin{vmatrix} 1 & 2 \\ -1 & 19 \end{vmatrix}}{\begin{vmatrix} 1 & 2 \\ -1 & 5 \end{vmatrix}}$

$x = \frac{-28}{7}$ $\quad y = \frac{21}{7}$

$x = -4$ $\quad y = 3$

③ → $(-4, 3, 0)$

$D = \langle 17, 2, 7 \rangle$

$L:$ $\left.\begin{aligned} x &= 17t - 4 \\ y &= 2t + 3 \\ z &= 7t \end{aligned}\right\}$ $t \in \mathbb{R}$

QUESTION 9. (5 points). Can we draw the entire line $L : x = 2t, y = -3t + 1, z = 11t + 4$ inside the plane $2x - 6y - 2z = 20$? EXPLAIN

$N_{plane} \cdot D_{line}$ must $= 0$

$N = \langle 2, -6, -2 \rangle$

$D = \langle 2, -3, 11 \rangle$

$N \cdot D = 4 + 18 - 22$
$= 0$ //

take a point on L and check if the point lies in the plane or not

NO

yes the line can be entirely drawn on the plane because the dot product of the normal and directional vector is 0

2

Name NADIN ELSHIRBINI, ID 76707

Exam I: MTH 111, Spring 2018

Ayman Badawi Nadin El Shirbini

Points = $\frac{80}{80}$

QUESTION 1. a) **(3 points)** Are the points $q_1 = (1, 2, -2)$, $q_2 = (3, 3, 1)$, and $q_3 = (5, 4, 4)$ co-linear? Show the work

$\overrightarrow{Q_1Q_2} = \langle 2, 1, 3 \rangle$

$\overrightarrow{Q_1Q_3} = \langle 4, 2, 6 \rangle$

$$\overrightarrow{Q_1Q_2} \times \overrightarrow{Q_1Q_3} = \begin{vmatrix} i & j & k \\ 2 & 1 & 3 \\ 4 & 2 & 6 \end{vmatrix} = \left\langle \begin{vmatrix} 1 & 3 \\ 2 & 6 \end{vmatrix}, \begin{vmatrix} 2 & 3 \\ 4 & 6 \end{vmatrix}, \begin{vmatrix} 2 & 1 \\ 4 & 2 \end{vmatrix} \right\rangle = \langle 0, 0, 0 \rangle$$

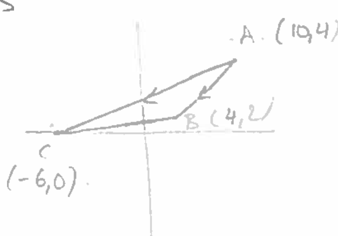
cross product is zero $\Rightarrow$ they are colinear

b) **(3 points)** Given $A = (10, 4)$, $B = (4, 2)$, and $C = (-6, 0)$ are the vertices of a triangle. Roughly, sketch the triangle ABC. Find the area of the triangle ABC.

$\overrightarrow{AB} = \langle -6, -2 \rangle$

$\overrightarrow{AC} = \langle -16, -4 \rangle$

$$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} i & j & k \\ -6 & -2 & 0 \\ -16 & -4 & 0 \end{vmatrix} = \langle 0, 0, -8 \rangle$$



$$A_{\Delta ABC} = \frac{1}{2} |\overrightarrow{AB} \times \overrightarrow{AC}| = \frac{1}{2}\sqrt{(-8)^2} = \boxed{4 \text{ units}^2}$$

c) **(3 points)** Find a vector F that is perpendicular to both vectors $V = \langle 2, -1, 4 \rangle$ and $W = \langle 0, 4, 2 \rangle$

$$\vec{F} = \vec{V} \times \vec{W} = \begin{vmatrix} i & j & k \\ 2 & -1 & 4 \\ 0 & 4 & 2 \end{vmatrix} = \boxed{\langle -18, -4, 8 \rangle}$$

d) **(2 points)** Let V, W as in (c). Find a vector F that is perpendicular to both V and W such that $|F| = 2$.(hint: Just think a little)

$$|F| = \sqrt{18^2 + 4^2 + 8^2} = 2\sqrt{101}$$

$$(2)\frac{1}{|F|} \cdot F = \frac{\cancel{2}}{\cancel{2}\sqrt{101}} \cdot F = \frac{1}{\sqrt{101}} \cdot F = \boxed{\frac{1}{\sqrt{101}} \langle -18, -4, 8 \rangle}$$

$$\boxed{F \left\langle \frac{-18}{\sqrt{101}}, \frac{-4}{\sqrt{101}}, \frac{8}{\sqrt{101}} \right\rangle}$$

check if $|F| = 2$:

$$|F| = \sqrt{\left(\frac{-18}{\sqrt{101}}\right)^2 + \left(\frac{-4}{\sqrt{101}}\right)^2 + \left(\frac{8}{\sqrt{101}}\right)^2} = 2$$

QUESTION 3. (i) (4 points) (1) Convince me that the line $L: x = 4t, y = -4t + 1, z = 2t + 1$ is perpendicular to the plane $P: 2x + -2y + z = 12$ (If you think that I am wrong, then state your reason). (2) Can we draw the vector $V = \langle 1, -2, -6 \rangle$ inside P?

L: x = 4t
y = -4t + 1
z = 2t + 1

P: 2x + -2y + z = 12

$D_2 = \langle 2, -2, 1 \rangle$

① $D_1 = \langle 4, -4, 2 \rangle$

$D_1 \times D_1 = \begin{vmatrix} i & j & k \\ 4 & -4 & 2 \\ 2 & -2 & 1 \end{vmatrix} = \begin{vmatrix} -4 & 2 \\ -2 & 1 \end{vmatrix}, -\begin{vmatrix} 4 & 2 \\ 2 & 1 \end{vmatrix}, \begin{vmatrix} 4 & -4 \\ 2 & -2 \end{vmatrix} = -4+4, -(4-4), -8+8$

$D_1 \times D_2 = \langle 0, 0, 0 \rangle$ → Plane and line are perpendicular.

② $V = \langle 1, -2, -6 \rangle$

$D_2: \langle 2, -2, 1 \rangle$

$V \cdot D_2 = 0$ → Yes $V \cdot D_2 \neq 0$ → No

$V \cdot D_2 = \langle 1, -2, -6 \rangle \cdot \langle 2, -2, 1 \rangle$

$V \cdot D_2 = 2 + 4 - 6 = 0$ → Yes, we can draw V inside P.

(ii) (3 points) Find the distance between $Q = (10, 10, 33)$ and the plane $P: 2x - 2y + z = 21$.

Q: (10, 10, 33)

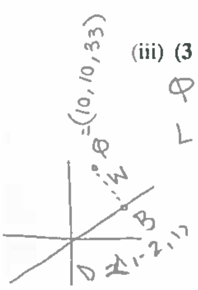
P: 2x - 2y + z = 21

↳ 2x - 2y + z - 21 = 0

$QP = \dfrac{|2(10) - 2(10) + 33 - 21|}{\sqrt{(2)^2 + (-2)^2 + (1)^2}}$

$QP = \dfrac{12}{3} = \boxed{4 \text{ units}}$

(iii) (3 points) Find the distance between $Q = (10, 10, 33)$ and the line $L: x = t + 1, y = -2t + 3, z = t$



Q: (10, 10, 33)

L: x = t + 1
y = -2t + 3
z = t

D = ⟨1, -2, 1⟩
I = ⟨1, 3, 0⟩

W = |QB| = 9, 7, 33

$QL = \dfrac{|W \times D|}{|D|}$

$|W \times D| = \begin{vmatrix} i & j & k \\ 9 & 7 & 33 \\ 1 & -2 & 1 \end{vmatrix}$

$= \begin{vmatrix} 7 & 33 \\ -2 & 1 \end{vmatrix}, -\begin{vmatrix} 9 & 33 \\ 1 & 1 \end{vmatrix}, \begin{vmatrix} 9 & 7 \\ 1 & -2 \end{vmatrix}$

$= \langle 7 + 66, -(9 - 33), -18 - 7 \rangle = \langle 73, 24, -25 \rangle$

$\dfrac{|W \times D|}{|D|} = \dfrac{\sqrt{73^2 + 24^2 + 25^2}}{\sqrt{1^2 + 2^2 + 1^2}} = \boxed{32.99 \text{ units}}$

(iv) (6 points) The two planes $P_1: x + 2y + z = 10$ and $P_2: -x + 2y - z = 6$ intersects in a line L. Find a parametric equations of L.

P_1: x + 2y + z = 10 → $N_1 = \langle 1, 2, 1 \rangle$

P_2: -x + 2y - z = 6 → $N_2 = \langle -1, 2, -1 \rangle$

$N_1 \times N_2 = \begin{vmatrix} i & j & k \\ 1 & 2 & 1 \\ -1 & 2 & -1 \end{vmatrix} = \begin{vmatrix} 2 & 1 \\ 2 & -1 \end{vmatrix}, -\begin{vmatrix} 1 & 1 \\ -1 & -1 \end{vmatrix}, \begin{vmatrix} 1 & 2 \\ -1 & 2 \end{vmatrix}$

$= \langle -2 - 2, -(-1 + 1), 2 + 2 \rangle$

$\boxed{N_1 \times N_2 = \langle -4, 0, 4 \rangle}$

Let $\boxed{z = 0}$ in P_1 and P_2

x + 2y = 10 → x = 10 - 2y → 10 - 2(4) = 10 - 8 = $\boxed{2 = x}$

-x + 2y = 6

↓

-(10 - 2y) + 2y = 6

-10 + 2y + 2y = 6

-10 + 4y = 6

4y = 6 + 10

4y = 16 ⇒ y = 16/4 ⇒ $\boxed{y = 4}$

Parametric eqns:

x: -4t - 2

y: 4

z: 4t

QUESTION 3. (i) **(4 points)** The line $L : x = 2w, y = -w + 1, z = 3$ intersects the plane $4x + 7y + z = 12$ in a point Q. Find Q.

$$L: \begin{cases} x = 2w \\ y = -w + 1 \quad ; w \in \mathbb{R} \\ z = 3 \end{cases}$$

$P: 4x + 7y + z = 12$

$4(2w) + 7(-w+1) + 3 = 12$

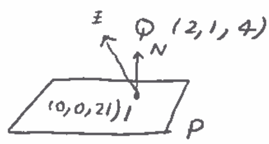
$8w - 7w + 7 + 3 = 12$

$w + 10 = 12$

$\boxed{w = 2}$ → The plane and the line intersect when $w = 2$

$\Rightarrow \boxed{Q(4, -1, 3)}$

(ii) **(4 points)** Find the distance between $Q = (2, 1, 4)$ and the plane $2x - 2y + z = 21$.



$I\ (0, 0, 21)$

$Q\ (2, 1, 4)$

$\vec{IQ} = \langle 2, 1, -17 \rangle$

$N = \langle 2, -2, 1 \rangle$

$$d = \frac{|\vec{IQ} \cdot N|}{|N|} = \frac{|2(2) + 1(-2) + 1(-17)|}{\sqrt{4 + 4 + 1}}$$

$$d = \frac{15}{\sqrt{9}} = \frac{15}{3} = 5 \text{ units}$$

(iii) **(6 points)** The two planes $P_1 : x + y + z = 2$ and $P_2 : -x + y - z = 6$ intersects in a line L. Find a parametric equations of L.

$N_1 = \langle 1, 1, 1 \rangle$

$N_2 = \langle -1, 1, -1 \rangle$

$$\vec{D} = \vec{N_1} \times \vec{N_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ -1 & 1 & -1 \end{vmatrix}$$

$$\vec{D} = -2\hat{i} + 0\hat{j} + 2\hat{k}$$

→ Let $z = 0$; find x and y :

$$\begin{cases} x + y = 2 \\ -x + y = 6 \end{cases}$$

$2y = 8$

$\boxed{y = 4}$

$x + 4 = 2$

$x = 2 - 4$

$\boxed{x = -2}$

The point is $(-2, 4, 0)$ and $D = \langle -2, 0, 2 \rangle$

* Parametric Eqns :

$$L: \begin{cases} x = -2 - 2t \\ y = 4 \quad ; t \in \mathbb{R} \\ z = 2t \end{cases}$$

$t\langle 1,-3,2\rangle + (2,0,1)$

Exam I: MTH 111, Spring 2017

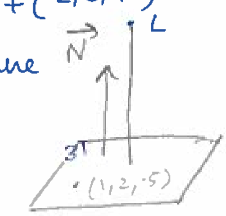
Ayman Badawi

Points = $\frac{58}{58}$

QUESTION 1. (4 points) Given that the line $L = 2+t, y = -3t, z = 1+2t$ is perpendicular to a plane, say P. If the point $(1,2,-5)$ lies in the plane P, find the equation of the plane P.

The parametric eqn can be written as $L: t\langle 1,-3,2\rangle + (2,0,1)$

since $L \perp$ to plane & pt $(1,2,-5)$ lies on the plane



$1(x-1) + -3(y-2) + 2(z+5) = 0$

$x - 1 - 3y + 6 + 2z + 10 = 0$

$x - 3y + 2z + 15 = 0$

4/4

QUESTION 2. (5 points) The two planes $P_1: 2x - y + z = 6$ and $P_2: -x + y + 4z = 4$ intersect in a line L. Find a parametric equations of L.

$P_1: 2x - y + z = 6 \quad \langle 2,-1,1\rangle \rightarrow \vec{N_1}$

$P_2: -x + y + 4z = 4 \quad \langle -1,1,4\rangle \rightarrow \vec{N_2}$

$$\vec{N_1} \times \vec{N_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ -1 & 1 & 4 \end{vmatrix} = \hat{i}(-4-1) - \hat{j}(8+1) + \hat{k}(2-1)$$

$= -5\hat{i} - 9\hat{j} + \hat{k} \rightarrow \langle -5,-9,1\rangle$

Assume $z = 0$

$2x - y = 6$

$+ \quad -x + y = 4$

$x = 10$

$-10 + y = 4$

$y = 14$

pt $(10,14,0)$

$L: t\langle -5,-9,1\rangle + (10,14,0)$

$: \langle -5t, -9t, t\rangle + (10,14,0)$

$x = -5t + 10$; $y = -9t + 14$; $z = t$

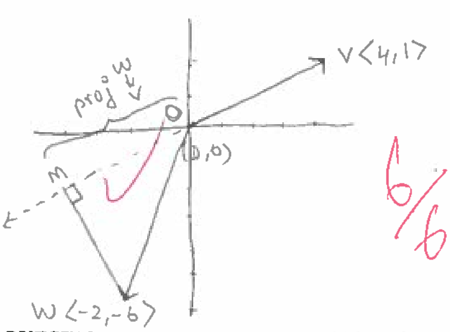
5/5

QUESTION 3. (6 points) From the origin (i.e, (0, 0)) draw the two vectors $V = \langle 4,1\rangle$, $W = \langle -2,-6\rangle$. First draw $Proj_V^W$. Then find $Proj_V^W$ and its length.

$proj_V^W = \frac{V \cdot W}{|V|^2} \cdot V = \frac{-8-6}{17} \langle 4,1\rangle$

$= \frac{-14}{17}\langle 4,1\rangle$

$= \left\langle \frac{-56}{17}, \frac{-14}{17} \right\rangle$



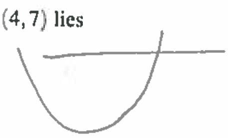
3136
+ 196
3332
289
11.52

$|proj_V^W| = \sqrt{\left(\frac{-56}{17}\right)^2 + \left(\frac{-14}{17}\right)^2}$

$= \sqrt{11.52} = 3.39$

6/6

QUESTION 4. (3 points) Given that $y = -2$ is the directrix of a parabola that has focus F. If the point $Q = (4,7)$ lies on the curve of the parabola, find $|QF|$ (i.e., find the distance between F and Q).



$|QL| = 9$

since $|QF| = |QL|$

$\therefore |QF| = 9$ units

3/3

(diagram on next page)

3.7 Notes on Trig. Functions, area and volume

· Trig. Functions

$\cancel{y = si}$ $y = a\sin(bx)$

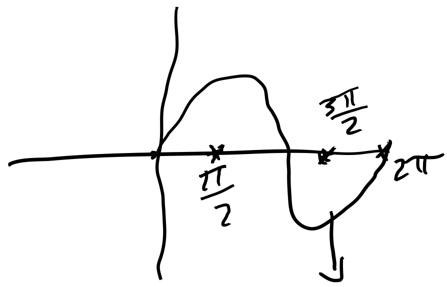
$y' = ab\cos(bx)$

$y = 5\sin(7x)$

why! → $y' = 35\cos(7x)$

$y = 3\sin(-8x)$

$y' = -24\cos(-8x)$



$y = \sin(x)$

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

→ $y = f(x) = a\cos(bx)$

$y' = f'(x) = -ab\sin(bx)$

$y = 10\cos(-3x)$

$y' = -(-30)\sin(-3x)$

$y' = 30\sin(-3x)$

$y = 3\cos(x)$

$y' = -3\sin(x)$

$f(x) = 3\cos(x)$

$f'(x) = -3\sin(x)$

① $y = a\sin(bx)$

$y' = ab\cos(bx)$

② $y = a\cos(bx)$

$y' = -ab\sin(bx)$

always

$\int 3\cos(2x)\,dx = \boxed{\text{Answer}}' = \frac{3\sin(2x)}{2} + C$

$= \frac{3}{2}\sin(2x) + C$

$\int 5\cos(7x)\,dx = \frac{5}{7}\sin(7x) + C$

③ $\int a\cos(bx)\,dx = \frac{a}{b}\sin(bx) + C$

---

④ $\int a\sin(bx)\,dx = -\frac{a}{b}\cos(bx) + C$

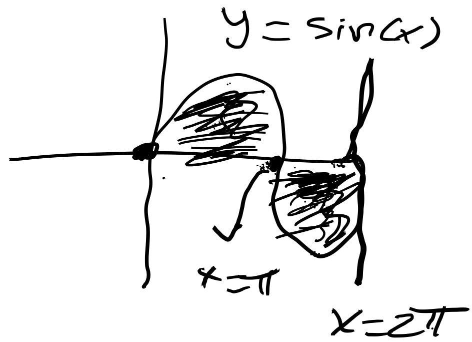
$\int 3\sin(10x) = -\frac{3}{10}\cos(10x) + C$

$\int 12\sin(2x) = -\frac{12}{2}\cos(2x) + C$

$f(x) = x^3 + e^{4x} + 3\cos(10x)$

$f'(x) = 3x^2 + 4e^{4x} + -30\sin(10x)$

Q. Area is bounded by $y_1 = \sin(x)$, $y_2 = 0$ (x-axis), $x = 0$, and $x = 2\pi$



A: $\displaystyle\int_{x=0}^{x=\pi}$ bigger − smaller

$\displaystyle\int a\sin bx = -\frac{a}{b}\cos(bx)$

$$\text{Area} = \int_{x=0}^{x=\pi} (\sin(x) - 0)\,dx + \int_{x=\pi}^{2\pi} \overset{-\sin(x)}{\underline{0 - \sin(x)}}\,dx$$

$$= -\cos x \Big|_{x=0}^{x=\pi} + \cos(x) \Big|_{x=\pi}^{x=2\pi}$$

$$-\cos(\pi) - (-\cos(0)) + \cos(2\pi) - \cos(\pi)$$

minus

$$1 + 1 + 1 + 1 = 4\ \text{unit}^2$$

Some basic Facts

→ (1) $\sin(2x) = 2\sin(x)\cos(x)$

(1.5) → $\sin^2(x) + \cos^2(x) = 1$

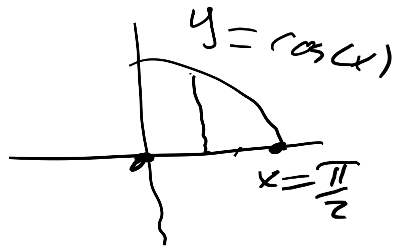
→ (2) $\cos^2(x) = \dfrac{1+\cos(2x)}{2}$

(3) $\sin^2(x) = \dfrac{1-\cos(2x)}{2}$

(4) $\cos^2(\underline{bx}) = \dfrac{1+\cos(2bx)}{2}$

(5) $\sin^2(bx) = \dfrac{1-\cos(2bx)}{2}$

Q. Find the volume of ~~un~~ the object when we rotate $y=\cos(x)$ about x-axis $(y=0)$



A: $\pi \displaystyle\int_{x=0}^{x=\frac{\pi}{2}} \left[\cos(x) - 0\right]^2 dx$

$$A = \pi \int_0^{\frac{\pi}{2}} \cos^2 x \, dx \qquad \cos^2(x) = \frac{1}{2} + \frac{\cos(2x)}{2}$$

$$= \pi \int_0^{\frac{\pi}{2}} \frac{1}{2} + \frac{1}{2}\cos(2x) \, dx$$

$$= \pi \left[\frac{1}{2}x + \frac{1}{4}\sin(2x) \right] \Big|_{x=0}^{\frac{\pi}{2}}$$

$$= \pi \left[\frac{1}{2} \cdot \frac{\pi}{2} + \underbrace{\frac{1}{4} \cdot 0 - 0}_{} \right] = \frac{\pi^2}{4}$$

3.8 Notes on Integration by Substitution

Integration by substitution

$\int \frac{1}{x}\,dx = [\text{Answer}]' = \ln(|x|) + C$ $\quad \ln(x) + C$

$\frac{1}{x}$

$\ln(-x) + C$

$\frac{-1}{-x} = \frac{1}{x}$

$\int \frac{1}{x}\,dx = \ln(|x|) + C$

$\int x^{-1}\,dx =$

$\int a x^{m \neq -1}\,dx = \frac{a}{m+1} x^{m+1} + C$

$y = ((1+2x^3))^4$ → chain Rule

$y' = 4(1+2x^3)^3(6x^2)$ Power formula

$\int \frac{1}{x}\,dx = \ln(|x|) + C$

$y = \frac{1}{x} = x^{-1}$

$y' = -1x^{-2} = \frac{-1}{x^2}$

$y = (1+2x^3)^4$

$y' = 24x^2(1+2x^3)^3$

$\int (24x^2)(1+2x^3)^3\,dx = (1+2x^3)^4 + C$

Integration by substitution

$[f(x)]^{n \neq -1} \longrightarrow n f(x)^{n-1} [f'(x)]$

$$\frac{1}{8}\int 8x\,(1+4x^2)^5\,dx = \frac{1}{8}\,\frac{(1+4x^2)^6}{6}$$

$$u = 1+4x^2$$

$$\frac{du}{dx} = u' = 8x$$

$$\frac{1}{3}\int 3(1+x^2)\,[3x+x^3]^{10}\,dx = \frac{1}{3}\,\frac{(3x+x^3)^{11}}{11} + C$$

$$u = 3x+x^3$$

$$u' = 3+3x^2$$

$$= \frac{1}{33}(3x+x^3)^{11} + C$$

$$\int \cos(x)\,[1+\sin(x)]^3\,dx = \frac{[1+\sin(x)]^4}{4} + C$$

$$u = 1+\sin x$$

$$u' = \cos(x)$$

$$\int f'(x)\,[f(x)]^{n\neq -1}\,dx = \frac{f(x)^{n+1}}{n+1} + C$$

$$\int 6e^x\,(3+e^x)^{10}\,dx = 6\,\frac{(3+e^x)^{11}}{11} + C$$

$$u = 3+e^x$$

$$u' = e^x$$

$$\int \frac{x+1}{x^2+2x+3}\,dx$$

$$\frac{1}{2}\int 2(x+1)\left((x^2+2x+3)\right)^{-1} dx = \frac{1}{2}\ln|x^2+2x+3| + c$$

n

$u = x^2+2x+3$

$u' = 2x+2$

$$\to \int f'(x)\,[f(x)]^{-1}\,dx = \ln|f(x)| + C$$

$$\int \frac{\cancel{\cos} -\sin x+3}{2\cos(x)+6x}\,dx$$

$$\frac{1}{2}\int 2[-\sin(x)+3]\left((2\cos(x)+6x)\right)^{-1} dx$$

$u = 2\cos(x)+6x$

$u' = -2\sin(x)+6$

$$= \frac{1}{2}\ln|2\cos(x)+6x| + C$$

$$\int \tan(x)\,dx = \boxed{}$$

$$\int \frac{\sin(x)}{\cos(x)}\,dx =$$

$$-\int -\sin(x)\,[\cos(x)]^{-1}\,dx = -\ln|\cos(x)| + c$$

$u = \cos(x)$

$u' = -\sin x$

$$= \ln\left|\frac{1}{\cos(x)}\right| + C$$

$= \ln|\sec(x)| + C$

$$3\int \frac{(9e^{3x}+12\cos(x))}{3} \left(\left(e^{3x}+4\sin(x)\right)\right)^3 dx$$

$u = e^{3x} + 4\sin x$

$u' = 3e^{3x} + 4\cos(x)$

$$= 3\,\frac{\left(e^{3x}+4\sin(x)\right)^4}{4} + C$$

3.9 Open Questions-Solutions Last lecture

Questions

$\sqrt[n]{(\;)^m} = (\;)^{\frac{m}{n}}$

Q. $\int (x+1)\sqrt[3]{(x^2+2x+1)}\,dx$

A. $\frac{1}{2}\int 2\underbrace{(x+1)}\,(x^2+2x+1)^{\frac{1}{3}}\,dx =$

$u = x^2+2x+1$

$u' = 2x+2$

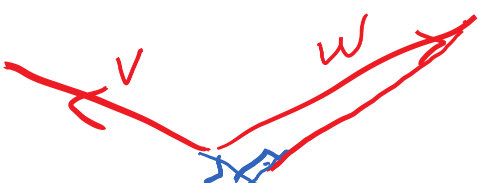
$\frac{\frac{1}{2}(x^2+2x+1)^{\frac{4}{3}}}{\frac{4}{3}} + C$

$\frac{3}{8}(x^2+2x+1)^{\frac{4}{3}} + C$

Q. 

draw proj_{w}^{v}

A. black = proj_{w}^{v}

Q. 

draw proj_{v}^{w}

blue = proj_{v}^{w}

Know

$\sin^2(\theta) + \cos^2(\theta) = 1$

$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$

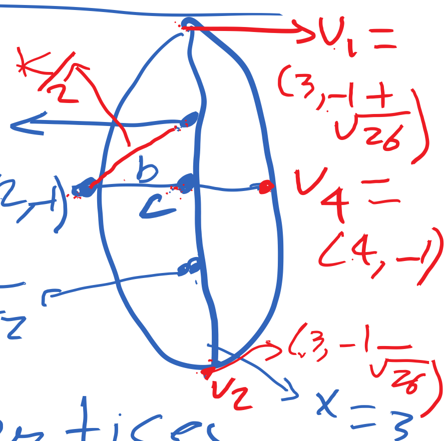
$\sin^2(a\theta) = \frac{1}{2} - \frac{1}{2}\cos(2a\theta)$

$\cos^2(a\theta) = \frac{1}{2} + \frac{1}{2}\cos(2a\theta)$

Q. $c = (3, -1)$

~~$F = (5$~~ $F_1 = (a, 4)$, $a = 3$

one of the vertices is $(2, -1)$



Find F_2, Find all vertices, Find ellipse-constant, Find the equation.

$|F_1c| = 5$, $F_2 = (3, -6)$ ✓

$|cV_3| = b = 3 - 2 = 1$, $\frac{K}{2} = \sqrt{|F_1c|^2 + |cV_3|^2} = \sqrt{25+1} = \sqrt{26}$

$K = 2\sqrt{26}$

$$\frac{(x-3)^2}{b^2} + \frac{(y+1)^2}{\left(\frac{K}{2}\right)^2} = 1$$

$$\frac{(x-3)^2}{1} + \frac{(y+1)^2}{26} = 1$$ ✓

Questions

Q. $y = \left(5e^{2x} + 3\cos(5x) + 3\right)^{10}$

A. $y' = 10\left(5e^{2x} + 3\cos(5x) + 3\right)^{9}\left(10e^{2x} - 15\sin(5x)\right)$

Q. $\int \frac{1}{x\left[\ln(x)\right]^2}\,dx$

A. $\int \frac{1}{x}\left[\ln(x)\right]^{-2}\,dx = \frac{\left[\ln(x)\right]^{-1}}{-1} + C$

$u = \ln(x)$

$u' = \frac{1}{x}$

Q. $y = \ln\left((x^2+1)^3(5x+1)^{10}\right)$. Find y'

A.

$y = \ln\left((x^2+1)^3\right) + \ln\left((5x+1)^{10}\right)$

$y = 3\ln(x^2+1) + 10\ln(5x+1)$

$y' = \frac{3(2x)}{x^2+1} + \frac{10(5)}{5x+1}$ ✓

3.10 Exam1-Review from previous semesters

Name NADIN ELSHIRBINI, ID 72731

Exam I: MTH 111, Spring 2018

Ayman Badawi Nadin El Shirbini

Points = $\frac{80}{80}$

QUESTION 1. a) **(3 points)** Are the points $q_1 = (1, 2, -2)$, $q_2 = (3, 3, 1)$, and $q_3 = (5, 4, 4)$ co-linear? Show the work

$\overrightarrow{Q_1Q_2} = \langle 2, 1, 3 \rangle$

$\overrightarrow{Q_1Q_3} = \langle 4, 2, 6 \rangle$

$$\overrightarrow{Q_1Q_2} \times \overrightarrow{Q_1Q_3} = \begin{vmatrix} i & j & k \\ 2 & 1 & 3 \\ 4 & 2 & 6 \end{vmatrix} = \left\langle \begin{vmatrix} 1 & 3 \\ 2 & 6 \end{vmatrix}, \begin{vmatrix} 2 & 3 \\ 4 & 6 \end{vmatrix}, \begin{vmatrix} 2 & 1 \\ 4 & 2 \end{vmatrix} \right\rangle = \langle 0, 0, 0 \rangle$$

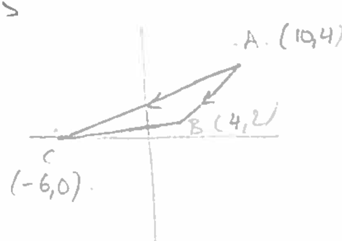
cross product is zero $\Rightarrow$ they are colinear

b) **(3 points)** Given $A = (10, 4)$, $B = (4, 2)$, and $C = (-6, 0)$ are the vertices of a triangle. Roughly, sketch the triangle ABC. Find the area of the triangle ABC.

$\overrightarrow{AB} = \langle -6, -2 \rangle$

$\overrightarrow{AC} = \langle -16, -4 \rangle$

$$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} i & j & k \\ -6 & -2 & 0 \\ -16 & -4 & 0 \end{vmatrix} = \langle 0, 0, -8 \rangle$$



$$A_{\Delta ABC} = \frac{1}{2} |\overrightarrow{AB} \times \overrightarrow{AC}| = \frac{1}{2}\sqrt{(-8)^2} = \boxed{4 \text{ units}^2}$$

c) **(3 points)** Find a vector F that is perpendicular to both vectors $V = \langle 2, -1, 4 \rangle$ and $W = \langle 0, 4, 2 \rangle$

$$\vec{F} = \vec{V} \times \vec{W} = \begin{vmatrix} i & j & k \\ 2 & -1 & 4 \\ 0 & 4 & 2 \end{vmatrix} = \boxed{\langle -18, -4, 8 \rangle}$$

d) **(2 points)** Let V, W as in (c). Find a vector F that is perpendicular to both V and W such that $|F| = 2$.(hint: Just think a little)

$$|F| = \sqrt{18^2 + 4^2 + 8^2} = 2\sqrt{101}$$

$$(2)\frac{1}{|F|} \cdot F = \frac{2}{2\sqrt{101}} \cdot F = \frac{1}{\sqrt{101}} \cdot F = \boxed{\frac{1}{\sqrt{101}} \langle -18, -4, 8 \rangle}$$

$$\boxed{F \left\langle \frac{-18}{\sqrt{101}}, \frac{-4}{\sqrt{101}}, \frac{8}{\sqrt{101}} \right\rangle}$$

(check if $|F| = 2$:

$$|F| = \sqrt{\left(\frac{-18}{\sqrt{101}}\right)^2 + \left(\frac{-4}{\sqrt{101}}\right)^2 + \left(\frac{8}{\sqrt{101}}\right)^2} = 2$$)

QUESTION 2. a) **(4 points)** Does the line $L_1 : x = 5t - 20, y = -t + 3, z = 3t - 27$ ($t \in R$) intersect the line $L_2 : x = -2w + 20, y = -4w - 5, z = 2w - 3$ ($w \in R$)? If yes find the intersection point Q.

$$L_1: \begin{cases} x = 5t - 20 \\ y = -t + 3 \\ z = 3t - 27 \end{cases} \qquad L_2: \begin{cases} x = -2w + 20 \\ y = -4w - 5 \\ z = 2w - 3 \end{cases}$$

$$5t - 20 = -2w + 20 \Rightarrow 5t + 2w = 40$$
$$-t + 3 = -4w - 5 \Rightarrow -t + 4w = -8$$
$$t = 8 \quad w = 0$$

check for z:

$z = 3t - 27 = 3(8) - 27 = -3$
$z = 2w - 3 = 2(0) - 3 = -3$
} they are equal $\Rightarrow$ L_1 and L_2 intersect

The point of intersection

$x = 2w + 20 = 2(0) + 20 = 20$
$y = -4w - 5 = -4(0) - 5 = -5$
$z = 2w - 3 = 2(0) - 3 = -3$

point of intersection is $(20, -5, -3)$

b) **(2 points)** Are the lines in (a) perpendicular? Explain Yes.

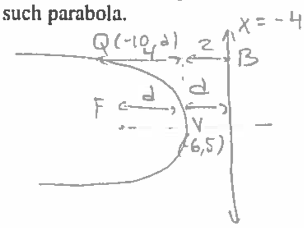
$D_1 = <5, -1, 3>$ $\quad D_2 = <-2, -4, 2>$

$D_1 \cdot D_2 = 5(-2) - 1(-4) + 3(2) = 0$

dot product $= 0 \Rightarrow$ They are perpendicular.

QUESTION 3. Given $x = -4$ is the directrix of of a parabola that has the point $(-6, 5)$ as its vertex point.

a) **(2 points)** Roughly, sketch such parabola.



$|d| = 2$

b) **(4 points)** Find the equation of the parabola

$$4d(x - x_0) = (y - y_0)^2$$
$$-4(2)(x + 6) = (y - 5)^2$$
$$\boxed{-8(x + 6) = (y - 5)^2}$$

c) **(2 points)** Find the focus of the parabola, say F.

$\boxed{F(-8, 5)}$

d) **(2 points)** Given $Q = (-10, b)$ is a point on the curve of the parabola. Find $|QF|$ (HINT: You should know how to do this QUICKLY!, you do not need the value of b)

$|QL| = |QB| = |QF| = \boxed{6}$

QUESTION 4. Given $y = x^2 - 6x - 1$ is an equation of a parabola.

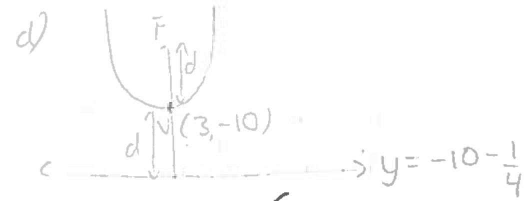
a)(**3 points**) Write the equation in the standard form.

$y = (x-3)^2 - 9 - 1$

$y = (x-3)^2 - 10$

$\boxed{(y+10) = (x-3)^2}$

$4d = 1 \Rightarrow d = 1/4$



b) (**2 points**) Find the equation of the directrix line.

$\boxed{y = -10 - \frac{1}{4} = -\frac{41}{4}}$

c)(**2 points**) Find the focus, say F

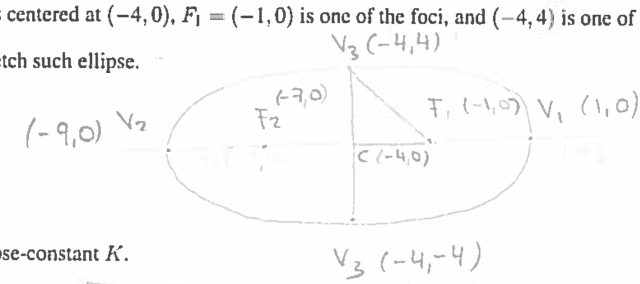
$\boxed{F(3, -10 + \frac{1}{4})} \rightarrow \boxed{F(3, -\frac{39}{4})}$

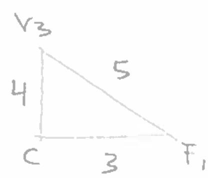
d)(**2 points**) Roughly, sketch the graph of such parabola.

(see picture)

QUESTION 5. An ellipse is centered at $(-4,0)$, $F_1 = (-1,0)$ is one of the foci, and $(-4,4)$ is one of the vertices.

(i) (**2 points**) Roughly, sketch such ellipse.





(ii) (**3 points**) Find the ellipse-constant K.

$|V_3F_1| = \frac{k}{2} = 5 \Rightarrow \boxed{K = 10}$

(iii) (**2 points**) Find the second foci of the ellipse.

$\boxed{F_2(-7,0)}$

(iv) (**3 points**) Find the remaining three vertices of the ellipse

$\boxed{V_3(-4,-4)}$

$\boxed{V_1(1,0)}$

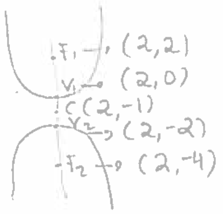
$\boxed{V_2(-9,0)}$

(v) (**3 points**) Find the equation of the ellipse.

$$\boxed{\frac{(x+4)^2}{25} + \frac{y^2}{16} = 1}$$

QUESTION 6. Consider the hyperbola $(y+1)^2 - \frac{(x-2)^2}{8} = 1.$

a) **(2 points)** Draw the hyperbola, roughly



$|CF_1| = \sqrt{1+8} = 3$

b) **(2 points)** Find the hyperbola-constant K.

$\left(\frac{K}{2}\right)^2 = 1$

$\frac{K}{2} = 1 \Rightarrow \boxed{K = 2}$ ✓

c)**(3 points)** Find the two vertices of the hyperbola.

$\boxed{V_1\ (2,0)}$

$\boxed{V_2\ (2,-2)}$ ✓

d) **(3 points)** Find the foci of the hyperbola.

$\boxed{F_1\ (2,2)}$

$\boxed{F_2\ (2,-4)}$ ✓

QUESTION 7. Given two lines $L_1 : x = t+1, y = 2t+4, z = -5t+3$ and $L_2 : x = 2w+7, y = 4w+16, z = -10w-27$.

(i) **(3 points)** Find the symmetric equation of L_1.

$\boxed{x-1 = \frac{y-4}{2} = \frac{-z+3}{5}}$ ✓

(ii) **(3 points)** Is D_1 parallel to D_2? (note that D_1 is the directional vector of L_1 and D_2 is the directional vector of L_2)
Show the work

$D_1 < 1,2,-5 >$
$D_2 < 2,4,-10 >$

$D_1 = c D_2$
$<1,2,-5> = c <2,4,-10>$
$c = \frac{1}{2}$ ✓

$D_1 = \frac{1}{2} D_2 \Rightarrow$ They are parallel

(iii) **(2 points)** Is L_1 parallel to L_2? Explain (show the work)

Take $t=0 \rightarrow (1,4,3)$
check if $(1,4,3) \in L_2$

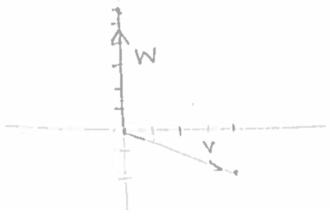
$1 = 2w+7 \Rightarrow w = -3$
$4 = 4w+16 \Rightarrow w = -3$
$3 = -10w-27 \Rightarrow w = -3$

$\Rightarrow$ it $\in$ to L_2. ✓

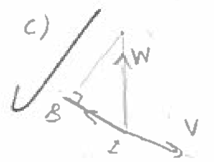
$\Rightarrow L_1$ and L_2 intersect and they are NOT parallel. They are collinear (same line/ on top of each other)

QUESTION 8. Let (0, 0) be the initial point of the two vectors $V = <4, -2>$, and $w = <0, 6>$.

a) **(2 points)** Draw V and W in the xy-plane.



$proj_W^V = \vec{1B}$



$proj_V^W = \vec{1B}$

b) **(2 points)** Use the picture that you draw in (a) in order to draw $Proj_W^V$

c)**(2 points)** Use the picture that you draw in (a) in order to draw $Proj_v^w$

d) **(4 points)** Find $Proj_w^v$ and find its length.

$$proj_W^V = \frac{V \cdot W}{|W|^2} \cdot W = \frac{-12}{36} \cdot W = -\frac{1}{3} <0, 6> = \boxed{<0, -2>}$$

$$\boxed{|proj_W^V| = \sqrt{2^2} = 2}$$

e)**(3 points)** Find the angle between V and W

$$\cos\theta = \frac{V \cdot W}{|V||W|} = \frac{-12}{(6)(2\sqrt{5})} = -\frac{\sqrt{5}}{5}$$

$$\theta = \cos^{-1}\left(-\frac{\sqrt{5}}{5}\right) = \boxed{116.565°}$$

Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: abadawi@aus.edu, www.ayman-badawi.com

Name Krstin Raed, ID 78656

MTH 111 Math for Architects Spring 2019, 1–6

Exam I: MTH 111, Spring 2019

$F = V \times W$

Ayman Badawi

Points = $\frac{87}{87}$

QUESTION 1. b) **(4 points)** Given $A = (6, 10)$, $B = (-7, 3)$, and $C = (-4, -2)$ are the vertices of a triangle. Find the area of the triangle ABC.

Area of the triangle ABC $= \frac{1}{2} |AB \times AC|$

$AB = \langle -13, -7 \rangle$ (B-A)

$AC = \langle -10, -12 \rangle$ (C-A)

$AB \times AC = \begin{vmatrix} i & j & k \\ -13 & -7 & 0 \\ -10 & -12 & 0 \end{vmatrix} = 0i - 0j + \underline{86k} = \underline{86}$

Area of $\Delta ABC = \frac{1}{2}\, 86 = \boxed{43 \text{ units}^2}$

c) **(3 points)** Find a vector F that is perpendicular to both vectors $V =< 2, 6, -3 >$ and $W =< 5, -4, 1 >$ such that $|F| = 111$.

$F = V \times W = \begin{vmatrix} i & j & k \\ 2 & 6 & -3 \\ 5 & -4 & 1 \end{vmatrix} = -6i - 17j - 38k$

$|F| = 111 \quad = \frac{111}{|F|} F$

$= \boxed{\frac{111}{42} \langle -6, -17, -38 \rangle}$

QUESTION 2. a) **(4 points)** The line $L_1 : x = -2t - 3, y = -3t + 3, z = 4t - 2$ $(t \in R)$ intersects the line $L_2 : x = 2w - 13, y = 4w - 15, z = 4w - 6$ $(w \in R)$ in a point Q. Find Q.

$L_1 : x = -2t - 3$
$y = -3t + 3$
$z = 4t - 2$

$L_2 : x = 2w - 13$
$y = 4w - 15$
$z = 4w - 6$

use substitution method

find pt of intersection: $-2t - 3 = 2w - 13$

$\frac{-2t}{-2} = \frac{2w - 13 + 3}{-2}$

$\boxed{t = -w + 5}$

sub in second equ

$-3(-w+5)+3 = 4w-15$

$3w - 15 + 3 = 4w - 15$

$4w - 3w = -15 + 15 + 3$

$\boxed{1w = 3}$

$t = -3 + 5$

$\boxed{t = 2}$

- now sub in each line to get intersection pt

$-2(2) - 3 = 2(3) - 13$

$\boxed{-7 = -7}$

$-3(2) + 3 = 4(3) - 15$

$\boxed{-3 = -3}$

$4(2) - 2 = 4(3) - 6$

$\boxed{6 = 6}$

$\boxed{\text{Intersection pt} = Q = (-7, -3, 6)}$

b) **(2 points)** Are the lines in (a) perpendicular? Explain

$D_1 = \langle -2, -3, 4 \rangle$

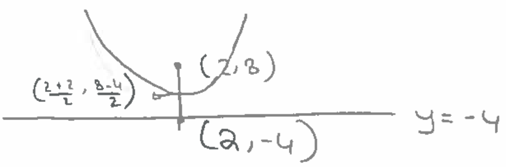
$D_2 = \langle 2, 4, 4 \rangle$

$D_1 \cdot D_2 = (-2 \times 2) + (-3 \times 4) + (4 \times 4)$
$= 0$

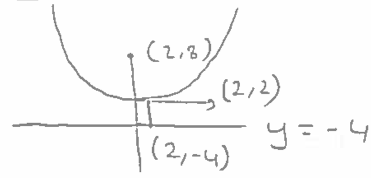
So they are perpendicular because their dot product is zero & they intersect

QUESTION 3. Given $y = -4$ is the directrix of a parabola that has the point $F = (2, 8)$ as its focus point.

a) **(2 points)** Roughly, sketch such parabola.



2



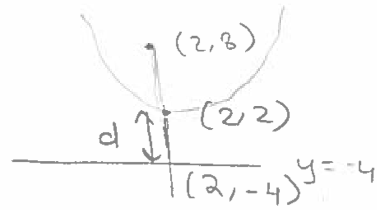
d = 6 & its up

4

b)**(4 points)** Find the equation of the parabola

$$4d(y-2) = (x-2)^2$$
$$4(6)(y-2) = (x-2)^2$$
$$24(y-2) = (x-2)^2$$

$\boxed{d=6}$



$d = (-4-2$
-6

c) **(2 points)** Find the vertex of the parabola, say V.

$$V = (2,2)$$

QUESTION 4. Given $y = 4x^2 + 24x - 3$ is an equation of a parabola.

a)**(3 points)** Write the equation in the standard form.

$$y = 4x^2 + 24x - 3$$
$$y = 4(x^2 + 6x) - 3$$
$$y = 4((x+3)^2 - 9) - 3$$
$$y = 4(x+3)^2 - 36 - 3$$
$$y = 4(x+3)^2 - 39$$
$$\frac{1}{4}(y+39) = \frac{4(x+3)^2}{4}$$
$$\frac{1}{4}(y+39) = (x+3)^2$$

$4d = \frac{1}{4}$

$d = \frac{1}{4 \times 4}$

$\boxed{d = \frac{1}{16}}$

so +

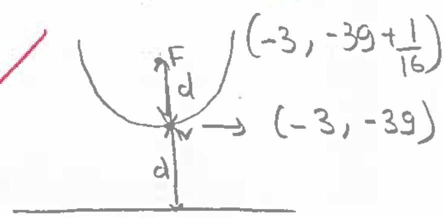
b) **(2 points)** Find the equation of the directrix line.

$$y = -\frac{625}{16}$$

c)**(2 points)** Find the focus, say F

$$F = (-3, -39 + \tfrac{1}{16}) = (-3, -\tfrac{623}{16})$$

d)**(2 points)** Roughly, sketch the graph of such parabola.

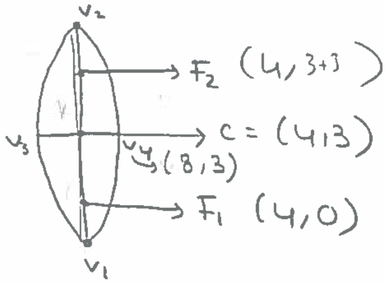


$(-3, -39 - \frac{1}{16})$

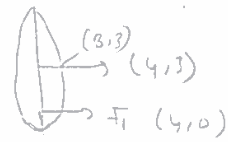
$(-3, -\frac{625}{16})$

QUESTION 5. An ellipse is centered at $(4,3)$, $F_1 = (4,0)$ is one of the foci, and $(8,3)$ is one of the vertices.

(i) **(2 points)** Roughly, sketch such ellipse.



x does not change



$CF^2 = \left(\frac{k}{2}\right)^2 - b^2$

$3^2 = \left(\frac{k}{2}\right)^2 - 4^2$

$25 = \left(\frac{k}{2}\right)^2$

$\boxed{CF = 3}$

$\boxed{b = 4}$

(ii) **(3 points)** Find the ellipse-constant K.

$$CF^2 = \left(\frac{k}{2}\right)^2 - b^2$$

$$3^2 = \left(\frac{k}{2}\right)^2 - 4^2$$

$$\boxed{k = 10}$$

(iii) **(2 points)** Find the second foci of the ellipse.

$$F_2 = (4, 3+3)$$
$$(4, 6)$$

(iv) **(3 points)** Find the remaining three vertices of the ellipse

$$V_1 = \left(4, 3 - \frac{10}{2}\right) \quad \boxed{(4, -2)}$$

$$V_2 \left(4, 3 + \frac{10}{2}\right) \quad (4, 8)$$

$$V_3 \ (0, 3)$$

(v) **(3 points)** Find the equation of the ellipse.

$$\frac{(y-3)^2}{\left(\frac{10}{2}\right)^2} + \frac{(x-4)^2}{4^2} = 1$$

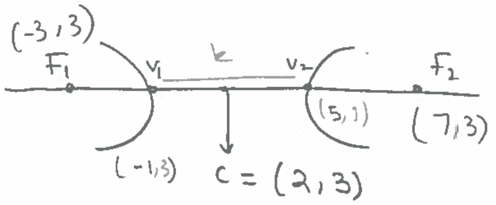
$$\frac{(y-3)^2}{25} + \frac{(x-4)^2}{16} = 1$$

QUESTION 6. Consider the hyperbola $\frac{(x-2)^2}{9} - \frac{(y-3)^2}{16} = 1$.

$(\frac{k}{2})^2$ b^2

a) **(2 points)** Draw the hyperbola, roughly.

under x so right left



b) **(2 points)** Find the hyperbola-constant K.

$(\frac{k}{2})^2 = 9$

$\frac{k}{2} = \sqrt{9}$

$k = 3 \times 2$

$\boxed{k = 6}$

c) **(3 points)** Find the two vertices of the hyperbola.

$V_2 = (2+3, 3)$
$(5,3)$

$V_1 = (2-3, 3)$
$(-1,3)$

d) **(3 points)** Find the foci of the hyperbola.

$CF^2 = (\frac{k}{2})^2 + b^2$

$F_1 = (2-5, 3)$ $(-3,3)$

$CF^2 = 9 + 16$

$F_2 = (2+5, 3)$ $(7,3)$

$= 25$

$\boxed{CF = 5}$

QUESTION 7. (4 points) Given two lines $L_1: x = t+1, y = 2t+4, z = -5t+3$ ($t \in R$) and $L_2: x = 2w-1, y = 4w+1, z = -10w+13$ ($w \in R$). Is L_1 parallel to L_2? Explain (show the work)

· 2 lines are // If they have cst & they do not intersect

$L_1: x = t+1$
$y = 2t+4$
$z = -5t+3$

$L_2: x = 2w-1$
$y = 4w+1$
$z = -10w+13$

take $t = 0$

$1 = 2w - 1$
$4 = 4w + 1$
$3 = -10w + 13$

$2w = 2$, $w = 1$
$4w = 4-1$, $w = \frac{3}{4}$
$10w = 13-3$, $10w = 10$, $w = 1$

$D_1 \langle 1, 2, -5 \rangle$

$D_2 \langle 2, 4, -10 \rangle$

$1 = c2$
$2 = c4$
$-5 = c(-10)$

$c = \frac{1}{2}$
$c = \frac{1}{2}$
$c = \frac{1}{2}$

they have a cst

$2w = 2$
$\boxed{w = 1}$
$4 - 1 = 4w$
$3 = 4w$
$\boxed{w = \frac{3}{4}}$
$3 - 13 = -10w$
$-10 = -10w$

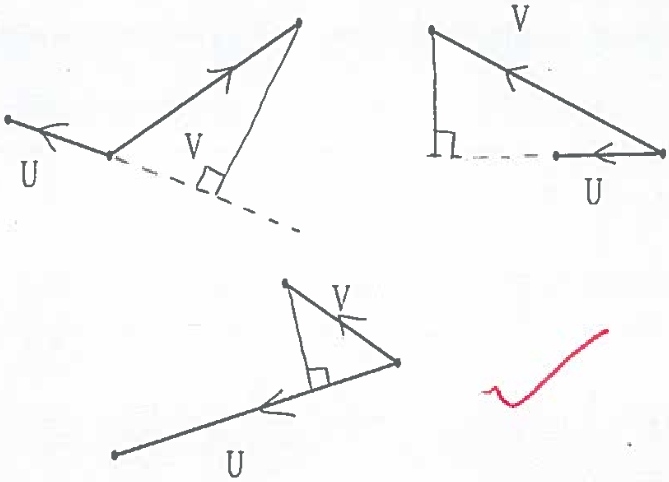
they do not intersect

$L_1 // L_2$

QUESTION 8. (6 points)

proj_U^V

Stare at the below. Then find Projection of V over U



QUESTION 9. (4 points) Find the equation of the plane that contains the points $Q_1 = (4,4,0)$. $Q_2 = (0,2,6)$ and $Q_3 = (4,0,8)$.

$N = \overrightarrow{Q_1Q_2} \times \overrightarrow{Q_1Q_3}$

$\langle -4,-2,6\rangle \times \langle 0,-4,8\rangle$

$$\begin{vmatrix} i & j & k \\ -4 & -2 & 6 \\ 0 & -4 & 8 \end{vmatrix} = 8i + 32j + 16k$$

$\langle 8, 32, 16\rangle$

choose a pt

$Q_1 = (4,4,0)$

$8(x-4)+32(y-4)+16(z-0)=0$

$\boxed{8(x-4)+32(y-4)+16z=0}$

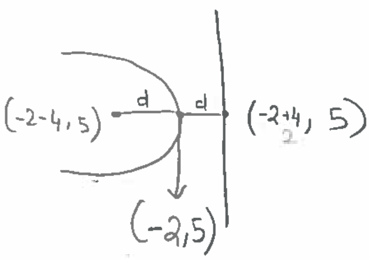
QUESTION 10. (6 points) Consider the parabola $-16(x+2) = (y-5)^2$.

(i) Sketch the parabola

$4d = -16$

$d = -4$

& before x so its left



(ii) Find the equation of the directrix line

$X = -2+4$

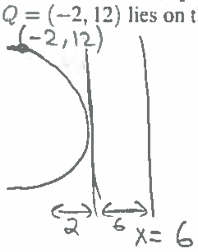
$\boxed{X = 2}$

(iii) Find the focus point.

Focus $= (-2-4, 5)$

$(-6, 5)$

QUESTION 11. **(4 points)** Given that $x = 6$ is the directrix line of a parabola that has F as its focus point. If the point $Q = (-2, 12)$ lies on the parabola. Find $|QF|$ (i.e., the distance between Q and F).

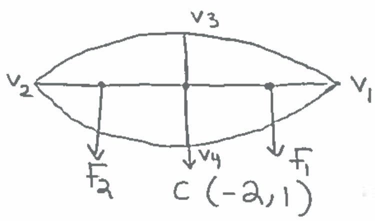


$|QF| = |QL| = 8$

QUESTION 12. **(6 points)** Consider the ellipse $\dfrac{(y-1)^2}{9} + \dfrac{(x+2)^2}{25} = 1.$

$9 \to b^2$; $25 \to$ big # so its $\left(\frac{k}{2}\right)^2$ so the shape is

(i) Sketch (roughly)



(ii) Find the foci of the ellipse

$CF^2 = \left(\frac{k}{2}\right)^2 - b^2$
$= 25 - 9$
$= 16$

$CF^2 = 16$ so $\boxed{CF = 4}$

so $F_1\ (-2+4,\ 1)$
$(2, 1)$

$F_2\ (-2-4,\ 1)$
$(-6, 1)$

(iii) Find all four vertices of the ellipse.

$\left(\frac{k}{2}\right)^2 = 25$ $\quad \boxed{\pm \frac{k}{2}}$

$\boxed{\frac{k}{2} = 5}$ $\quad \boxed{\pm b}$

$b^2 = 9 \quad \boxed{b = 3}$

$V_1 = (-2+5,\ 1)$
$(3, 1)$

$V_2 = (-2-5,\ 1)$
$(-7, 1)$

$V_3 = (-2,\ 1+3)$ $\quad (-2, 4)$

$V_4 = (-2,\ 1-3)$ $\quad (-2, -2)$

QUESTION 13. **(4 points)** Given $Q = (1, 6, 4)$ is not on the line $L : x = t + 1, y = 2t + 4, z = -5t + 3$ $(t \in R)$. Find $|QL|$.

$D = \langle 1, 2, -5 \rangle$

$I = \langle 1, 4, 3 \rangle$

$\underset{Q-I}{IQ} = \langle 0, 2, 1 \rangle$

$$|QL| = \frac{|D \times IQ|}{|D|} = \frac{\sqrt{12^2 + 1^2 + 2^2}}{\sqrt{1^2 + 2^2 + 5^2}} = \frac{\sqrt{149}}{\sqrt{30}}$$

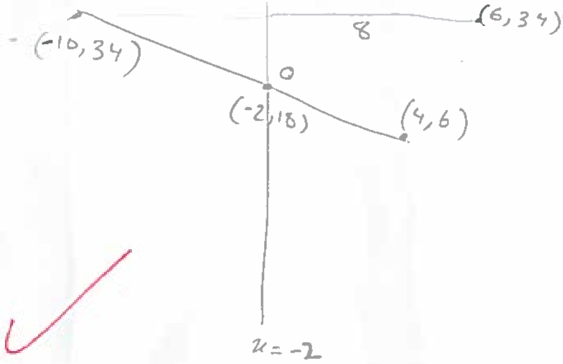
$$IQ \times D = \begin{vmatrix} i & j & k \\ 0 & 2 & 1 \\ 1 & 2 & -5 \end{vmatrix} = -12i + 1j - 2k$$

Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: abadawi@aus.edu, www.ayman-badawi.com

3.11 Exam2-Review from previous semesters

QUESTION 6. (5 points). Let $H = (4, 6)$, $F = (6, 34)$. Find a point Q on the line $x = -2$ such that $|HQ| + |FQ|$ is minimum.



$y = mx + b$

$m = \frac{6-34}{4+10} = -2$

$6 = -2(4) + b$

$b = 14$

$y = -2x + 14$

$y = -2(-2) + 14$
$= 18$

$Q = (-2, 18)$

QUESTION 7. (4 points). For what values of x does the tangent line to the curve $y = ln(4x+1) + 7x + 2$ have slope equal 8?

$y' = 8$

$y' = \frac{4}{4x+1} + 7 = 8$

$\frac{4}{4x+1} = 1$

$4 = 4x + 1$

$4x = 4 - 1$

$x = 3/4$

check $\frac{4}{4(\frac{3}{4})+1} + 7 =$

$1 + 7 = 8$ //

the T line has slope 8 at $x = \frac{3}{4}$

QUESTION 8. (6 points). The plane $P_1 : x + 2y - 3z = 2$ intersects the plane $P_2 : -x + 5y + z = 19$ in a line L. Find a parametric equations of L.

① → $N_1 \times N_2 = D$

$N_1 = \langle 1, 2, -3 \rangle$

$N_2 = \langle -1, 5, 1 \rangle$

$D = (2+15)i - (1-3)j + (5+2)k$
$= \langle 17, 2, 7 \rangle$

② → $z = 0$

$x + 2y = 2$

$-x + 5y = 19$

$x = \frac{\begin{vmatrix} 2 & 2 \\ 19 & 5 \end{vmatrix}}{\begin{vmatrix} 1 & 2 \\ -1 & 5 \end{vmatrix}}$ $\quad y = \frac{\begin{vmatrix} 1 & 2 \\ -1 & 19 \end{vmatrix}}{\begin{vmatrix} 1 & 2 \\ -1 & 5 \end{vmatrix}}$

$x = \frac{-28}{7}$ $\quad y = \frac{21}{7}$

$x = -4$ $\quad y = 3$

③ → $(-4, 3, 0)$

$D = \langle 17, 2, 7 \rangle$

$L: \left.\begin{aligned} x &= 17t - 4 \\ y &= 2t + 3 \\ z &= 7t \end{aligned}\right\} t \in \mathbb{R}$

QUESTION 9. (5 points). Can we draw the entire line $L: x = 2t, y = -3t + 1, z = 11t + 4$ inside the plane $2x - 6y - 2z = 20$? EXPLAIN

$N_{plane} \cdot D_{line}$ must $= 0$

$N = \langle 2, -6, -2 \rangle$

$D = \langle 2, -3, 11 \rangle$

$N \cdot D = 4 + 18 - 22$
$= 0$ //

NO

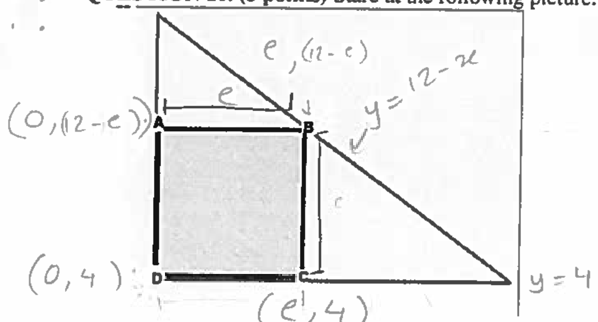
yes the line can be entirely drawn on the plane because the dot product of the normal and directional vector is 0

take a point on L and check if the point lies in the plane or not

2

Haya

QUESTION 10. (8 points) Stare at the following picture.



We want to construct a rectangle ABCD of largest area as in the picture above. Note that A and D lie on the y-axis, D and C lie on the line y = 4 (note that y = 4 intersects the y-axis at D) , and B lies on the line y = 12 - x. Find |DC| and |BC|.

$|BC| = (12-e) - 4$

$|DC| = e$

$A = |BC| \cdot |DC|$

$= [(12-e)-4] \cdot e$

$= (-e+8)e$

$= -e^2 + 8e$

$A' = -2e + 8$

① → $-2e + 8 = 0$

$e = 4$

② → $|BC| = (12-4) - 4$

$= 8 - 4$

$= 4$ units

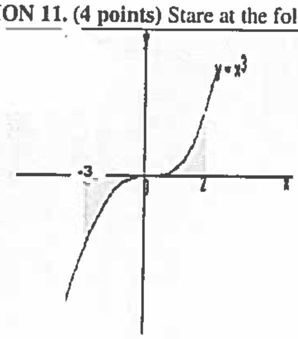
$|DC| = e$

$= 4$ units

Area $= 4 \times 4$

$= 16$ units^2

QUESTION 11. (4 points) Stare at the following picture.



Find the area of the shaded region. Note that $y = f(x) = x^3$ and x is between -3 and 2.

$$A = \left[\int_{-3}^{0} x^3\, dx\right] + \int_{0}^{2} x^3\, dx$$

$$= \left[\int_{-3}^{0} \frac{1}{4}x^4\right] + \int_{0}^{2} \frac{1}{4}x^4$$

$$= \left[\left[\frac{1}{4}0^4\right] - \left[\frac{1}{4}(-3)^4\right]\right] + \left[\left[\frac{1}{4}(2)^4\right] - \left[\frac{1}{4}(0)^4\right]\right]$$

$$= [0 + 20.25] + [4 - 0]$$

$$= 24.25 \text{ units}^2$$

Exam II: MTH 111, Fall 2017

Ayman Badawi

Points = $\frac{47}{47}$

QUESTION 1. (8 points) Find y' and DO NOT SIMPLIFY

(i) $y = 2x^3 + 10x - 7$

$y' = 6x^2 + 10.$

(ii) $y = \sqrt{x} + (3x-1)^{11}$ $\rightarrow y = x^{1/2} + (3x-1)^{11}$

$y' = \frac{1}{2\sqrt{x}} + 11(3x-1)^{10}(3).$

$y' = \frac{1}{2\sqrt{x}} + 33(3x-1)^{10}.$

(iii) $y = \frac{4}{x^2} + \frac{2}{\sqrt{x^2+4}}$

$y = 4x^{-2} + 2(x^2+4)^{-1/2}$

$y' = -8x^{-3} + \cancel{2}\left(-\frac{1}{\cancel{2}}\right)(x^2+4)^{-3/2}(2x)$

$y' = -\frac{8}{x^3} - 2x(x^2+4)^{-3/2}$

(iv) Given $y = k(4x^2 - x)$ such that $k'(3) = -7$. Find $y'(1)$ (i.e., evaluate y' when $x = 1$.)

$y' = (8x-1)k'(4x^2-x)$

$y'|_{x=1} = 7k'(3) = 7(-7) = -49.$

QUESTION 2. (i) **(3 points)** Can we draw the vector $v = < 3, -5, 2 >$ inside the plane $x - 4y - 11z = 7$? explain

$V = <3, -5, 2>$

$N = <1, -4, -11>$

$N \cdot V = 3(1) - 5(-4) + 2(-11)$

$N \cdot V = 3 + 20 - 22 = 1 \neq 0$

NO. The two vectors are not perpendicular, hence v can't be drawn inside the plane.

(ii) **(4 points)** Given $N = < 4, 6, 2 >$ is perpendicular to the plane P and the point $(4, 1, 1)$ lies inside the plane P. Find the equation of the plane P.

$N = <4,6,2>$
$<a,b,c>$

$Q(4,1,1)$
$Q(x_0, y_0, z_0)$

$a(x-x_0) + b(y-y_0) + c(z-z_0) = 0$

$4(x-4) + 6(y-1) + 2(z-1) = 0$

$4x - 16 + 6y - 6 + 2z - 2 = 0$

$\boxed{4x + 6y + 2z = 24}$

(iii) **(6 points)** Find the equation of the plane that contains the points $Q_1 = (1, 1, 4)$, $Q_2 = (2, 3, 6)$ and $Q_3 = (1, 1, 8)$.

$Q_1(1,1,4)$
$Q_2(2,3,6)$
$Q_3(1,1,8)$

$\overrightarrow{Q_1Q_2} = <1, 2, 2>$

$\overrightarrow{Q_1Q_3} = <0, 0, 4>$

$\vec{N} = \overrightarrow{Q_1Q_2} \times \overrightarrow{Q_1Q_3} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 2 \\ 0 & 0 & 4 \end{vmatrix}$

$\vec{N} = 8\hat{i} - 4\hat{j} + 0\hat{k}$

$\boxed{\vec{N} = <8, -4, 0>}$

$\cancel{8(x-1) - 4(y+4) + 0(z)}$

$8(x-1) - 4(y-1) + 0(z-4) = 0$

$8x - 8 - 4y + 4 = 0$

$\boxed{8x - 4y = 4}$

$\boxed{2x - y = 1}$

QUESTION 3. (i) (4 points) The line $L : x = 2w, y = -w+1, z = 3$ intersects the plane $4x+7y+z = 12$ in a point Q. Find Q.

$$L: \begin{cases} x = 2w \\ y = -w+1 \; ; \; w \in \mathbb{R} \\ z = 3 \end{cases}$$

$P: 4x + 7y + z = 12$

$4(2w) + 7(-w+1) + 3 = 12$

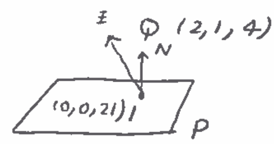
$8w - 7w + 7 + 3 = 12$

$w + 10 = 12$

$\boxed{w = 2}$ → The plane and the line intersect when $w = 2$

$\Rightarrow \boxed{Q(4, -1, 3)}$

(ii) (4 points) Find the distance between $Q = (2, 1, 4)$ and the plane $2x - 2y + z = 21$.



$I\ (0, 0, 21)$

$Q\ (2, 1, 4)$

$\vec{IQ} = \langle 2, 1, -17 \rangle$

$N = \langle 2, -2, 1 \rangle$

$$d = \frac{|\vec{IQ} \cdot N|}{|N|} = \frac{|2(2) + 1(-2) + 1(-17)|}{\sqrt{4+4+1}}$$

$$d = \frac{15}{\sqrt{9}} = \frac{15}{3} = 5 \text{ units}$$

(iii) (6 points) The two planes $P_1 : x+y+z = 2$ and $P_2 : -x+y-z = 6$ intersects in a line L. Find a parametric equations of L.

$N_1 = \langle 1, 1, 1 \rangle$

$N_2 = \langle -1, 1, -1 \rangle$

$$\vec{D} = \vec{N_1} \times \vec{N_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ -1 & 1 & -1 \end{vmatrix}$$

$$\vec{D} = -2\hat{i} + 0\hat{j} + 2\hat{k}$$

→ Let $z = 0$; find x and y :

$$\begin{cases} x + y = 2 \\ -x + y = 6 \end{cases}$$

$2y = 8$

$\boxed{y = 4}$

$x + 4 = 2$

$x = 2 - 4$

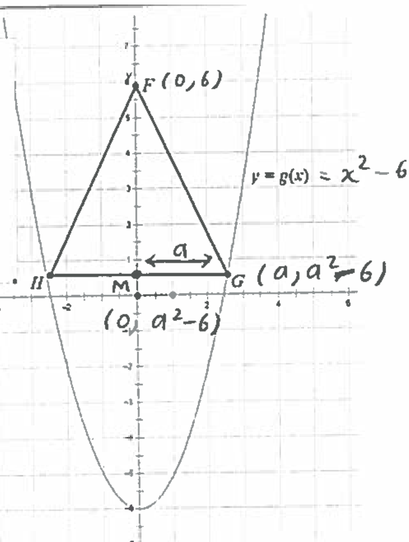
$\boxed{x = -2}$

The point is $(-2, 4, 0)$ and $D = \langle -2, 0, 2 \rangle$

* Parametric Eqns :

$$L: \begin{cases} x = -2 - 2t \\ y = 4 \qquad ; \; t \in \mathbb{R} \\ z = 2t \end{cases}$$

Given the curve of y = g(x) = x^2 - 6. The point F = (0, 6). We want to find two points G and H on the curve of g(x) so that the area of the triangle FGH is maximum. Find the length of the base and the length of the height of such triangle (note that G, H) lies on the same horizontal line.



QUESTION 4. (6 points)

Base $= \overline{GH} = 2a$; where $a > 0$.

Height $= \overline{FM} = 6 - (a^2 - 6)$

$\overline{FM} = 6 + 6 - a^2 = 12 - a^2$

$A\Delta = \frac{1}{2} bh$

$A\Delta = \frac{1}{2}(2a)(12 - a^2) = 12a - a^3$

$A' = 12 - 3a^2$

$A' = 0$

$12 = 3a^2$

$a^2 = 4 \rightarrow a = \pm 2$

$\boxed{a = +2}$ because $a > 0$.

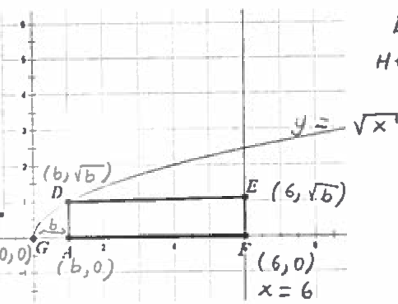
$A'' = -6a$

$A''|_{a=2} = -12 < 0 \rightarrow \cap$ curve MAXIMU when $\boxed{a = 2}$

Base $= \overline{GH} = 2(2) = 4$

Height $= \overline{FM} = 12 - 4 = 8$

given the curve of f(x) = sqrt(x). The vertical line x = 6. We want to find a point D on the curve of f(x), a point A on the x-axis, two points F and E on the vertical line x = 6 so that the area of the rectangle ADEF is maximum. Find the length (AF) and the width (AD) of such rectangle. Note that D, E are on the same horizontal line and A, F on the same horizontal line as well. Note G = (0, 0) and if |GA| = b, then |AF| = 6 - b



$A(b, 0)$, $F(6, 0)$

$D(b, \sqrt{b})$, $E(6, \sqrt{b})$

QUESTION 5. (6 points)

width $= \overline{AF} = 6 - b$

$\overline{AD} =$ ~~height~~ length $= \sqrt{b}$

$A_{\square} = l \times w$

$A_{\square} = \sqrt{b}(6 - b) = b^{1/2}(6 - b)$

$A_{\square} = 6b^{1/2} - b^{3/2}$

$A' = \frac{6}{2\sqrt{b}} - \frac{3\sqrt{b}}{2}$

$A' = \frac{3}{\sqrt{b}} - \frac{3\sqrt{b}}{2}$

$A' = \frac{(2)3 - 3b}{2\sqrt{b}}$

$A' = \frac{6 - 3b}{2\sqrt{b}}$

$A' = 0.$

$6 - 3b = 0$

$3b = 6$

$\boxed{b = 2}$

CHECK A'':

$A'' = \frac{-3(b^{-3/2})}{2} - \frac{3}{4\sqrt{b}}$

$A''|_{b=2} < 0$ $\cap$ MAX. when $\boxed{b = 2}$

$\overline{AF} =$ width $= 6 - 2 = 4$.

$\overline{AD} =$ length $= \sqrt{2}$.

Faculty information

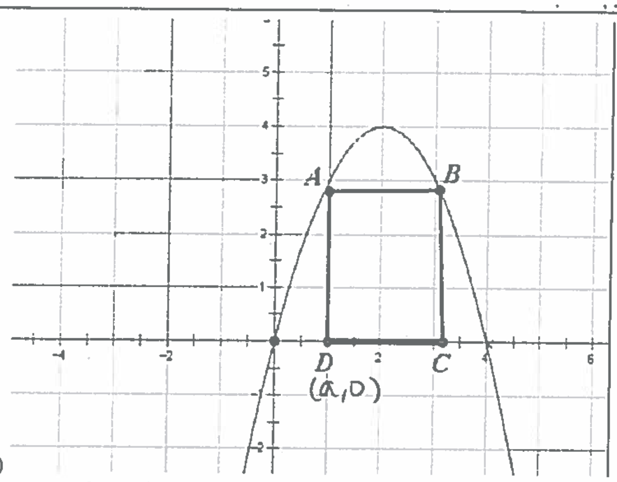
Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: abadawi@aus.edu, www.ayman-badawi.com

Side Note:

x		$+2$	
$f'(x)$	$+$	0	$-$
$f(x)$	$\nearrow$		$\searrow$

The curve Max

$\times$ Area $\square = 4\sqrt{2}$ units^2.



$Y = -x^2 + 4x$

$D(a,0) \to A = (a, -a^2+4a)$

$L = -a^2+4a$

$|OD| = |CF| = a$

$|CD| = W = 4-2a$

QUESTION 9. (8 points)

We want to construct a rectangle ABCD (see picture) of maximum area between the x-axis and the curve $y = -x^2 + 4x$. Find the length and the width of such rectangle. (Hint: Note that the curve intersects x-axis at $x = 0$ and at $x = 4$. Let O be the origin (0, 0) and F be (4, 0). Then |OD| = |CF|)

$A = W \times L$

$L = -a^2+4a$, $W = 4-2a$ $\to$ $A = (4-2a)(-a^2+4a)$

$A = -4a^2 + 16a + 2a^3 - 8a^2 = 2a^3 - 12a^2 + 16a$

$A' = 6a^2 - 24a + 16 = 0 \to 2(3a^2 - 12a + 8) = 0$

$\to 3a^2 - 12a + 8 = 0 \to a = \dfrac{12 \pm \sqrt{48}}{2(3)}$

$A'' = 12a - 24 \to a = \dfrac{12+\sqrt{48}}{6} \to A'' > 0$

$a = \dfrac{12-\sqrt{48}}{6} \to A'' < 0 \to$ Area max when $a = \dfrac{12-\sqrt{48}}{6}$

$\boxed{L = \dfrac{8}{3}}$, $\boxed{W = \dfrac{4\sqrt{3}}{3}}$ $\to$ $\boxed{A = \dfrac{32\sqrt{3}}{9} \text{ units}^2}$

Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: abadawi@aus.edu, www.ayman-badawi.com

Name Haya Sujaa, ID g00082558

Exam II: MTH 111, Spring 2018

Ayman Badawi

Points = $\frac{62}{62}$

QUESTION 1. (12 points) Find y' and DO NOT SIMPLIFY

(i) $y = 4e^{(2x^2-4x)} + 2x - 5$

$y' = 4e^{(2x^2-4x)} \cdot (4x-4) + 2$

(ii) $y = (5x^2 + 3x)\sqrt{5x+10}$

$y = (5x^2+3)(5x+10)^{1/2}$

$y' = \left[(5x^2+3) \cdot \frac{1}{2}(5x+10)^{-1/2} \cdot 5\right] + \left[(5x+10)^{1/2} \cdot (10x)\right]$

(iii) $y = ln[(2x^5 + 4x^3 - 3x)(2x+7)^5]$

$y = \ln(2x^5+4x^3-3x) + \ln(2x+7)^5$

$y' = \frac{10x^4 + 12x^2 - 3}{2x^5+4x^3-3x} + \frac{10}{2x+7}$

(iv) $y = 3(e^{(3x+2)} + 7x^4 + 5x + 2)^4$

$y' = 12(e^{(3x+2)} + 7x^4 + 5x + 2)^3 \cdot (3e^{(3x+2)} + 28x^3 + 5)$

 Haya

QUESTION 2. (i) **(6 points)** Does the line $L: x = 2t+1, y = 5t-1, z = -2t+3$ lie entirely inside the plane $x+2y+z = 23$? If not, does it intersect the plane? If yes, then find the intersection point.

$L: \left.\begin{array}{l} x = 2t+1 \\ y = 5t-1 \\ z = -2t+3 \end{array}\right\} t \in \mathbb{R}$

$P \Rightarrow x + 2y + z = 23$

it doesn't lie interiorly on the plane but intersects it at (5.2, 9.5, −1.2)

$P(L) \Rightarrow (2t+1) + 2(5t-1) + (-2t+3) = 23 \leftarrow 1$

$\cancel{2t} + 1 + 10t - 2 - \cancel{2t} + 3 = 23$

$10t + 2 = 23$

$10t = 21$

$t = 21/10$

$= 2.1$

$x = 2(2.1) + 1 = 5.2$

$2) \rightarrow y = 5(2.1) - 1 = 9.5$

$z = -2(2.1) + 3 = -1.2$

$Q = (5.2, 9.5, -1.2)$

the point of intersection is $(5.2, 9.5, -1.2) \leftarrow 3$

(ii) **(4 points)** Given $N = <-2, 3, 2>$ is perpendicular to the plane P and the point $(-1, 4, 2)$ lies inside the plane P. Find the equation of the plane P.

$N_x(x - P_x) + N_y(y - P_y) + N_z(z - P_z) = 0$

$-2(x+1) + 3(y-4) + 2(z-2) = 0 \Leftrightarrow$ plane.

(iii) **(4 points)** Find the distance between $Q = (10, 10, 33)$ and the plane $P: -2x + 2y - 5z = 21$.

$D = \frac{|P(Q)|}{|N|} \quad \frac{|-2(10) + 2(10) - 5(33) - 21|}{\sqrt{4+4+25}} = \frac{186}{\sqrt{33}}$ units.

(iv) **(6 points)** The two planes $P_1: x + 4y + z = 10$ and $P_2: -x + 2y - z = 8$ intersects in a line L. Find a parametric equations of L.

$N_1 \times N_2 = D$

$N_1 = \langle 1, 4, 1 \rangle$

$N_2 = \langle -1, 2, -1 \rangle$

$\begin{array}{ccc} i & j & k \\ 1 & 4 & 1 \\ -1 & 2 & -1 \end{array}$

$(-4-2)i - (-1+1)j + (2+4)k$

$1 \rightarrow D = \langle -6, 0, 6 \rangle$

let $z = 0$

$-x + 2y = 8$

$x + 4y = 10$

$x = -2$

$y = 3$

$z = 0$

$2 \rightarrow Q = (-2, 3, 0)$

$D = \langle -6, 0, 6 \rangle$

$3 \rightarrow L: \left.\begin{array}{l} x = -6t - 2 \\ y = 3 \\ z = 6t \end{array}\right\} t \in \mathbb{R}$

(v) (4 points) Can we draw the vector $V =< 1, -2, -6 >$ inside $P : 5x + 7y - 3z = 19$? explain

$V \cdot N$ must $= 0$

$\langle 1, -2, -6 \rangle$
$\langle 5, 7, -3 \rangle$

$V \cdot N = 5 + -14 + 18 = 9$

$\therefore$ NO you cannot draw V on the plane because the Dot product of V and the Normal is not 0

QUESTION 3. (7 points) Let $f(x) = e^{(x^2+2x+1)} + 3$.

(i) For what values of x does $f(x)$ increase?

$f'(x) = e^{(x^2+2x+1)} \cdot (2x+2)$

$e^{(x^2+2x+1)} \cdot (2x+2) = 0$

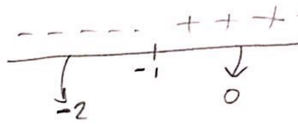
$\frac{e^{(x^2+2x+1)} \cdot (2x+2)}{(2x+2)} = \frac{0}{(2x+2)}$

$e^{(x^2+2x+1)} = 0$

$\ln e^{(x^2+2x+1)} = 0$

$x^2 + 2x + 1 = 0$

1 → $x = -1$ ← critical value



← 2

$f'(-2) = -$

$f'(0) = +$

$\therefore$ $f(x)$ increases from ← 3

$(-1, +\infty)$

(ii) For what values of x does $f(x)$ decrease?

$f(x)$ is decreasing from $(-\infty, -1)$

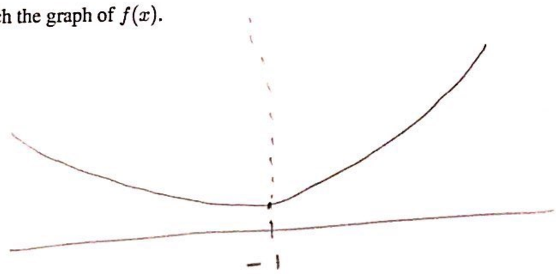
(iii) Find all local minimum, maximum points of $f(x)$ (just find the x-values where local min. and local max exist).

[No local or absolute maximum]

[local and absolute minimum at $x = -1$
point $(-1, 4)$]

$f(-1) = 4$

(iv) Roughly, sketch the graph of $f(x)$.



 Haya

QUESTION 4. (5 points) Let $f(x) = ln(5x-4)+4$. Find the equation of the tangent line to the curve of $f(x)$ at $x = 1$.

$f(1) = \ln(5-4)+4$
$= 4$
point $= (1, 4)$

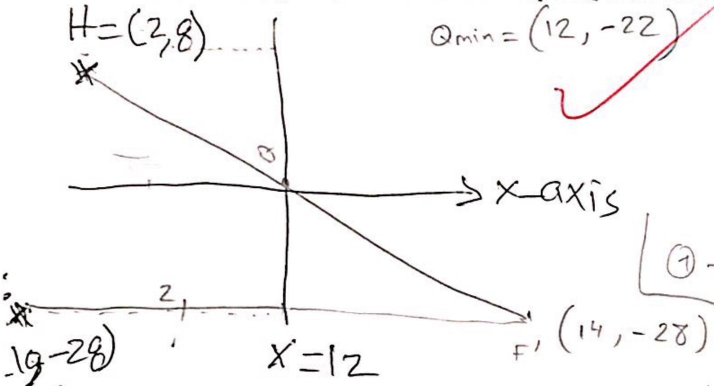
$f'(x) = \frac{5}{5x-4}$

① → $f'(1) = m = \frac{5}{1}$

$y = 5x + b$
$4 = 5(1) + b$
② → $b = 4 - 5$
$= -1$

③ → $\boxed{y = 5x - 1}$ is the equation of the tangent line.

QUESTION 5. (7 points) Given H and F. Find a point Q on the line $x = 12$ such that $|HQ| + |FQ|$ is minimum.



$Q_{min} = (12, -22)$

(H=(2,8),
F=(10,-28)

$m = \frac{-28-8}{14-2} = -3$

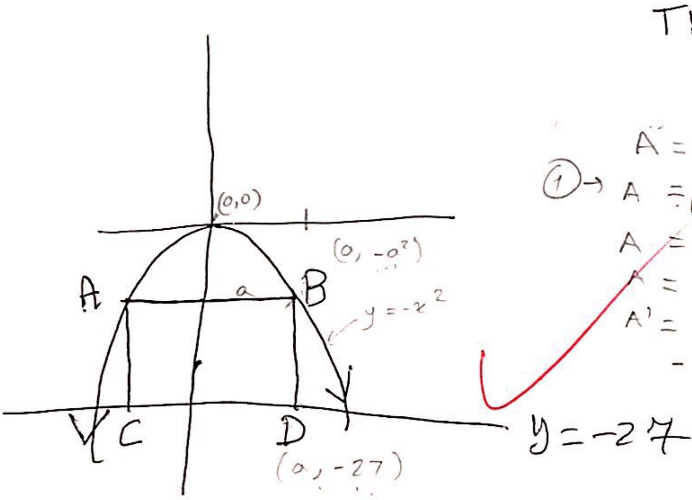
① → $y = -3x + b$
$8 = -3(2) + b$
$b = 8 + 6$
$= 14$
$y = -3x + 14$

|AC|

$Q = (12, y)$
$y = -3(12) + 14$
$= -22$

$Q_{min} = (12, -22)$

QUESTION 6. (7 points) Consider the following picture. Find $|AB|$ and $|\cancel{CD}|$ so that the area is MAXIMUM.



The curve is $y = -x^2$,
the line $y = -27$

$A = |BD||AB|$

① → $A = [-a^2 - (-27)] \cdot 2a$
$A = (-a^2 + 27) \cdot 2a$
$A = -2a^3 + 54a$
$A' = -6a^2 + 54$
$-6a^2 + 54 = 0$
$-6a^2 = -54$
$a^2 = -54/-6$
$a = \sqrt{9}$
$a = +3$

$A'' = -12a$
② → $A''(3) = - \therefore$ max.

$A = (-3^2 + 27)(6)$
$= 108$ units^2

$|AB| = 2a$
$= 6$ units

$|BD| = (-a^2 + 27)$
$= 18$ units

Faculty information
Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: abadawi@aus.edu, www.ayman-badawi.com

Rania Hegab

Exam II: MTH 111, Spring 2018

Ayman Badawi

Points = $\frac{55}{55}$ 47

Excellent !!

QUESTION 1. (8 points) Find y' and DO NOT SIMPLIFY

(i) $y = 6e^{(3x^2+6x+1)}$

$\boxed{y' = 6e^{(3x^2+6x+1)}\cdot(6x+6)}$

(ii) $y = (2x+3)\sqrt{7x+2}$

$y = (2x+3)(7x+2)^{\frac{1}{2}}$

$y' = (1)'(2) + (2)'(1)$

$\boxed{y' = 1(7x+2)^{\frac{1}{2}} + \frac{1}{2}(7x+2)^{-\frac{1}{2}}(2x+3)}$

(iii) $y = ln[\frac{(3x+2)^3(2x+7)^2}{(7x+12)^4}]$

$y = 3\ln(3x+2) + 2\ln(2x+7) - 4\ln(7x+12)$

$y' = \frac{3(3)}{3x+2} + \frac{2(2)}{2x+7} - \frac{4(7)}{7x+12}$

$\boxed{y' = \frac{9}{3x+2} + \frac{4}{2x+7} - \frac{28}{7x+12}}$

(iv) $y = 2(3x^2+5x)^{12}$

$\boxed{y = 24(3x^2+5x)^{11}\cdot(6x' + 5)}$

QUESTION 2. (i) **(3 points)** What can you say about the line $L: x = 2t+1, y = t-1, z = -2t+3$ and the plane $x + 2y + z = 16$? (i.e., Doe L lie inside the plane? Does L intersect the plane exactly in one point? or neither?

$L: x = 2t + 1$
$\quad y = t - 1$
$\quad z = -2t + 3$

$P: x + 2y + z = 16$

$(2t+1) + 2(t-1) - 2t + 3 = 16$

$2t + 1 + 2t - 2 - 2t + 3 = 16$

$2t = 14 \Rightarrow t = 14/2 \Rightarrow \boxed{t = 7}$

$x: 2(7)+1 = 15$

$y: 7 - 1 = 6$

$z: -2(7)+3 = -11$

$\boxed{Q\text{: intersection point}: (15, 6, -11)}$

(ii) **(4 points)** Given $N = < -2, 3, 2 >$ is perpendicular to the plane P and the point $(-1, 4, 2)$ lies inside the plane P. Find the equation of the plane P.

Find eqn → Directional vector, point Q

$N = <-2, 3, 2> \perp P$ at $Q(-1, 4, 2)$

$P: -2(x+1) + 3(y-4) + 2(z-2) = 0$

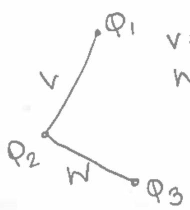
$P: -2x - 2 + 3y - 12 + 2z - 4 = 0$

$\boxed{P: -2x + 3y + 2z = 18}$

(iii) **(6 points)** Find the equation of the plane that contains the points $Q_1 = (4, 4, 0)$, $Q_2 = (0, 2, 6)$ and $Q_3 = (4, 0, 8)$.

Eqn of Plane → directional vector and point Q_1

$Q_1: (4, 4, 0)$
$Q_2: (0, 2, 6)$
$Q_3: (4, 0, 8)$



$V = Q_1Q_2 = <4, 2, -6>$

$W = Q_3Q_2 = <4, -2, 2>$

$V \times W = \begin{vmatrix} i & j & k \\ 4 & 2 & -6 \\ 4 & -2 & 2 \end{vmatrix} = \begin{vmatrix} 2 & -6 \\ -2 & 2 \end{vmatrix}, -\begin{vmatrix} 4 & -6 \\ 4 & 2 \end{vmatrix}, \begin{vmatrix} 4 & 2 \\ 4 & -2 \end{vmatrix}$

$= <4 - 12, -(8 + 24), -8 - 8>$

$= <-8, -32, -16>$

$P: -8(x-4) - 32(y-4) - 16(z+0) = 0$

$P: -8x + 32 - 32y + 128 - 16z = 0$

$\boxed{P: -8x - 32y - 16z = -160}$

QUESTION 3. (i) **(4 points)** (1) Convince me that the line $L: x = 4t, y = -4t+1, z = 2t+1$ is perpendicular to the plane $P: 2x + -2y + z = 12$ (If you think that I am wrong, then state your reason). (2) Can we draw the vector $V = <1, -2, -6>$ inside P?

1) $L: x = 4t$
$y = -4t + 1$
$z = 2t + 1$

$P: 2x + -2y + z = 12$

$D_2 = <2, -2, 1>$

$D_1 = <4, -4, 2>$

2) $V = <1, -2, -6>$

$D_2: <2, -2, 1>$

$V \cdot D_2 = 0 \rightarrow$ Yes $\quad V \cdot D_2 \neq 0 \rightarrow$ No

$V \cdot D_2 = <1, -2, -6> \cdot <2, -2, 1>$

$V \cdot D_2 = 2 + 4 - 6 = 0 \rightarrow$ Yes, we can draw V inside P.

$$D_1 \times D_1 = \begin{vmatrix} i & j & k \\ 4 & -4 & 2 \\ 2 & -2 & 1 \end{vmatrix} = \begin{vmatrix} -4 & 2 \\ -2 & 1 \end{vmatrix}, - \begin{vmatrix} 4 & 2 \\ 2 & 1 \end{vmatrix}, \begin{vmatrix} 4 & -4 \\ 2 & -2 \end{vmatrix} = -4+4, -(4-4), -8+8$$

$D_1 \times D_2 = <0, 0, 0> \rightarrow$ Plane and line are perpendicular.

(ii) **(3 points)** Find the distance between $Q = (10, 10, 33)$ and the plane $P: 2x - 2y + z = 21$.

$Q: (10, 10, 33)$

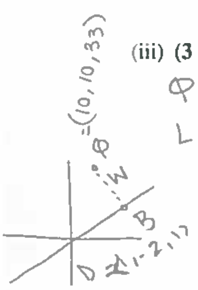
$P: 2x - 2y + z = 21$

$\hookrightarrow 2x - 2y + z - 21 = 0$

$$QP = \frac{|2(10) - 2(10) + 33 - 21|}{\sqrt{(2)^2 + (-2)^2 + (1)^2}}$$

$QP = \frac{12}{3} = \boxed{4 \text{ units}}$

(iii) **(3 points)** Find the distance between $Q = (10, 10, 33)$ and the line $L: x = t+1, y = -2t+3, z = t$



$Q: (10, 10, 33)$

$L: x = t + 1$
$y = -2t + 3$
$z = t$

$D = <1, -2, 1>$
$I = <1, 3, 0>$

$W = |QB| = 9, 7, 33$

$$QL = \frac{|W \times D|}{|D|} = \quad |W \times D| = \begin{vmatrix} i & j & k \\ 9 & 7 & 33 \\ 1 & -2 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} 7 & 33 \\ -2 & 1 \end{vmatrix}, - \begin{vmatrix} 9 & 33 \\ 1 & 1 \end{vmatrix}, \begin{vmatrix} 9 & 7 \\ 1 & -2 \end{vmatrix}$$

$= <7 + 66, -(9 - 33), -18 - 7> = <73, 24, -25$

$$\frac{|W \times D|}{|D|} = \frac{\sqrt{73^2 + 24^2 + 25^2}}{\sqrt{1^2 + 2^2 + 1^2}} = \boxed{32.99 \text{ units}}$$

(iv) **(6 points)** The two planes $P_1: x + 2y + z = 10$ and $P_2: -x + 2y - z = 6$ intersects in a line L. Find a parametric equations of L.

$P_1 = x + 2y + z = 10 \rightarrow N_1 = <1, 2, 1>$

$P_2: -x + 2y - z = 6 \rightarrow N_2 = <-1, 2, -1>$

$$N_1 \times N_2 = \begin{vmatrix} i & j & k \\ 1 & 2 & 1 \\ -1 & 2 & -1 \end{vmatrix} = \begin{vmatrix} 2 & 1 \\ 2 & -1 \end{vmatrix}, - \begin{vmatrix} 1 & 1 \\ -1 & -1 \end{vmatrix}, \begin{vmatrix} 1 & 2 \\ -1 & 2 \end{vmatrix}$$

$= <-2 - 2, -(-1 + 1), 2 + 2>$

$\boxed{N_1 \times N_2 = <-4, 0, 4>}$

Let $\boxed{z = 0}$ in P_1 and P_2

$x + 2y = 10 \rightarrow x = 10 - 2y \rightarrow 10 - 2(4) = 10 - 8 = \boxed{2 = x}$

$-x + 2y = 6$

$\downarrow$

$-(10 - 2y) + 2y = 6$

$-10 + 2y + 2y = 6$

$-10 + 4y = 6$

$4y = 6 + 10$

$4y = 16 \Rightarrow y = 16/4 \Rightarrow \boxed{y = 4}$

Parametric eqns:
$x: -4t - 2$
$y: 4$
$z: 4t$

QUESTION 4. (7 points) Let $f(x) = -x^3 + 6x^2 + 15x + 1$.

(i) For what values of x does $f(x)$ increase?

$f'(x) = -3x^2 + 12x^1 + 15$

$\boxed{\begin{array}{l} x = 5 \\ x = -1 \end{array}}$

$f(x)$ increases $\rightarrow (-1, 5)$

(ii) For what values of x does $f(x)$ decrease?

$f(x)$ decreases $\rightarrow (-\infty, -1) \cup (5, +\infty)$

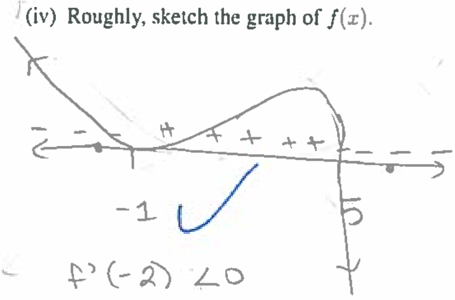
(iii) Find all minimum, maximum points of $f(x)$.

min at $x = -1 \rightarrow (-1, -7)$

max at $x = 5 \rightarrow (5, -43) \rightarrow 101$

$-(5)^3 + 6(25) + 15(5) + 1 =$ 101

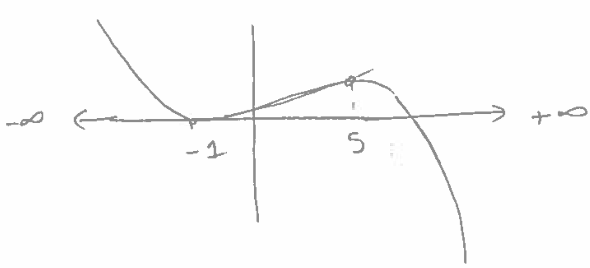
(iv) Roughly, sketch the graph of $f(x)$.



$f'(-2) < 0$

$f'(0) > 0$

$f'(6) < 0$



QUESTION 5. (4 points) Let $f(x) = 2xe^{(x-1)} + ln(2x-1) + 4$. Find the equation of the tangent line to the curve of $f(x)$ at $x = 1$.

$f(x) = 2xe^{(x-1)} + \ln(2x-1) + 4$

$P: (1, f(1)) = (1, 6)$

$f(1) = 2(1)e^{(1-1)} + \ln(2(1)-1) + 4 = 6$

$f'(x) = (1)'(2) + (2)'(1) + \dfrac{\log(2x-1)}{\log(10)} + 0$

$f'(x) = 2e^{(x-1)} + e^{(x-1)}(1)(2x) + \log(2x-1) \cdot \dfrac{1}{\log 10}$

$f'(x) = 2e^{(x-1)} + 2xe^{(x-1)} + \dfrac{2}{\log(10)} \Rightarrow f'(1) = 6$

$y = mx + b$

$6 = 6(1) + b$

$6 = 6 + b$

$6 - 6 = b$

$b = 0$

$\boxed{y = 6x}$

QUESTION 6. (7 points) Consider $f(x) = 4 - \sqrt{x}$, $k(x) = -2$. Find the length and the width of the largest rectangle that you can draw between $f(x)$ and $k(x)$, see picture.

Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: abadawi@aus.edu, www.ayman-badawi.com

$\rightarrow A = l \cdot w$

$A = m(6 - \sqrt{m})$

$A = m(6 - m^{1/2})$

$A = 6m - m^{3/2}$

$\rightarrow A' = 6 - \frac{3}{2}m^{1/2}$

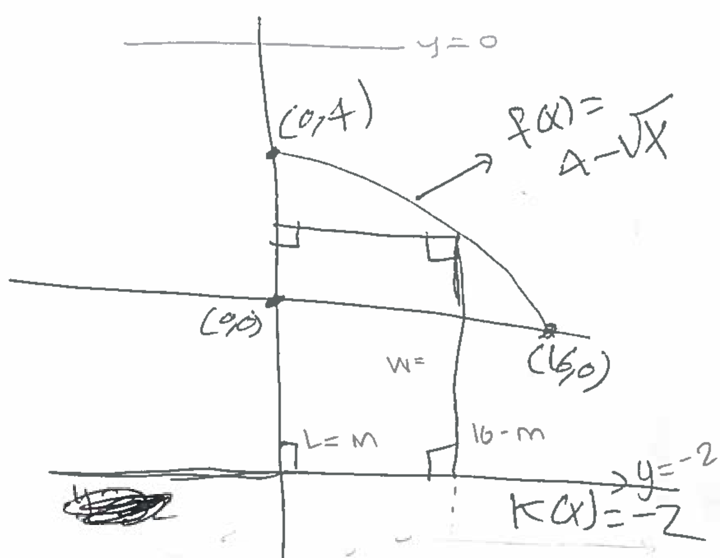
$\rightarrow 0 = 6 - \frac{3}{2}m^{1/2}$

$6 = \frac{3}{2}m^{1/2}$

$\dfrac{6}{3/2} = \dfrac{3/2}{3/2}m^{1/2}$

$\sqrt{4} = \sqrt{m^{1/2}}$

$\boxed{m = 16}$



$L = m$

$W = 4 - \sqrt{m} + 2 = 6 - \sqrt{m}$

$L = m = \boxed{16}$

$W = 6 - \sqrt{m} = 6 - \sqrt{16} = \boxed{2}$

$\rightarrow A'' = -\frac{3}{4}m^{-1/2}$

$A''(16) = -\frac{3}{4}(16)^{-1/2} < 0 \rightarrow \boxed{\text{max.}}$

3.12 Questions with solutions on Derivative, Integration, Volume, and reviews for the final exam from previous semesters

Q. $\displaystyle\int \frac{6\cos(2x)}{1+\sin(2x)}\,dx$

A. $\displaystyle\int 6\cos(2x)\,[1+\sin(2x)]^{-1}\,dx =$

$6 = 3 \cdot 2$

$u = 1+\sin(2x)$

$u' = 2\cos(2x)$

$3\displaystyle\int 2\cos(2x)\,[1+\sin(2x)]^{-1}\,dx =$

$u = 1+\sin(2x)$

$u' = 2\cos(2x)$

$= 3\ln|1+\sin(2x)| + C$

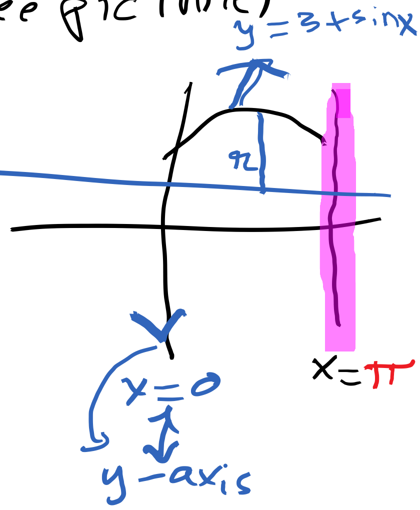
Q. $\displaystyle\int (2x+5e^{5x}-\sin(x))\,[x^2+e^{5x}+\cos(x)]^4\,dx$

$u = x^2+e^{5x}+\cos(x)$

$u' = 2x+5e^{5x}-\sin(x)$

$= \dfrac{(x^2+e^{5x}+\cos(x))^5}{5} + C$

Find the volume of the object when we rotate $y = 3+\sin(x)$ about $y=1$, where $0 \le x < \pi$ (see picture)



A: $$\pi \int_{x=0}^{x=\pi} [3+\sin(x)-1]^2\,dx \qquad y=1$$

$$= \pi \int_0^{\pi} [2+\sin(x)]^2\,dx$$

$$= \pi \int_{x=0}^{x=\pi} [4+4\sin(x)+\sin^2(x)]\,dx$$

Now $\sin^2(x) = \frac{1}{2} - \frac{1}{2}\cos(2x)$

$$= \pi \int_{x=0}^{x=\pi} 4+4\sin(x)+\frac{1}{2}-\frac{1}{2}\cos(2x)\,dx =$$

$$= \pi \left[4x - 4\cos(x) + \frac{1}{2}x - \frac{1}{4}\sin(2x) \Big|_{x=0}^{x=\pi} \right]$$

(note $\int 4\sin x = -4\cos x$

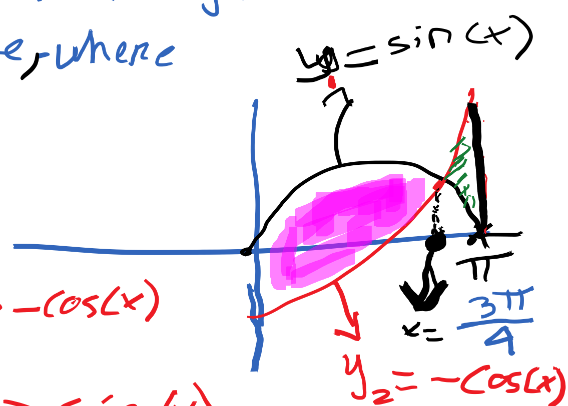
$\int -\frac{1}{2}\cos(2x) = -\frac{1}{2}\cdot\frac{\sin(2x)}{2}$
$= -\frac{1}{4}\sin(2x)$)

$$= \pi \left[4\pi - 4\cos(\pi) + \frac{1}{2}\pi - \frac{1}{4}\sin(2\pi) - \left(0 - 4\cos(0) + \frac{1}{2}(0) - \frac{1}{4}\sin(0)\right) \right]$$

$= \pi \left[4\pi + 4 + \frac{1}{2}\pi - 0 - 0 + 4 - 0 + 0 \right]$

$= \pi \left[4.5\pi + 8 \right] = (4.5\pi^2 + 8\pi)$ unit^3.

Q. Find the area of the region as in the picture, where $0 \le x \le \pi$



Solution: by staring.

$0 \le x \le \frac{3\pi}{4}$, $\sin(x) > -\cos(x)$

$\frac{3\pi}{4} \le x \le \pi$, $-\cos(x) > \sin(x)$

so

$$\text{Area} = \int_{x=0}^{x=\frac{3\pi}{4}} (\sin(x) - (-\cos(x)))\,dx + \int_{x=\frac{3\pi}{4}}^{\pi} -\cos(x) - \sin(x)\,dx$$

$$= -\cos(x) + \sin x \Big|_{x=0}^{x=\frac{3\pi}{4}} + -\sin(x) + \cos(x) \Big|_{x=\frac{3\pi}{4}}^{x=\pi} =$$

$$\underbrace{-\cos(\tfrac{3\pi}{4}) + \sin(\tfrac{3\pi}{4}) - (-\cos(0) + \sin(0))}\ + \underbrace{-\sin(\pi) + \cos(\pi)}$$

$$\underbrace{-\left(-\sin(\tfrac{3\pi}{4}) + \cos(\tfrac{3\pi}{4})\right)}$$

$= \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} + 1 - 0 - 1 + \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}$

$= 2\sqrt{2}$ unit^2

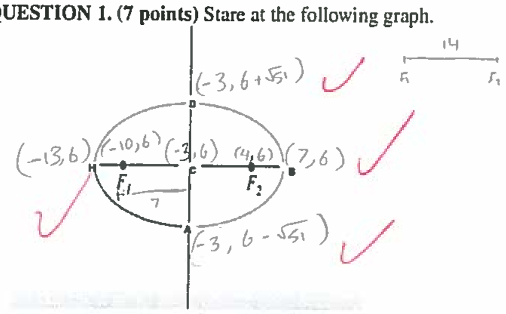
Name Haya Sujaa , ID g00082558

Final Exam, MTH 111, Spring 2019

Ayman Badawi

$$Score = \frac{75}{78}$$

QUESTION 1. (7 points) Stare at the following graph.



(i) Given $F1 = (-10, 6)$, $F2 = (4, 6)$ and the ellipse-constant is 20.

(ii) Find the center $c =$

$|F_2F| = 7 \quad \therefore C = (-3, 6)$

(iii) Find the vertices $A = (-3, -1.14)$, $D = (-3, 13.14)$, $H = (-13, 6)$, and $B = (7, 6)$

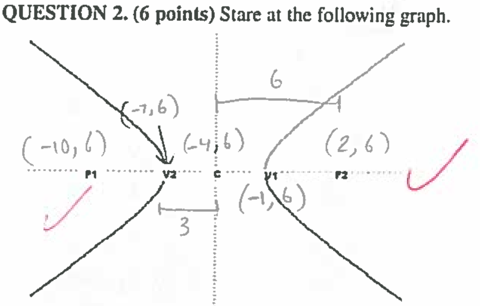
$100 - 49 = b^2$

$b = \sqrt{51}$

(iv) Find the equation of the ellipse.

$$\frac{(x+3)^2}{100} + \frac{(y-6)^2}{51} = 1$$

QUESTION 2. (6 points) Stare at the following graph.



Given $c = (-4, 6)$, $|cv2| = 3$, and $F2 = (2, 6)$.

(i) Find $v1 = (-1, 6)$ $F1 = (-10, 6)$, $v2 = (-7, 6)$, and the hyperbola-constant $k = 6$

$|CF_1| = \sqrt{(\tfrac{6}{2})^2 + b^2} = 6$

$\sqrt{9 + b^2} = 6$

$9 + b^2 = 36$

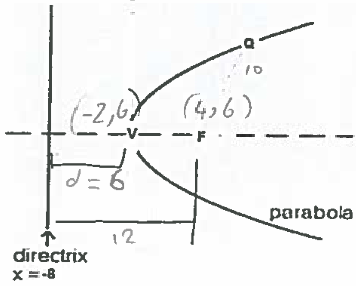
$b^2 = 36 - 9$

$b^2 = 27$

(ii) Find the equation of the hyperbola

$$\frac{(x+4)^2}{9} - \frac{(y-6)^2}{27} = 1$$

QUESTION 3. (4 points) Stare at the following graph.



Given $F = (4,6)$, the directrix line, L is $x = -8$, and $|QF| = 10$.

(i) Find $|QL| = |QF| = 10$

(ii) Find $v = (-2, 6)$

(iii) Find the equation of the parabola

$$24(x+2) = (y-6)^2$$

QUESTION 4. (6 points). Find y' and do not simplify

(i) $y = ln[(4x+3)^{10}(-5x+30)^3]$

$y = \ln(4x+3)^{10} + \ln(-5x+30)^3$

$y = 10\ln(4x+3) + 3\ln(-5x+30)$

$y' = \dfrac{10 \cdot 4}{4x+3} + \dfrac{3 \cdot -5}{-5x+30}$

$y' = \dfrac{40}{(4x+3)} + \dfrac{-15}{(-5x+30)}$

(ii) $y = e^{(6x^3+x^2-1)} + 10x^2 - x + 23$

$y' = \left[\left[e^{(6x^3+x^2-1)} \cdot (18x^2+2x)\right] + 20x - 1\right]$

(iii) $y = (21 + 5x - 6x^3)^7$

$y' = 7(21+5x-6x^3)^6 \cdot (5 - 18x^2)$

QUESTION 5. (6 points).

(i) Find $\int x e^{(x^2+1)}\, dx$

$u = x^2+1$

$u' = 2x$

$\frac{1}{2}\left(e^{(x^2+1)}\right) + C$

(ii) Find $\int \frac{e^{2x}+1}{(e^{2x}+2x-5)^3}\, dx$

$\int (e^{2x}+1)(e^{2x}+2x-5)^{-3}\, dx$

$u = e^{2x} + 2x - 5$

$u' = 2e^{2x} + 2$

$\frac{1}{2}\int 2(e^{2x}+1)(e^{2x}+2x-5)^{-3}\, dx$

$\frac{1}{2} \cdot \frac{1}{-2}(e^{2x}+2x-5)^{-2} + C$

(iii) Find $\int (6x+3)(x^2+x-5)^{11}\, dx$

$u = x^2 + x - 5$

$u' = 2x + 1$

$3 \cdot \frac{1}{12}(x^2+x-5)^{12} + C$

QUESTION 6. (5 points). Let $H = (4,6)$, $F = (6,34)$. Find a point Q on the line $x = -2$ such that $|HQ| + |FQ|$ is minimum.

$y = mx + b$

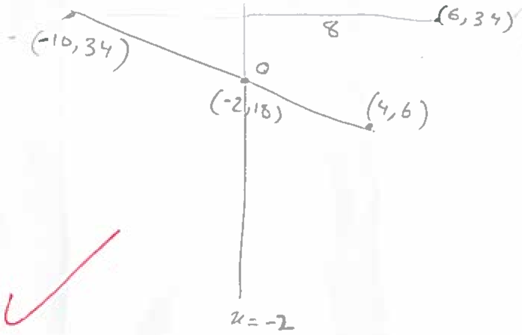
$m = \frac{6-34}{4+10} = -2$

$6 = -2(4) + b$

$b = 14$

$y = -2x + 14$

$y = -2(-2) + 14$
$= 18$



$Q = (-2, 18)$

QUESTION 7. (4 points). For what values of x does the tangent line to the curve $y = ln(4x+1) + 7x + 2$ have slope equal 8?

$y' = 8$

$y' = \frac{4}{4x+1} + 7 = 8$

$\frac{4}{4x+1} = 1$

$4 = 4x + 1$

$4x = 4 - 1$

$x = 3/4$

check $\frac{4}{4(\frac{3}{4})+1} + 7 =$

$1 + 7 = 8$ //

the T line has slope 8 at $x = \frac{3}{4}$

QUESTION 8. (6 points). The plane $P_1 : x + 2y - 3z = 2$ intersects the plane $P_2 : -x + 5y + z = 19$ in a line L. Find a parametric equations of L.

① → $N_1 \times N_2 = D$

$N_1 = \langle 1, 2, -3 \rangle$

$N_2 = \langle -1, 5, 1 \rangle$

$D = (2 + 15)i - (1 - 3)j + (5 + 2)k$

$= \langle 17, 2, 7 \rangle$

② → $z = 0$

$x + 2y = 2$

$-x + 5y = 19$

$x = \frac{\begin{vmatrix} 2 & 2 \\ 19 & 5 \end{vmatrix}}{\begin{vmatrix} 1 & 2 \\ -1 & 5 \end{vmatrix}}$ $\quad y = \frac{\begin{vmatrix} 1 & 2 \\ -1 & 19 \end{vmatrix}}{\begin{vmatrix} 1 & 2 \\ -1 & 5 \end{vmatrix}}$

$x = \frac{-28}{7}$ $\quad y = \frac{21}{7}$

$x = -4$ $\quad y = 3$

③ → $(-4, 3, 0)$

$D = \langle 17, 2, 7 \rangle$

$L:$ $\left.\begin{array}{l} x = 17t - 4 \\ y = 2t + 3 \\ z = 7t \end{array}\right\} t \in \mathbb{R}$

QUESTION 9. (5 points). Can we draw the entire line L: $x = 2t, y = -3t + 1, z = 11t + 4$ inside the plane $2x - 6y - 2z = 20$? EXPLAIN

$N_{plane} \cdot D_{line}$ must $= 0$

$N = \langle 2, -6, -2 \rangle$

$D = \langle 2, -3, 11 \rangle$

$N \cdot D = 4 + 18 - 22$
$= 0$ //

take a point on L and check if the point lies in the plane or not

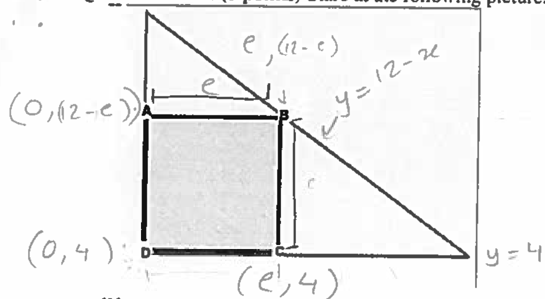
No

yes the line can be entirely drawn on the plane because the dot product of the normal and directional vector is 0

3

Haya

QUESTION 10. (8 points) Stare at the following picture.



We want to construct a rectangle ABCD of largest area as in the picture above. Note that A and D lie on the y-axis, D and C lie on the line y = 4 (note that y = 4 intersects the y-axis at D), and B lies on the line y = 12 - x. Find |DC| and |BC|.

$|BC| = (12 - e) - 4$

$|DC| = e$

$A = |BC| \cdot |DC|$

$= [(12-e)-4] \cdot e$

$= (-e+8)e$

$= -e^2 + 8e$

$A' = -2e + 8$

① → $-2e + 8 = 0$

$e = 4$

② → $|BC| = (12-4) - 4$

$= 8 - 4$

$= 4$ units

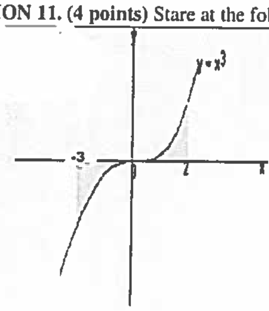
$|DC| = e$

$= 4$ units

Area $= 4 \times 4$

$= 16$ units^2

QUESTION 11. (4 points) Stare at the following picture.



Find the area of the shaded region. Note that $y = f(x) = x^3$ and x is between -3 and 2.

$$A = \left[\int_{-3}^{0} x^3\,dx\right] + \int_{0}^{2} x^3\,dx$$

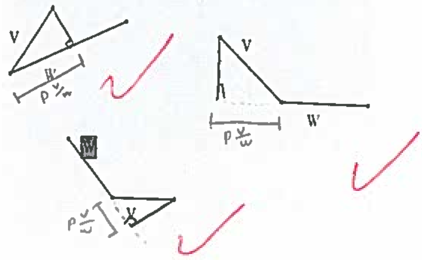
$$= \left[\int_{-3}^{0} \frac{1}{4}x^4\right] + \int_{0}^{2} \frac{1}{4}x^4$$

$$= \left[\left[\frac{1}{4}0^4\right] - \left[\frac{1}{4}(-3)^4\right]\right] + \left[\left[\frac{1}{4}(2)^4\right] - \left[\frac{1}{4}(0)^4\right]\right]$$

$$= [0 + 20.25] + [4 - 0]$$

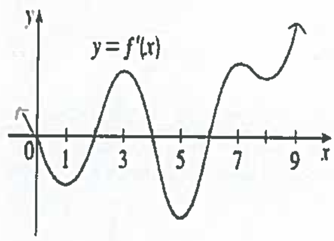
$$= 24.25 \text{ units}^2$$

QUESTION 12. (4.5 points) Stare at the following picture.



Draw the projection of V over W.

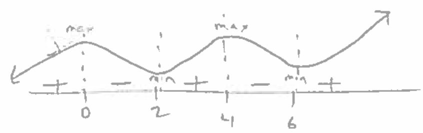
QUESTION 13. (7.5 points) Stare at the following graph of $y = f'(x)$.



critical values $= 0, 2, 4, 6$

$\nearrow = (-\infty, 0), (2, 4), (6, +\infty)$

$\searrow = (0, 2), (4, 6)$



(i) At what value(s) of x does $f(x)$ have local max.?

at $x = 0$ and $x = 4$

(ii) At what value(s) of x does $f(x)$ have local min.?

at $x = 2$ and $x = 6$

(iii) For what values of x does $f(x)$ increase?

$(-\infty, 0) \cup (2, 4) \cup (6, +\infty)$

(iv) For what values of x does $f(x)$ decrease?

$(0, 2) \cup (4, 6)$

(v) For what values of x will the normal lines have positive slope.

Normal line will have a + slope when the tangent line has − slope

$\therefore$ when the function x is decreasing $\therefore (0, 2) \cup (4, 6)$

QUESTION 14. (5 points) Given $L_1 : x = 2t, y = t + 1, z = 3t$ is perpendicular to $L_2 : x = 4w + 6, y = -2w, z = aw + 1$ and they intersect at a point Q. Find the value of a and find the point Q.

$L_1 : \left.\begin{array}{l} x = 2t \\ y = t + 1 \\ z = 3t \end{array}\right\} t \in \mathbb{R}$ $\quad L_2 : \left.\begin{array}{l} x = 4w + 6 \\ y = -2w \\ z = aw + 1 \end{array}\right\} w \in \mathbb{R}$

$Q = (2, 2, 3)$

$a = -2$

$x = x \quad y = y$

$2t = 4w + 6 \quad | \quad t + 1 = -2w$

$t + 2w = -1$

$2t - 4w = 6$

$t + 2w = -1$

$t = \dfrac{\begin{vmatrix} 6 & -4 \\ -1 & 2 \end{vmatrix}}{\begin{vmatrix} 2 & -4 \\ 1 & 2 \end{vmatrix}} \qquad w = \dfrac{\begin{vmatrix} 2 & 6 \\ 1 & -1 \end{vmatrix}}{\begin{vmatrix} 2 & -4 \\ 1 & 2 \end{vmatrix}}$

$t = 1$

$w = -1$

$x = 2$

$y = 2$

$z = 3$

$z = aw + 1$

$3 = a(-1) + 1$

$3 - 1 = a(-1)$

$2 = a \cdot -1$

$a = 2/-1 \qquad a = -2$

Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: abadawi@aus.edu, www.ayman-badawi.com

AUS | American University of Sharjah

Score:

American University of Sharjah
Department of Mathematics and Statistics

Final Exam - spring 2018
MTH 111 – Math for Architects

Instructor Name: **Ayman Badawi**

➔**The name above *must be* the name of *your* instructor**🡸

SCORE (---98---)
100

Student Name: NADIN ELSHIRBINI
Student ID Number: 72434

1. *No Questions are allowed during the examination.*
2. *This exam has 6 pages plus this cover page .*
3. *Do not separate the pages of the exam.*
4. *Scientific calculator are allowed but cannot be shared.*
   Graphing Calculators are not allowed.
5. *Take off your cap. Turn off all cell phones and remove all headphones.*
6. *No communication of any kind is permitted.*
7. *All working must be shown*

Student signature: ______________________________

Name——————————, ID ——————

Final Exam: MTH 111, Spring 2018

Ayman Badawi

Points = $\frac{\quad}{100}$

QUESTION 1. (9 points) Find y' and DO NOT SIMPLIFY

(i) $y = (x+1)e^{(3x+2)}$

$y' = e^{3x+2} + (3x+3)e^{3x+2} = e^{3x+2}(3x+4)$

(ii) $y = ln[(3x-2)^4(2x+1)^7]$

$y' = \frac{12}{3x-2} + \frac{14}{2x+1}$

(iii) $y = (7x+2)^9$

$y' = 63(7x+2)^8$

QUESTION 2. (i) **(6 points)** Does the line line $L_1: x = t+1, y = t-1, z = 7$ intersect the line $L_2: x = -w+4, y = w-2, z = 2w+3$? If yes, then find the intersection point. Is L_1 perpendicular to L_2?

$D_1 <1,1,0>$ $D_2 <-1,1,2>$

$D_1 \neq c D_2 \Rightarrow$ $\boxed{L_1 \text{ and } L_2 \text{ intersect}}$

$D_1 \times D_2 = <2,-2,2>$
$\Rightarrow L_1$ not $\perp L_2$.

-2

$t+1 = -w+4 \rightarrow t+w = 3$
$t-1 = w-2 \rightarrow t-w = -1$
$t = 1 \quad w = 2$

Using $t=1$:
$x = 1+1 = 2$
$y = 1-1 = 0$
$z = 7$

or

Using $w=2$:
$x = -2+4 = 2$
$y = 2-2 = 0$
$z = 2(2)+3 = 7$

$\boxed{\text{point of intersection } (2,0,7)}$

(ii) **(4 points)** Convince me that $L_1: x = t, y = 10, z = -t+4$ is parallel to $L_2: x = 4w+1, y = 7, z = -4w+2$

$D_1 = <1,0,-1>$ $D_2 = <4,0,-4>$

$D_2 = 4D_1$

$t=0 \rightarrow (0,10,4)$

$x: \; 0 = 4w+1 \rightarrow w = \frac{-1}{4}$
$z: \; 4 = -4w+2 \rightarrow w = -\frac{1}{4}$
$y: \; w = 0$

diff. $w \Rightarrow L_1$ and L_2 are parallel.

(iii) Let $Q_1 = (1,1,0)$, $Q_2 = (0,-1,2)$ and $Q_3 = (2,2,2)$.

a. **(5 points)** Find the equation of the plane that contains Q_1, Q_2, Q_3.

$\overrightarrow{Q_1Q_2} \langle -1,-2,2\rangle \quad \overrightarrow{Q_1Q_3} \langle 1,1,2\rangle$

$N = |Q_1Q_2 \times Q_1Q_3| = \begin{vmatrix} i & j & k \\ -1 & -2 & 2 \\ 1 & 1 & 2 \end{vmatrix} = \langle -6,4,1\rangle$

$P: -6(x-2)+4(y-2)+1(z-2)=0$

b. **(2 points)** Find the area of the triangle that has Q_1, Q_2, Q_3 as vertices.

$A = \frac{1}{2}|\overrightarrow{Q_1Q_2} \times \overrightarrow{Q_1Q_3}| = \frac{\sqrt{6^2+4^2+1^2}}{2} = \frac{\sqrt{53}}{2}$ units^2

(iv) **(4 points)** Given $L: x = t+1, y = 8, z = 4t+1$ lies entirely inside the plane $P: ax+2y+z=b$ Find the values of a, b. $D\langle 1,0,4\rangle \quad N\langle a,2,1\rangle$

$N \cdot D = 0$

$a+4=0$

$\boxed{a=-4}$

$-4(t+1)+2(8)+4t+1=b$

$-4t-4+16+4t+1=b$

$\boxed{b=13}$

(v) **(4 points)**item Find the distance between the point $(1,-1,1)$ and the line $L: x = t+1, y = 2t+3, z = -2t+10$

$Q(1,-1,1) \quad I(1,3,10)$

$V = \overrightarrow{IQ} = \langle 0,-4,-9\rangle \quad D\langle 1,2,-2\rangle$

$V \times D = \begin{vmatrix} i & j & k \\ 0 & -4 & -9 \\ 1 & 2 & -2 \end{vmatrix} = \langle 26,-9,4\rangle$

$d = \frac{|V\times D|}{|D|} = \frac{\sqrt{26^2+9^2+4^2}}{\sqrt{1^2+2^2+2^2}} = \frac{\sqrt{773}}{3}$ units

(vi) **(3 points)** For what values of x will the tangent line to the curve $f(x) = e^x - 4x + 2$ be horizontal? (Hint: Note that horizontal lines have slope 0)

$f'(x) = e^x - 4$

$0 = e^x - 4$

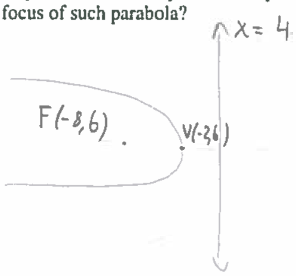
$e^x = 4$

$\ln e^x = \ln 4$

$x \ln e = \ln 4$

$\boxed{x = \ln 4}$

(vii) **(5 points)** Find the equation of a parabola that has x = 4 as its directrix line and (-2, 6) as its vertex. What is the focus of such parabola?



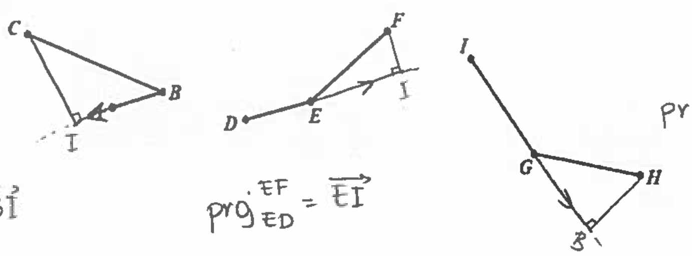
$d = |-2-4| = 6$

$-4d(x-x_0) = (y-y_0)^2$

$\boxed{-24(x+2) = (y-6)^2}$

$\boxed{F(-8,6)}$

(viii) **(6 points)**



$\text{proj}_{BA}^{BC} = \vec{BI}$ $\text{proj}_{ED}^{EF} = \vec{EI}$ $\text{proj}_{GI}^{GH} = \vec{GB}$

Use the pictures above
1. Draw the projection vector of BC over BA
2. Draw the projection vector of EF over ED
3. Draw the projection vector of GH over GI

(ix) Let $f(x) = (x^2 - 6x + 5)^4$.

a. **(3 points)**Find $f'(x)$. Then find the sign of $f'(x)$.

$f'(x) = 4(2x-6)(x^2-6x+5)^3$
$0 = 4(2x-6)(x^2-6x+5)^3$
$2x-6=0$ $\quad x^2-6x+5=0$
$x=3$ $\quad x=5 \quad x=1$

$f'(x)$ negative for $(-\infty, 1) \cup (3,5)$
$f'(x)$ positive for $(1,3) \cup (5,+\infty)$

x	$-\infty$		1		3		5		$+\infty$
$f'(x)$		$-$	0	$+$	0	$-$	0	$+$	
$f(x)$		↘		↗		↘		↗	

b. **(2 points)**For what values of x does $f(x)$ increase?

$(1,3) \cup (5,+\infty)$

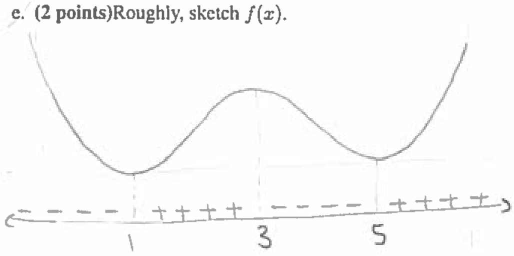
c. **(2 points)**For what values of x does $f(x)$ decrease?

$(-\infty, 1) \cup (3,5)$

d. **(2 points)**Find all local min (max) points of $f(x)$ if possible

min at $x=1$ and $x=5$ MIN : $(1,0)$ and $(5,0)$
max at $x=3$ MAX : $(3,256)$

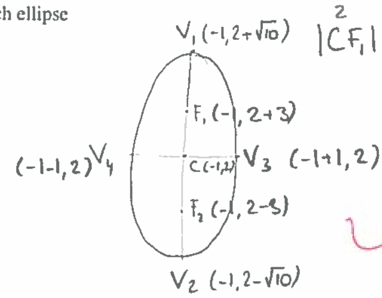
e. **(2 points)**Roughly, sketch $f(x)$.



(x) Consider the ellipse $(x+1)^2 + \frac{(y-2)^2}{10} = 1$

$C(-1,2)$
$\frac{k}{2} = \sqrt{10}$
$|CF_1| = \sqrt{10-1} = 3$

a. (2 points) Roughly, draw such ellipse



b. (2 points)Find the foci

$F_1(-1,5)$
$F_2(-1,-1)$

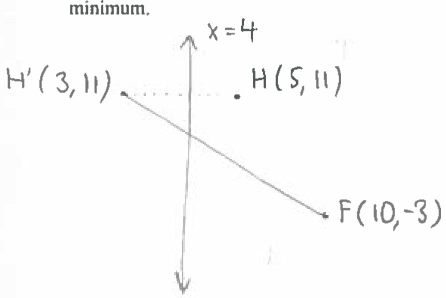
c. (2 points)Find the ellipse constant

$\boxed{k = 2\sqrt{10}}$

d. (2 points)Find all four vertices

$V_1(-1, 2+\sqrt{10})$ $V_3(0,2)$
$V_2(-1, 2-\sqrt{10})$ $V_4(-2,2)$

(xi) (6 points) Let $H = (5, 11)$ and $F = (10, -3)$. Find a point Q on the vertical line $x = 4$ such that $|HQ| + |QF|$ is minimum.



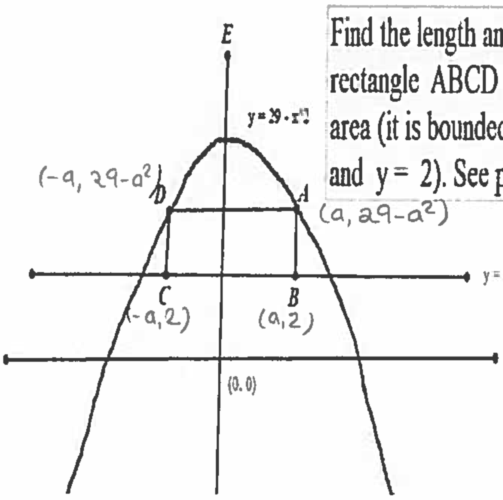
$m = \frac{-3-11}{10-3} = -2$

$11 = -2(3) + b$
$b = 17$

$y = -2x + 17$
$y = -2(4) + 17 = 9$

$Q(4,9)$

(xii) (8 points)



Find the length and the width of the rectangle ABCD that has maximum area (it is bounded by y = 29 - x^2 and y = 2). See picture

$W = |BC| = 2a$

$L = |AB| = 29 - a^2 - 2 = 27 - a^2$

$A = LW = 2a(27 - a^2)$

$A = 54a - 2a^3$

$A' = 54 - 6a^2$

$0 = 54 - 6a^2$

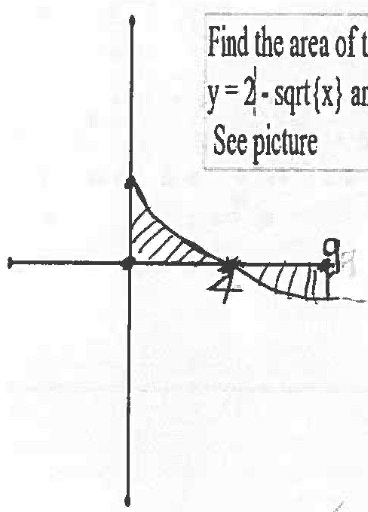
$54 = 6a^2 \Rightarrow a = 3$

$A'' = -12a \big|_{a=3} < 0 \Rightarrow$ max. at $a = 3$

$W = 2a = 6$ units

$L = 27 - a^2 = 18$ units

(xiii) (6 points)



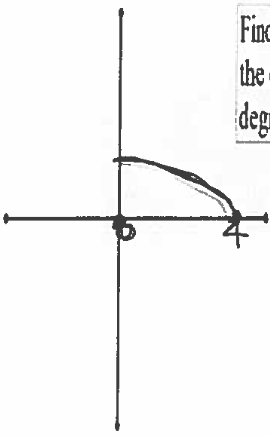
Find the area of the shaded region that is bounded by y = 2 - sqrt{x} and x-axis, where x is between 0 and 9. See picture

$$A = \int_0^9 2 - \sqrt{x}\, dx = \int_0^4 2 - \sqrt{x}\, dx - \int_4^9 2 - \sqrt{x}\, dx$$

$$= \left[2x - \frac{2}{3} x^{\frac{3}{2}} \Big|_0^4 \right] - \left[2x - \frac{2}{3} x^{\frac{3}{2}} \Big|_4^9 \right]$$

$$= \frac{8}{3} - \left(0 - \frac{8}{3}\right) = \frac{8}{3} + \frac{8}{3} = \frac{16}{3} \text{ unit}^2$$

(xiv) (4 points)



Find the volume of the solid object that is obtained by rotating the curve of y = sqrt{4 - x}, where x is between 0 and 4, 360 degrees about the x-axix

$$V = \pi \int_0^4 (\sqrt{4-x})^2 dx = \pi \int_0^4 4-x\, dx$$

$$= \pi \left(4x - \frac{x^2}{2} \Big|_0^4 \right) = \pi(8-0)$$

$$= 8\pi \text{ units}^3$$

(xv) (3 points) $\frac{1}{6}\int 6x^2(2x^3+7)^9\, dx$

$$\frac{(2x^3+7)^{10}}{60} + C$$

(xvi) (3 points) $\frac{1}{2}\int \frac{2(x+1)}{x^2+2x+3}\, dx$

$$\frac{\ln|x^2+2x+3|}{2} + C$$

(xvii) (3 points) $\frac{1}{4}\int 4(x+5)e^{(2x^2+20x+1)}\, dx$

$$\frac{1}{4} e^{2x^2+20x+1} + C$$

Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: abadawi@aus.edu, www.ayman-badawi.com

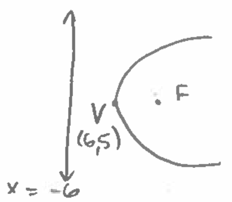
Final Exam: MTH 111, Fall 2017

Ayman Badawi

$$\text{Points} = \frac{81}{82}$$

QUESTION 1. (6 points) Given $x = -6$ is the directrix of of a parabola that has the point $(6, 5)$ as its vertex point.

a) Find the equation of the parabola



$|VL| = |-6-6| = |-12| = 12$

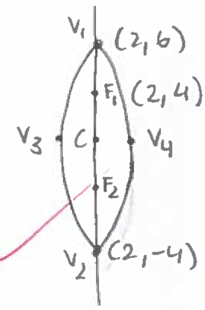
$4(12)(x-6) = (y-5)^2 \Rightarrow 48(x-6) = (y-5)^2$

b) Find the focus of the parabola.

$|VF| = 12 \rightarrow F(18, 5)$

QUESTION 2. (8 points) Given $(2, -4), (2, 6)$ are the vertices of the major axis of an ellipse (recall major axis is the longer axis) and $(2, 4)$ is one of the foci.

(i) Find the vertices of the minor axis (shorter axis). (you may want to draw such ellipse (roughly)).



$|V_1V_2| = K = |6+4| = 10 \rightarrow \frac{K}{2} = 5 = |V_1C|$

$C = (2, 1) \rightarrow |F_1C| = |4-1| = 3 \rightarrow b^2 = \left(\frac{K}{2}\right)^2 - |F_1C|^2$

$b^2 = 5^2 - 3^2 = 16 \rightarrow V_3(18, 1),\ V_4(-14, 1)$

(ii) Find the ellipse-constant K.

$K = 10$

(iii) Find the second foci of the ellipse.

$F_2(2, -2)$

(iv) Find the equation of the ellipse.

$$\frac{(x-2)^2}{16} + \frac{(y-1)^2}{25} = 1$$

QUESTION 3. (5 points) Given $y = 3x^2 + 12x + 9$ is an equation of a parabola. Write the equation of the parabola in the standard form and find the equation of its directrix.

$y = 3x^2 + 12x + 9 \rightarrow y = 3(x^2 + 4x + 3) \rightarrow y = 3\left[(x+2)^2 - 4 + 3\right]$

$y = 3(x+2)^2 - 1(3) \rightarrow \frac{1}{3}(y+3) = (x+2)^2$

$4d = \frac{1}{3} \rightarrow d = \frac{1}{12}$

$V = (-2, -3) \rightarrow$ directrix $\rightarrow x = -2 - \frac{1}{12} \Rightarrow \boxed{\frac{-25}{12} = x}$

$y = -3 - \frac{1}{12}$

QUESTION 4. a) (4 points) Given two lines $L_1 : x = 2t, y = -2t + 3, z = -t + 1$ and $L_2 : x = -4w - 12, y = 4w + 15, z = 2w + 7$. Is L_1 parallel to L_2? EXPLAIN clearly.

$L_1: \left.\begin{array}{l} x = 2t \\ y = -2t+3 \\ z = -t+1 \end{array}\right\} t \in \mathbb{R}$

$L_2: \left.\begin{array}{l} x = -4w-12 \\ y = 4w+15 \\ z = 2w+7 \end{array}\right\} w \in \mathbb{R}$

$D_1 = \langle 2, -2, -1 \rangle$, $D_2 = \langle -4, 4, 2 \rangle$

$D_2 = C D_1 \to C = -2 \to D_1 \parallel D_2$

Q Lies on L_2.

intersection : $L_1 \to t = 0 \to Q(0, 3, 1)$

$L_2 \to x: 0 = -4w - 12 \to w = -3$ / $z: 1 = 2w + 7 \to w = -3$

$y: 3 = 4w + 15 \to w = -3$ / $\to$ $\boxed{L_1 \text{ not } \parallel L_2 \text{ they overlap}}$

b)(4 points) Let L be the line L_1 as in (a). Given that the point $Q = (2, 3, 4)$ does not lie on L. Find $|QL|$ (distance between Q and L).

$I = (0, 3, 1)$, $Q(2, 3, 4) \to \vec{IQ} = \langle 2, 0, 3 \rangle$

$|QL| = \dfrac{|\vec{IQ} \times D_1|}{|D_1|}$, $\vec{IQ} \times D = \begin{vmatrix} i & j & k \\ 2 & 0 & 3 \\ 2 & -2 & -1 \end{vmatrix} = \langle 6, 8, -4 \rangle$

$\to |QL| = \dfrac{\sqrt{6^2 + 8^2 + (-4)^2}}{\sqrt{2^2 + (-2)^2 + (-1)^2}} = \boxed{\dfrac{2\sqrt{29}}{3}}$

c) (6 points) Convince me that $q_1 = (1, 4, 2), q_2 = (2, 1, -1)$, and $q_3 = (3, 5, 2)$ are not co-linear. Then find the area of the triangle with vertices q_1, q_2, q_3.

$\vec{q_1q_2} = \langle 1, -3, -3 \rangle$

$\vec{Q_1Q_3} = \langle 2, 1, 0 \rangle$

$\to \vec{Q_1Q_2} \times \vec{Q_1Q_3} = \begin{vmatrix} i & j & k \\ 1 & -3 & -3 \\ 2 & 1 & 0 \end{vmatrix} = \langle 3, -6, 7 \rangle \Rightarrow$ $\vec{Q_1Q_3}$ not $\parallel$ $\vec{Q_1Q_2}$ $\Rightarrow$ not colinear

$A_\Delta = \frac{1}{2} |\vec{Q_1Q_2} \times \vec{Q_1Q_3}|$

$= \frac{1}{2}\sqrt{3^2 + (-6)^2 + 7^2}$

$= \dfrac{\sqrt{94}}{2}$ units^2

d)(6 points) The two planes $P_1 : 2x + y + 2z = 2$ and $P_2 : -x + y - z = 5$ intersects in a line L. Find a parametric equations of L.

$N_1 = \langle 2, 1, 2 \rangle$, $N_2 = \langle -1, 1, -1 \rangle$

$D = N_1 \times N_2 = \begin{vmatrix} i & j & k \\ 2 & 1 & 2 \\ -1 & 1 & -1 \end{vmatrix} = \langle -3, 0, 3 \rangle$

$\to$ Let $z = 0$ $\to$ $2x + y = 2$

$-1 \times [-x + y = 5]$

$\to$ $2x + y = 2$

$x - y = -5$

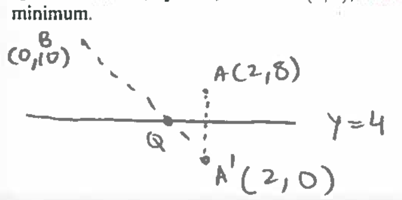
$3x = -3 \to \boxed{x = -1} \to 2(-1) + y + 2(0) = 2$

$\boxed{y = 4}$

$Q = (-1, 4, 0)$

$\to \boxed{L_1 : \left.\begin{array}{l} x = -3t - 1 \\ y = 4 \\ z = 3t \end{array}\right\} t \in \mathbb{R}}$

QUESTION 5. (6 points) Let $A = (2, 8)$, $B = (0, 10)$. Find a point Q on the line $y = 4$ such that $|BQ| + |QA|$ is minimum.



$|AL| = |8 - 4| = 4$

$\to m = \dfrac{y_1 - y_2}{x_1 - x_2} = \dfrac{10 - 0}{0 - 2} = \dfrac{10}{-2} = -5$.

$y = -5x + b \to 10 = -5(0) + b \to b = 10$

$y = -5x + 10 \to 4 = -5x + 10 \to 4 - 10 = -5x \to x = \dfrac{+6}{+5}$

$\boxed{Q = \left(\frac{6}{5}, 4\right)}$

QUESTION 6. (9 points)

(i) Given $f'(1) = 2$ and $y = f(x^2 + 2x - 7)$. Then $y'(2) =$

$$y' = \left[f'(x^2+2x-7)\right]\left[2x+2\right] = \left[f'(2^2+2(2)-7)\right]\left[2(2)+2\right] =$$

$$\left[f'(1)\right]\left[6\right] = 6(2) = \boxed{12}$$

(ii) Let $f(x) = -6e^{(x^3+6x-7)}$. Then $f'(x) =$

$$f(x) = -6e^{(x^3+6x-7)} \to f'(x) = -6(3x^2+6)(e^{x^3+6x-7})$$

$$f(x) = \ln(5x-9)^3 + \ln(2x-3)^7 = 3\ln(5x-9) + 7\ln(2x-3)$$

(iii) Let $f(x) = ln\left((5x-9)^3(2x-3)^7\right)$. Then $f'(x) =$ $\boxed{f'(x) = \frac{3(5)}{5x-9} + \frac{7(2)}{2x-3}}$

QUESTION 7. (10 points)

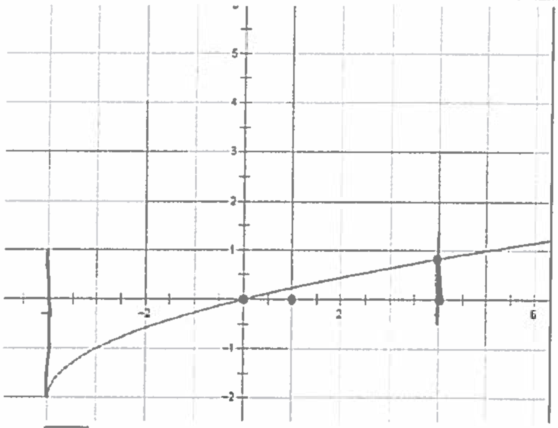
$$\int \frac{x+1}{x^2+2x+1}\,dx = \int (x+1)(x^2+2x+1)^{-1}\,dx = \boxed{\frac{1}{2}\ln\left|(x^2+2x+1)\right| + C}$$

(i) $\int \frac{e^x+3}{(e^x+3x+1)^2}\,dx = \int (e^x+3)(e^x+3x+1)^{-2}\,dx = \boxed{\frac{(e^x+3x+1)^{-1}}{-1} + C}$

(iii) $\int x^5(x+1)^2\,dx = \int x^5(x^2+2x+1)\,dx = \int x^7+2x^6+x^5\,dx =$

$$\int x^7 dx + 2\int x^6 dx + \int x^5 dx = \boxed{\frac{x^8}{8} + \frac{2x^7}{7} + \frac{x^6}{6} + C}$$

(iv) $\int 10(2x+7)^{11}\,dx = 5\int 2(2x+7)^{11}\,dx \Rightarrow \boxed{\frac{5(2x+7)^{12}}{12} + C}$



$$y = \sqrt{x+4} - 2$$
$$2 = \sqrt{x+4}$$
$$4 = x+4$$
$$x = 0$$

QUESTION 8.

Stare at $f(x) = \sqrt{x+4} - 2$ where $-4 \le x \le 4$. Then

a) **(6 points)** Find the area of the region bounded by the curve of $f(x)$, x-axis, and $-4 \le x \le 4$.

$$-\int_{-4}^{0} \sqrt{x+4} - 2\, dx + \int_{0}^{4} \sqrt{x+4} - 2\, dx = -\left(\frac{2}{3}(x+4)^{3/2} - 2x \Big|_{-4}^{0} \right) +$$

$$\left(\frac{2}{3}(x+4)^{3/2} - 2x \Big|_{0}^{4} \right) = -\left(\frac{16}{3} - 8 \right) + \left[\left(\frac{2}{3}(8)^{3/2} - 8 \right) - \frac{16}{3} \right]$$

$$\boxed{\text{Area} \approx 4.42 \text{ unit}^2}$$

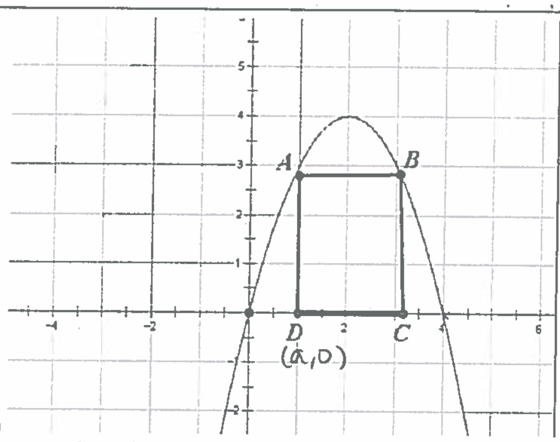
b) **(4 points)** Imagine that the region between x = 0 and x = 4 is rotated about x-axis 360 degrees. What is the volume of the object?

$$\pi \int_{0}^{4} (\sqrt{x+4} - 2)^2 dx \rightarrow \pi \int_{0}^{4} (x+4) - 4\sqrt{x+4} + 4\, dx$$

$$\Rightarrow \pi \left[\int_{0}^{4} x + 8\, dx - 4 \int_{0}^{4} \sqrt{x+4}\, dx \right] \Rightarrow \pi \left[\left(\frac{x^2}{2} + 8x \Big|_{0}^{4} \right) - 4 \left(\frac{2(x+4)^{3/2}}{3} \Big|_{0}^{4} \right) \right]$$

$$\Rightarrow \pi \left[(40 - 0) - 4 \left(\frac{2(8)^{3/2}}{3} - \frac{2(4)^{3/2}}{3} \right) \right]$$

$$\boxed{\text{Volume} \approx 0.99\pi \text{ units}^3}$$



$Y = -x^2 + 4x$

$D(a,0) \to A = (a, -a^2 + 4a)$

$L = -a^2 + 4a$

$|OD| = |CF| = a$

$|CD| = W = 4 - 2a$

QUESTION 9. (8 points)
We want to construct a rectangle ABCD (see picture) of maximum area between the x-axis and the curve $y = -x^2 + 4x$. Find the length and the width of such rectangle. (Hint: Note that the curve intersects x-axis at $x = 0$ and at $x = 4$. Let O be the origin (0, 0) and F be (4, 0). Then |OD| = |CF|)

$A = W \times L$

$L = -a^2 + 4a$, $W = 4 - 2a \to A = (4-2a)(-a^2+4a)$

$A = -4a^2 + 16a + 2a^3 - 8a^2 = 2a^3 - 12a^2 + 16a$

$A' = 6a^2 - 24a + 16 = 0 \to 2(3a^2 - 12a + 8) = 0$

$\to 3a^2 - 12a + 8 = 0 \to a = \dfrac{12 \pm \sqrt{48}}{2(3)}$

$A'' = 12a - 24 \to a = \dfrac{12 + \sqrt{48}}{6} \to A'' > 0$

$a = \dfrac{12 - \sqrt{48}}{6} \to A'' < 0 \to$ Area max. when $a = \dfrac{12 - \sqrt{48}}{6}$

$\boxed{L = \dfrac{8}{3}}$, $\boxed{W = \dfrac{4\sqrt{3}}{3}}$ $\to$ $\boxed{A = \dfrac{32\sqrt{3}}{9} \text{ units}^2}$

Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: abadawi@aus.edu, www.ayman-badawi.com

4 Worked out Solutions for all Assessment Tools

4.1 Solution for Quiz I

15/15

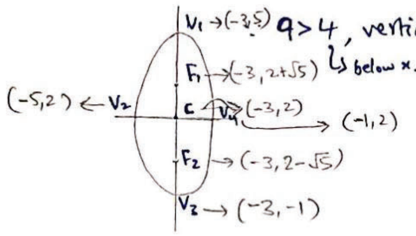
Name KHADEEJA MOOPAN, ID 87433

Quiz One, MTH 111 , Fall 2020

Ayman Badawi

QUESTION 1. consider the ellipse $\frac{(y-2)^2}{9} + \frac{(x+3)^2}{4} = 1$

(i) Roughly, sketch such ellipse.



$9 > 4$, vertical major axis

↳ below x.

✓

(ii) Find the center c

From eq. $x+3 = x - x_0$ and $y-2 = y-y_0$.

$\therefore$ $\boxed{\text{center} \to (-3,2)}$

✓

(iii) Find the ellipse constant k.

From eq. $\left(\frac{k}{2}\right)^2 = 9 \Rightarrow \frac{k}{2} = 3$

$\therefore$ $\boxed{k=6}$

✓

(iv) Find the foci, F_1, F_2

$|\overline{CF_1}|^2 = \left(\frac{k}{2}\right)^2 - b^2$. (From eq. $b^2 = 4 \Rightarrow b = 2$)

$|\overline{CF_1}|^2 = 9 - 4 = 5 \rightarrow |\overline{CF_1}|^2 = 5$ (or) $|\overline{CF_1}| = \sqrt{5}$.

$|\overline{CF_1}| = |\overline{CF_2}|$.

$\therefore$ $\boxed{F_1 \to (-3, 2+\sqrt{5}) \text{ and } F_2 \to (-3, 2-\sqrt{5})}$

✓

(v) Find all vertices.

$|\overline{V_1V_3}| = k \Rightarrow |\overline{V_1C}| = |\overline{V_3C}| = k/2 \Rightarrow |\overline{V_1C}| = |\overline{V_3C}| = 3$

and, $|\overline{V_2C}| = |\overline{V_4C}| = b = 2$

$\therefore$ $\boxed{V_1 \to (-3,5);\ V_3 \to (-3,-1);\ V_2 \to (-5,2);\ V_4 \to (-1,2)}$

✓

(vi) Given that $Q = (x_1, y_1)$ is a point on the ellipse and $|QF_1| = 2$. Find $|QF_2|$.

$|\overline{QF_1}| + |\overline{QF_2}| = k$.

In this question, $k = 6$. and given, $|\overline{QF_1}| = 2$

$\therefore$ $2 + |\overline{QF_2}| = 6 \Rightarrow |\overline{QF_2}| = 6 - 2$

$\therefore$ $\boxed{|\overline{QF_2}| = 4}$

✓

Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: abadawi@aus.edu, www.ayman-badawi.com

4.2 Solution for Quiz II

MATH QUIZ-2

(Parabola)

NAME: Afraa Parkar

AUSID: g00088916

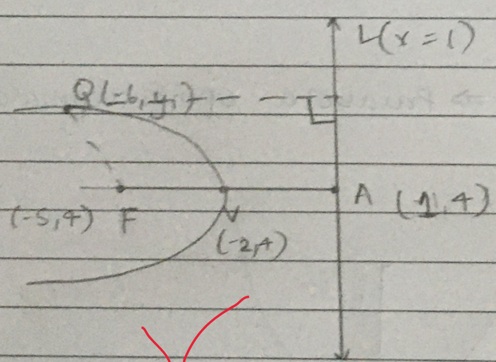
DATE: 17|09|2020

(1) (i) $-12(x+2) = (y-4)^2$

From equation, the parabola opens either towards right or left.

$4a = -12$

$\Rightarrow a = -12/4 = -3 < 0 \Rightarrow$ The parabola opens towards left.



3/3

(ii) From the equation,

Coordinates of vertex are : $\boxed{(-2, 4)}$ [Ans]

(iii) $4a = -12 \Rightarrow a = -3$

$|a| = 3$ units

Coordinates of focus F are : $(-2-3, 4)$

$\Rightarrow \boxed{(-5, 4)}$ [Ans]

2/2

(iv) $|FV| = |VA| = 3$ units.

Equation of the directrix line is $x = -2+3$

$\boxed{x = 1}$ [Ans]

PAGE No.
DATE / /

(v) For a parabola, $|QF| = |QL|$

$|QL| = |\Delta x| =$ 7 units.

$\therefore$ $\boxed{|QF| = 7\text{ units.}}$ [Ans]

2/2

(2) (i) $y = x^2 - 10x + 20$

$\Rightarrow$ $y - 20 = x^2 - 10x$

$\Rightarrow$ $y - 20 = (x-5)^2 - 25$

$\Rightarrow$ $y - 20 + 25 = (x-5)^2$

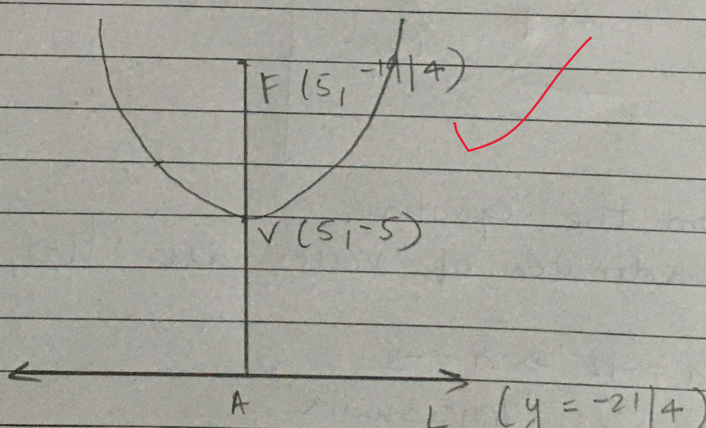
$\Rightarrow$ $\boxed{(y+5) = (x-5)^2}$ $\Rightarrow$ STANDARD FORM

3/3

From equation parabola opens either upwards or downwards.

$4a = 1$

$a = 1/4 > 0 \Rightarrow$ Parabola opens upwards.



(ii) From equation, ~~focus =~~ Vertex $= (5, -5)$

$4a = 1 \Rightarrow \underline{a = 1/4}$

2/2

Coordinates of focus F : $(5, -5 + \frac{1}{4})$

$\Rightarrow (5, -19/4)$ [Ans]

PAGE No.
DATE / /

(iii) $|FV| = |VA| = 1/4$

$\therefore$ Equation of directrix line is $y = -5 - \frac{1}{4}$

$\boxed{y = \frac{-21}{4}}$ [Ans]

2/2

$-5 + \frac{1}{4}$

$-20+1$

4.3 Solution for Quiz III

Name KHADEEJA MOOPAN, ID 87433

Quiz three, MTH 111 , Fall 2020

Ayman Badawi

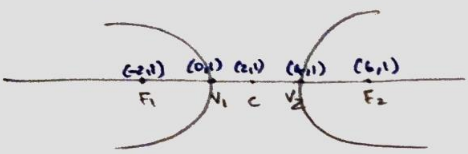
15/15

QUESTION 1. (SHOW THE WORK)

consider the hyperbola $\frac{(x-2)^2}{4} - \frac{(y-1)^2}{12} = 1$

(i) Roughly, sketch such hyperbola.

+ve x → right-left.



✓

(ii) Find the center c

From eq. center $c \to \boxed{(2,1)}$ ✓

(iii) Find the hyper-constant k.

From eq. $\left(\frac{k}{2}\right)^2 = 4$

$\Rightarrow \frac{k}{2} = 2$

$\Rightarrow \boxed{k = 4}$ ✓

(iv) Find the foci, F_1, F_2

From eq. $\left(\frac{k}{2}\right)^2 = 4$ and $b^2 = 12$.

$|\bar{C}F_1| = |\bar{C}F_2| = \sqrt{\left(\frac{k}{2}\right)^2 + b^2} = \sqrt{4+12} = \sqrt{16} = 4$

$\therefore |\bar{C}F_1| = |\bar{C}F_2| = 4$.

$\Rightarrow F_1 \to (2-4,1) \Rightarrow \boxed{F_1 \to (-2,1)}$

$\Rightarrow F_2 \to (2+4,1) \Rightarrow \boxed{F_2 \to (6,1)}$ ✓

(v) Find all vertices.

$|V_1V_2| = k = 4$. and $|CV_1| = |CV_2| = \frac{k}{2} = 2$

$\therefore V_1 \to (2-2,1) \Rightarrow \boxed{V_1 \to (0,1)}$

$V_2 \to (2+2,1) \Rightarrow \boxed{V_2 \to (4,1)}$ ✓

(vi) Given that $Q = (x_1, y_1)$ is a point on the hyperbola and $|QF_1| = 3$. Find $|QF_2|$.

For Hyperbola, we have $||QF_1| - |QF_2|| = k$.

Given $|QF_1| = 3$ and we know that $k = 4$.

So, $\cancel{3 - |QF_1| = 4}$ $||QF_2| - 3| = 4$.

$\therefore \boxed{|QF_2| = 7}$ ✓

Faculty information

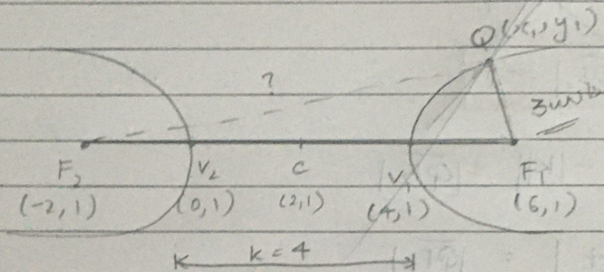
Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: abadawi@aus.edu, www.ayman-badawi.com

QUIZ #3
(Hyperbola)

15/15

NAME: Afraa Parkar DATE: 24/09/2020
~~CLASS:~~ AUS ID: g00088916

(A) (i) Since $(x-2)^2$ is a positive term the hyperbola is in the right-left direction.



3/3

(ii) From the equation,
The coordinates of centre are : $\boxed{(2,1)}$ [Ans]

2/2

(iii) From the given equation,
$\left(\frac{k}{2}\right)^2 = 4 \Rightarrow \frac{k}{2} = 2 \Rightarrow \boxed{k = 4}$ [Ans]

2/2

(iv) For a hyperbola,
$|\overline{CF_1}| = |\overline{CF_2}| = \sqrt{(k/2)^2 + b^2} = \sqrt{4+12} = \sqrt{16} = 4$ units

3/3

$\therefore$ Coordinates of F_1 : $(2+4, 1) \Rightarrow \boxed{(6,1)}$ [Ans]
$\therefore$ Coordinates of F_2 : $(2-4, 1) \Rightarrow \boxed{(-2,1)}$ [Ans]

(v) From equation, $k/2 = 2$
and, centre $= (2,1)$

3/3

$\therefore$ Coordinates of V_1 : $(2+2, 1) \Rightarrow \boxed{(4,1)}$ [Ans]
Coordinates of V_2 : $(2-2, 1) \Rightarrow \boxed{(0,1)}$ [Ans]

$1 = \cancel{|\overline{QF_2}|}$

$\therefore \boxed{|\overline{QF_2}| = \cancel{1}}$ ~~[Ans]~~

(vi) $\left.\begin{array}{l} Q = (x_1, y_1) \\ |\overline{QF_1}| = 3 \end{array}\right\}$ given.

$\left| |\overline{QF_1}| - |\overline{QF_2}| \right| = k.$ (k can be -4 or $+4$)

$\left| |\overline{QF_1}| - k \right| = |\overline{QF_2}|$ (Here, $k = -4$)

$\Rightarrow |3 - (-4)| = |\overline{QF_2}|$

$\Rightarrow 7 = |\overline{QF_2}|.$

$\therefore \boxed{|\overline{QF_2}| = \underline{\underline{7 \text{ units}}}}$ [Ans].

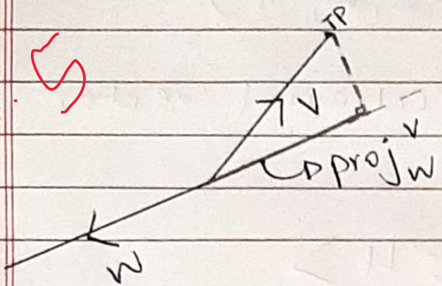
4.4 Solution for Quiz IV

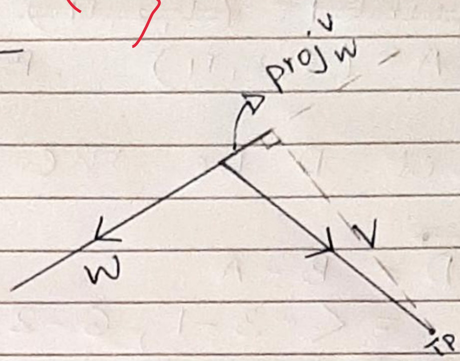
Name: Joan Dsilva
ID: 900087567
Course: MTH111
Section: 01
Date: 8th Oct 2020

15/15

Quiz 4

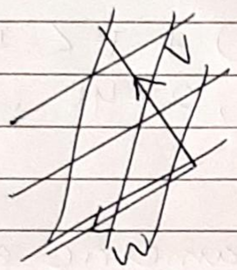
1. 1.5

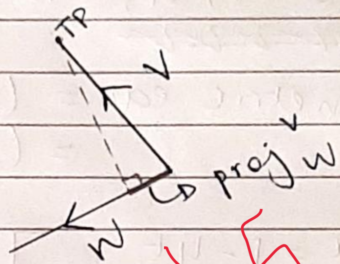




(TP = Terminal point)

1.5





1.5

4.5/4.5

2. $v = \langle 3, 4 \rangle \quad w = \langle -1, 4 \rangle$

$$\left| proj_v^w \right| = \frac{|w \cdot v|}{|v|}$$

$$= \frac{|(-1 \times 3) + (4 \times 4)|}{|\sqrt{9+16}|}$$

$$= \frac{|-3+16|}{|5|}$$

2/2

$$= \frac{13}{5}$$

✓

3. ~~Point 1 = (1, 2, 4)~~

$A = (1, 2, 4) \qquad B = (-3, 6, -8)$

Let D be the directional vector.

$\vec{D} = B - A$

$= \langle -3-1,\ 6-2,\ -8-4 \rangle$

$D = \langle -4, 4, -12 \rangle$

parametric eq $= (1, 2, 4) + t\langle -4, 4, -12 \rangle$

$= (1-4t,\ 2+4t,\ 4-12t)$

$$L: \left.\begin{aligned} x &= 1-4t \\ y &= 2+4t \\ z &= 4-12t \end{aligned}\right] \to \text{parametric equation.}$$

4. $L_1: \left.\begin{aligned} x &= t+2 \\ y &= -2t+1 \\ z &= -t+4 \end{aligned}\right\} t \in R \qquad L_2: \left.\begin{aligned} x &= -3w+13 \\ y &= 2w-9 \\ z &= 3w-7 \end{aligned}\right\} w \in R$

If L_1 intersects L_2, then L_1 values equal to L_2 values.

$\Rightarrow t+2 = -3w+13$

$\Rightarrow t+3w = 11$

$\Rightarrow 2w-9 = -2t+1$

$\Rightarrow 2t+2w = 10$

$\Rightarrow t+w = 5$

$$\begin{aligned} t+3w &= 11 \\ -\quad t+w &= 5 \\ \hline 2w &= 6 \\ w &= 3 \end{aligned}$$

$\therefore t = 2.$

$t + 3 \times 3 = 11$

$t = 2.$

$L_1 : z = -t + 4$
$z = -2 + 4$
$z = 2$

$L_2 : z = 3w - 7$
$z = 3 \times 3 - 7$
$z = 2$

~~Since the values of x, y, z of L_1 and L_2 are the same, L_1 and L_2 intersects~~

1.5/1.5

Intersecting point $= (4, -3, 2)$

$t + 2 = 4$
$-2 \times 2 + 1 = -3$
$-2 + 4 = 2$

If L_1 is perpendicular to L_2, then $D_1 \cdot D_2 = 0$.

~~$D_1 \cdot D_2 =$~~ $D_1 = \langle 1, -2, -1 \rangle$
$D_2 = \langle -3, 2, 3 \rangle$

$D_1 \cdot D_2 = (1 \times -3) + (-2 \times 2) + (-1 \times 3)$
$= \cancel{-3 + (-4)}\ -3 - 4 - 3$
$= -10.$

~~Since~~ $D_1 \cdot D_2 \neq 0$. $\therefore$ The lines L_1 and L_2 are not perpendicular.

QUIZ 8- VECTOR & 3D

14/15

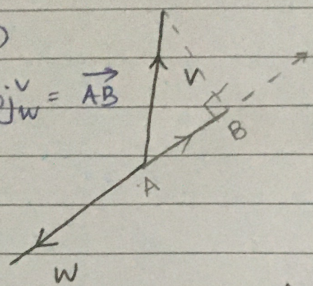
NAME: Afraa Parkar

AUS ID: g00088916

DATE: 8|10|20

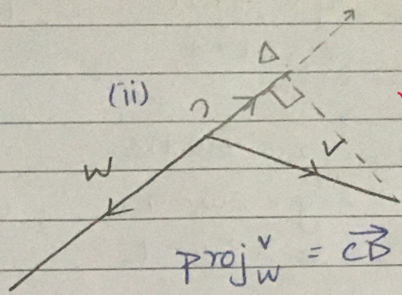
(1) (i) $proj_w^v = \vec{AB}$

1.5



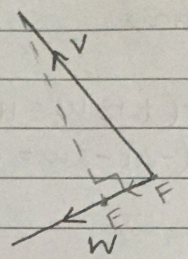
(ii) $proj_w^v = \vec{CD}$

1.5



(iii) $proj_w^v = \vec{FE}$

1.5



4.5/4.5

(2) $|v \cdot w| = 3(-1) + 4(4)$

$= -3 + 16 = \underline{\underline{13}}$

$|v| = \sqrt{(\Delta x)^2 + (\Delta y)^2} = \sqrt{9+16} = \cancel{25}\ \underline{\underline{5}}$

N/N

$\therefore \left|proj_v^w\right| = \dfrac{|v \cdot w|}{|v|} = \boxed{\dfrac{13}{5}}$ (Ans)

(3) Let Q_1 be $(1,2,4)$ and Q_2 be $(-3,6,-8)$

then,

$\overrightarrow{Q_1Q_2} = Q_2 - Q_1$

$= \langle -4, 4, -12 \rangle$

then,

$L_1 : (1,2,4) + t\langle -4, 4, -12 \rangle$

3/3

$$\boxed{L_1 : \begin{array}{l} x = -4t+1 \\ y = 4t+2 \qquad , t \in \mathbb{R} \\ z = -12t+4 \end{array}}$$

$\Rightarrow$ Parametric equation

(4) $L_1: x = t+2$

$y = -2t+1$, $t \in \mathbb{R}$

$z = -t+4$

$L_2: x = -3w+13$

$y = 2w-9$, $w \in \mathbb{R}$

$z = 3w-7$

Forming two equations,

$t+2 = -3w+13 \Rightarrow (t+3w=11) \times -2 \Rightarrow -2t - 6w = -22$ ①

$-2t+1 = 2w-9 \Rightarrow (-2t-2w=-10) \times 1 \Rightarrow -2t - 2w = -10$ ②

(+) (+) (+)

$-4w = -12$

$\Rightarrow w = 3$

Substituting w in eq. ①,

$-2t - 18 = -22$

$-2t = -22+18 = -4$

$t = 2$

$\therefore w = 3$ & $t = 2$

Now, substituting t & w in L_1 & L_2 respectively,

$L_1: x = 4$, $L_2: x = 4$

$y = -3$, $y = -3$

$z = 2$, $z = 2$

$\therefore$ The point of intersection of L_1 and L_2 is $\boxed{(4,-3,2)}$

[Ans]

5/6

L_1 is not perpendicular to L_2 since
D_1.D_2 = <1, -2, -1>.<-3, 2, 3> = -3 + -4 + -3 = -10 not equal 0

4.5 Solution for Quiz V

Name: Joan Dsilva ID: g00087567
Course: MTH 111
Section: 01
Date: 22nd Oct 2020

papergrid
Date: / /

Quiz 5.

15/15

1. $Q_1 = (-2, 1, 3)$
$Q_2 = (4, 2, 4)$
$Q_3 = (0, 5, 5)$

Let $V = \overrightarrow{Q_1Q_2}$ and $W = \overrightarrow{Q_1Q_3}$

$V = \overrightarrow{Q_1Q_2} = \langle 6, 1, 1 \rangle$
$W = \overrightarrow{Q_1Q_3} = \langle 2, 4, 2 \rangle$

$$V \times W = \begin{vmatrix} i & j & k \\ 6 & 1 & 1 \\ 2 & 4 & 2 \end{vmatrix}$$

$-66 - 4$
$= -70 + 10$
$= -60$

$= \langle 2 - 4, -(12 - 2), 24 - 2 \rangle$
$= \langle -2, -10, 22 \rangle = N$

Let $A = (x, y, z)$ be a point ~~on the~~

$\overrightarrow{Q_1A} = \langle x + 2, y - 1, z - 3 \rangle$

$\vec{N} \cdot \overrightarrow{Q_1A} = -2(x+2) - 10(y-1) + 22(z-3) = 0$
$= -2x - 4 - 10y + 10 + 22z - 66 = 0$
$= -2x - 10y + 22z = 60$ (simplified)

papergrid

Date: / /

2. P : $-2x + 6y + z = 2$.

(i) $V = \langle 8, 4, -8 \rangle$

Normal, $N = \langle -2, 6, 1 \rangle$

$\vec{N} \cdot \vec{V} = -16 + 24 - 8$

$= -24 + 24$

$= \underline{\underline{0}}$.

Since dot product of Normal vector of plane and the vector given is zero, the vector lies ~~entirely~~ inside the plane.

(ii) let $Q = (8, -4, -8)$

P: $-2x + 6y + z = 2$.

$\Rightarrow -2(8) + 6(-4) + (-8)$

$\Rightarrow -16 - 24 - 8$

$\Rightarrow -48 \neq 2$

Hence the point does not lie on the plane.

papergrid

Date: / /

(iii) $L: \left.\begin{aligned} x &= t-3 \\ y &= 2t-1 \\ z &= -9t-2 \end{aligned}\right\} t \in R.$

$P: -2x+6y+z=2.$

$12t-9t-2t$

t

$-2(t-3)+6(2t-1)-9t-2$

$\Rightarrow -2t+\cancel{6}+12t-\cancel{6}-9t-2$

$\Rightarrow t-2=2$

$\Rightarrow t=4.$

Hence the line intersects the plane at $t=4$.

$L: x=1$
$y=7$
$z=-38$

Therefore, intersection point of line and plane is $\underline{\underline{(1,7,-38)}}$

4.6 Solution for Quiz VI

Quiz 6 Dina Saed Abu Alfeilat.

11-05

15/15

Q1) $P_1: -2x+4y+3z=6$

$P_2: x-y+2z=4$

$N_1 = \langle -2, 4, 3 \rangle$

$N_2 = \langle 1, -1, 2 \rangle$

$$N_1 \times N_2 = \begin{vmatrix} i & j & k \\ -2 & 4 & 3 \\ 1 & -1 & 2 \end{vmatrix}$$

$$= \left\langle \begin{vmatrix} 4 & 3 \\ -1 & 2 \end{vmatrix}, -\begin{vmatrix} -2 & 3 \\ 1 & 2 \end{vmatrix}, \begin{vmatrix} -2 & 4 \\ 1 & -1 \end{vmatrix} \right\rangle$$

$= \langle 11, -(-7), -2 \rangle$

$= \langle 11, 7, -2 \rangle.$

Assume $z=0$.

$-2x+4y=6$

$(x-y=4) \times 2$

$\cancel{2x} - 2y = 8.$ +

$-\cancel{2x} + 4y = 6$

$2y = 14$ $\boxed{y=7}$

$-2x+4(7)=6$

$-2x+28=6$

$\cancel{-}2x = \cancel{-}22$

$\boxed{x=11}$

common point $(11, 7, 0)$.

$$\boxed{L: \begin{aligned} x &= 11t + 11 \\ y &= 7t + 7 \\ z &= -2t. \end{aligned}}$$

Is $P_1 \perp P_2$?

$N_1 = \langle -2, 4, 3 \rangle$

$N_2 = \langle 1, -1, 2 \rangle$

$N_1 \cdot N_2 = (-2)(1) + (4)(-1) + (3)(2)$

$= -2 - 4 + 6 =$

$-6 + 6 = 0.$

Yes they're perpendicular as dot product is zero.

Q2) Given $Q = (1, 2, 4)$

$$L: \begin{aligned} x &= t+1 \\ y &= -2t+3 \\ z &= 2t+5 \end{aligned}$$

$P_1: 2x+y+z=42.$

2i) $|QL| = \dfrac{|V \times D|}{|D|}$ $D = \langle 1, -2, 2 \rangle$

$I = (1, 3, 5)$

$V = \vec{IQ} = \langle 0, -1, -1 \rangle$

$$V \times D = \begin{vmatrix} i & j & k \\ 1 & -2 & 2 \\ 0 & -1 & -1 \end{vmatrix}$$

$$= \left\langle \begin{vmatrix} -2 & 2 \\ -1 & -1 \end{vmatrix}, -\begin{vmatrix} 1 & 2 \\ 0 & -1 \end{vmatrix}, \begin{vmatrix} 1 & -2 \\ 0 & -1 \end{vmatrix} \right\rangle$$

$= \langle 4, -(-1), -1 \rangle$

$= \langle 4, 1, -1 \rangle.$

$= \sqrt{(4)^2 + (1)^2 + (-1)^2}$

$= \sqrt{18}.$

$|D| = \sqrt{(1)^2 + (-2)^2 + (2)^2}$

$= \sqrt{9} = 3.$

$\boxed{|QL| = \dfrac{\sqrt{18}}{3} \text{ units}}$

2ii) $|QP_1| = \dfrac{|2(1) + (2) + (4) - 42|}{\sqrt{(2)^2 + (1)^2 + (1)^2}}$

$= \dfrac{|34|}{\sqrt{6}} = \boxed{\dfrac{34}{\sqrt{6}} \text{ units}}$

Q3) i) $y = 2x^3 + \sqrt[5]{x^3}$

$y = 2x^3 + x^{3/5}$

$\boxed{y' = 6x^2 + \tfrac{3}{5}x^{-2/5}}$

$\frac{3}{5} - \frac{1}{1} =$

$\frac{3}{5} - \frac{5}{5} = -2/5$

2/2

Q3ii) $y = \dfrac{2x+4}{x^5}$

$\frac{2x}{x^5} + \frac{4}{x^5}$

$= \frac{2}{x^4} + \frac{4}{x^5}$

$= 2x^{-4} + 4x^{-5}$

$= -8x^{-5} - 20x^{-6}$.

$\boxed{\Rightarrow y' = -8x^{-5} - 20x^{-6}}$

2/2

Name ~~Noor Abu El Soud~~, ID g00087597

MTH 111, Fall 2020, 1–1

Quiz Six, MTH 111 , Fall 2020

Ayman Badawi

Parametric equations

$$\boxed{\begin{aligned} x &= 11t+11 \\ y &= 7t+7 \\ z &= -2t \end{aligned}}$$

QUESTION 1. $P_1: -2x+4y+3z=6$ and $P_2: x-y+2z=4$ intersect at a line L find a parametric equations of L.

① $D = N_1 \times N_2 = \begin{vmatrix} i & j & k \\ -2 & 4 & 3 \\ 1 & -1 & 2 \end{vmatrix}$ $\left\langle \begin{vmatrix} 4 & 3 \\ -1 & 2 \end{vmatrix}, -\begin{vmatrix} -2 & 3 \\ 1 & 2 \end{vmatrix}, \begin{vmatrix} -2 & 4 \\ 1 & -1 \end{vmatrix} \right\rangle$

$N_1 = \langle -2, 4, 3 \rangle$

$N_2 \langle 1, -1, 2 \rangle$

$8 - -3,\ -(-4-3),\ 2-4$

② → then let $z=0$ ⇒ $-2x+4y=6$, $(x - y = 4)^{\times 4}$

$-2x+4y=6$

$4x-4y=16$

$2x=22$ $\boxed{x=11}$

$D = \langle 11, 7, -2 \rangle$

substitute: $-2(11)+4y=6$, $-22+4y=6$, $4y=28$, $\boxed{y=7}$

→ The point $(11,7,0)$ lies on intersection line

$-22-4+6=0$

Is P_1 perpendicular to P_2?

$N_1 \perp N_2$ $|N_1 \cdot N_2 = 0|$ $N_1 = \langle -2,4,3 \rangle \cdot N_2 \langle 1,-1,2 \rangle$

$-2-4+6 = 0$

$\boxed{P_1 \text{ is perpindicular to } P_2}$ $\boxed{P_1 \perp P_2}$ → yes!

QUESTION 2. Given $Q = (1,2,4)$ is not on the line $L: x = t+1, y = -2t+3, z = 2t+5$ and Q is not on the plane $P: 2x+y+z=42$

i) Find $|QL|$

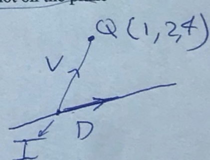
$V \times D = \begin{vmatrix} i & j & k \\ 0 & -1 & -1 \\ 1 & -2 & 2 \end{vmatrix} = \left\langle \begin{vmatrix} -1 & -1 \\ -2 & 2 \end{vmatrix}, -\begin{vmatrix} 0 & -1 \\ 1 & 2 \end{vmatrix}, \begin{vmatrix} 0 & -1 \\ 1 & 2 \end{vmatrix} \right\rangle$

$-2-2,\ -(0--1),\ 0--1$

$\langle -4, -1, 1 \rangle$

$V = \vec{IQ} = \langle 0, -1, -1 \rangle$

$D = \langle 1, -2, 2 \rangle$



when $t=0$

↳ $I = (1,3,5)$

$|V \times D| = \sqrt{16+1+1} = \sqrt{18}$

$|D| = \sqrt{1+4+4} = \sqrt{9} = 3$

$|QL| = \dfrac{|V \times D|}{|D|} = \dfrac{\sqrt{18}}{3} = \boxed{\sqrt{2}}$

ii) Find $|QP|$

$Q = (1,2,4)$ $P: 2x+y+z-42=0$ substitute point

$\dfrac{|2(1)+2+4-42|}{|N|}$ $N = \langle 2,1,1 \rangle$ $|N| = \sqrt{4+1+1} = \sqrt{6}$

$\boxed{|QP| = \dfrac{34}{\sqrt{6}}}$

QUESTION 3. i) Let $y = 2x^3 + \sqrt[5]{x^3} + 10$. Find y'.

$y = 2x^3 + x^{3/5} + 10$

$\frac{3}{5} - \frac{5}{5}$

$\boxed{y' = 6x^2 + \frac{3}{5}x^{-2/5}}$

ii) Let $\dfrac{2x+4}{x^5}$. Find y'.

$y = (2x+4)x^{-5}$ $y = 2x^{-4} + 4x^{-5}$ $\boxed{y' = -8x^{-5} + -20x^{-6}}$

Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: abadawi@aus.edu, www.ayman-badawi.com

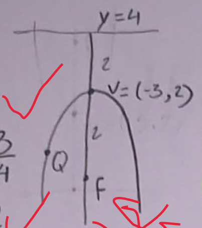
4.7 Solution for EXAM I

Exam 1, Fall 2020

Zuha Sabbagh
Oct. 13, 2020
g00088345

55/55

1) $-8(y-2)=(x+3)^2$

i) 

$\cancel{4}a=\frac{8}{4}$

$a=2$

2/2

ii) $V=(-3,2)$

iii) $F=(-3,0)$

iv) $y=2+2=4$

v) $|QF|=|QL|=-16-4=\boxed{20}$

2) $2y=x^2+6x+13$

i) $2y-13=x^2+6x$

$2y-13=(x+3)^2-9$

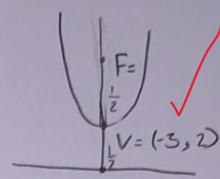
$2y-13+9=(x+3)^2$

$2y-4=(x+3)^2$

$2(y-2)=(x+3)^2$

$\cancel{4}a=\frac{2}{4}$

$a=\frac{1}{2}$



2/2

ii) $F=(-3,2+\frac{1}{2})$

$F=(-3,\frac{5}{2})$

2/2

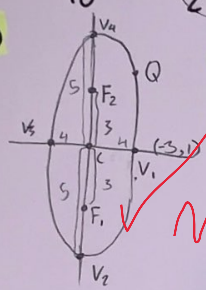
iii) $y=2-\frac{1}{2}=\frac{3}{2}$

2/2

3) $\frac{(x+3)^2}{16}+\frac{(y-1)^2}{25}=1$

$\sqrt{b^2}=\sqrt{16}$

$b=4$

i) 

ii) $C=(-3,1)$

iii) $\sqrt{\left(\frac{K}{2}\right)^2}=\sqrt{25}$

$K=5\times2=10$

2/2

iv) $\left(\frac{K}{2}\right)^2=b^2+|CF|^2$

$\sqrt{25-16}=CF$

$CF=3$

$F_1=(-3,1-3)=(-3,-2)$

$F_2=(-3,1+3)=(-3,4)$

2/2

v) $V_1=(-3+4,1)=(1,1)$ $V_2=(-3,1-5)=(-3,-4)$

$V_3=(-3-4,1)=(-7,1)$ $V_4=(-3,1+5)=(-3,6)$

2/2

vi) $|QF_1|=7$

$|QF_1|+|QF_2|=K$

$7+QF_2=10$

$\boxed{|QF_2|=3}$

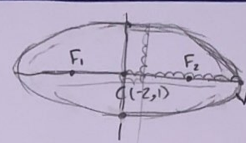
2/2

4) ellipse

$C=(-2,1)$

$V=(10,1)$

$V=(-2,6)$



$b=5$

$K=2\times12=24$

$\left(\frac{K}{2}\right)^2=b^2+|CF|^2$

$\sqrt{\left(\frac{24}{2}\right)^2-5^2}=CF$

$CF=10.9$

2/2

i) $F_1=(-2-10.9,1)$

$=(-12.9,1)$

$F_2=(-2+10.9,1)$

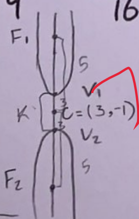
$=(8.9,1)$

2/2

ii) $\frac{(x+2)^2}{\left(\frac{24}{2}\right)^2}+\frac{(y-1)^2}{25}=1$

$\frac{(x+2)^2}{144}+\frac{(y-1)^2}{25}=1$

5) $\frac{(y+1)^2}{9}-\frac{(x-3)^2}{16}=1$

i) 

2/2

ii) $\sqrt{\left(\frac{K}{2}\right)^2}=\sqrt{9}$

$K=3\times2=6$

2/2

iii) $CF=\sqrt{9+16}$

$CF=5$

$F_1=(3,-1+5)=(3,4)$

$F_2=(3,-1-5)=(3,-6)$

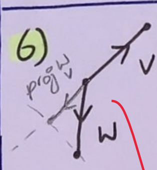
2/2

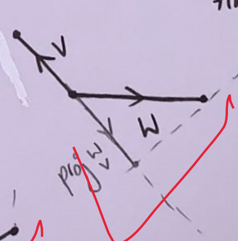
iv) $V_1=(3,-1+3)=(3,2)$

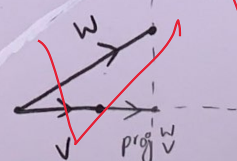
$V_2=(3,-1-3)=(3,-4)$

2/2

6) find $\text{Proj}_{\vec{v}}^{\vec{w}}$







3/3

Scanned with CamScanner

7) $L_1: x = 4t+2$ $\quad L_2: x = -2w+12$
$y = -2t+1$ $\quad y = -2w-1$
$z = t+4$ $\quad z = 4w+2$

① if L_1 intersects L_2, then find intersection point?

$4t+2 = -2w+12$
$2w = -4t-2+12$
$\frac{2w}{2} = \frac{-4t}{2} + \frac{10}{2}$
$\boxed{w = -2t+5}$

$-2t+1 = -2w-1$
$-2t+1 = -2(-2t+5)-1$
$-2t+1 = +4t-10-1$
$4t+2t = 10+1+1$
$\frac{6t}{6} = \frac{12}{6}$
$\boxed{t=2}$

$w = -2(2)+5$
$w = -4+5$
$\boxed{w=1}$

$x = 4(2)+2$
$x = 10$
$y = -2(2)+1$
$y = -3$
$z = 2+4$
$z = 6$

$x = -2(1)+12$
$x = 10$
$y = -2(1)-1$
$y = -3$
$z = 4(1)+2$
$z = 6$

yes L_1 intersects L_2 at point $(10,-3,6)$

4X

② is L_1 perpendicular to L_2?
$D_1 = \langle 4,-2,1\rangle$ $\quad D_2 = \langle -2,-2,4\rangle$

$4(-2) + (-2)(-2) + 1(4) = 0$
$-8+4+4 = 0$
$0 = 0$

L_1 is perpendicular to L_2 based off dot product.

1/1

③ Find the symmetric equation of the line L_1

$$\frac{x-2}{4} = \frac{y-1}{-2} = z-4$$

2/2

8) $a = (-4,2)$
$b = (1,-1)$
$c = (4,5)$

$V = (1--4, -1-2) = (5,-3)$ turn 3D $(5,-3,0)$
$W = (4--4, 5-2) = (8,3)$ turn 3D $(8,3,0)$

$$\begin{vmatrix} i & j & k \\ 5 & -3 & 0 \\ 8 & 3 & 0 \end{vmatrix}$$

$V\cdot W = \left\langle \begin{bmatrix} -3 & 0 \\ 3 & 0 \end{bmatrix}, -\begin{bmatrix} 5 & 0 \\ 8 & 0 \end{bmatrix}, \begin{bmatrix} 5 & -3 \\ 8 & 3 \end{bmatrix} \right\rangle$

$V\cdot W = \langle 0, 0, 5(3)-(-3)(8)\rangle = \langle 0,0,39\rangle$

$A = \frac{1}{2}|V\cdot W|$
$A = \frac{1}{2}\sqrt{0^2+0^2+39^2}$
$A = \frac{1}{2}(39) = \boxed{\frac{39}{2} \text{ units}}$

2/2

9) $L_1: x = 3t+2$ $\quad L_2: x = -6w+14$
$y = 2t+1$ $\quad y = -4w+9$
$z = 3t+4$ $\quad z = -6w+16$

$D_1 = \langle 3,2,3\rangle$ $\quad D_2 = \langle -6,-4,-6\rangle$

$D_1 = tD_2$
$\frac{3}{-6} = \frac{t\,\cancel{-6}}{\cancel{-6}}$
$t = -\frac{1}{2}$
$D_1 = -\frac{1}{2}D_2$

$D_1 \parallel D_2$

$t=0$:
$x=2$
$y=1$
$z=4$
$\Rightarrow$

$2 = -6w+14$
$\frac{-12}{-6} = \frac{\cancel{-6}w}{\cancel{-6}}$
$w=2$

$1 = -4w+9$
$\frac{-8}{-4} = \frac{\cancel{-4}w}{\cancel{-4}}$
$w=2$

$4 = -6w+16$
$\frac{-12}{-6} = \frac{\cancel{-6}w}{\cancel{-6}}$
$w=2$

2/2

L_1 is not parallel to L_2, they are collinear

Scanned with CamScanner

4.8 Solution for EXAM II

Sohila Salah Afia
900086497

Q1 i) $L: x = -2t+8$ $\quad P: x+2y+z=12.$

$y = t-1$

$z = t+3$

$(-2t+8)+2(t-1)+(t+3)=12.$

$\cancel{-2t}+8\cancel{+2t}-2+t+3=12.$

$t+9=12$

$\boxed{t=3}$

line L intersects Plane P when t is 3

$-2(3)+8=x$

$3-1=y$

$3+3=z$

$\boxed{(2,2,6)}$ → point of intersection.

Q ii) $N_1 \cdot N_2 = 0$

$N_1 = \langle 2,1,3 \rangle \quad N_2 = \langle 4,6,a \rangle$

$(2\times 4)+(1\times 6)+3a=0$

$8+6+3a=0.$

$14+3a=0$

$3a=-14$

$\boxed{a=-\frac{14}{3}}$

Q1 iii)

$P_1: 2x+y+3z=4$ $\qquad$ $P_2: 4x+2y+az=8$

$4x=8$

$4=8/x$

$N_1 = cN_2$

$N_1: \langle 2,1,3\rangle = c\, N_2 = \langle 4,2,a\rangle$

$2 = c4$ $\qquad$ $1 = c2$ $\qquad$ $3 = ac$

$c = \frac{1}{2}$ $\qquad$ $c = 1/2$ $\qquad$ $3 = a\,1/2$

$\boxed{a = 6}$

Check $\rightarrow$

$y=0$ $z=0$ for P_1

$2x=4$ $\qquad$ $(2,0,0)$

$x = 4/2$

$x=2$

P_2: $4(2)+2(0)+6(0)=8$

$8=8$

these 2 planes are colinear

to be parallel a = 6. However, since they share a point/ they are co-planar / so no values of a exists where P_1 || P_2

iv) $Q(4,4,-15)$ $\qquad$ Plane: $-2x+2y-z=21$

$N = \langle -2,2,-1\rangle$

$$\frac{|-2(4)+2(4)-(-15)-21|}{\sqrt{(-2)^2+(2)^2+1^2}} = \frac{6}{3} = 2 \text{ units}$$

v) $P_1: x+4y+z=10$ $\qquad$ $P_2: -x+3y-z=11$

$N_1 = \langle 1,4,1\rangle$ $\qquad$ $N_2 = \langle -1,3,-1\rangle$

$N_1 \times N_2 = D$

$$\begin{vmatrix} i & j & k \\ 1 & 4 & 1 \\ -1 & 3 & -1 \end{vmatrix}$$

$\langle ((4\times -1)-(3\times 1)),\ -[(1\times -1)-(-1\times 1)],\ ((1\times 3)-(4\times -1))\rangle$

$\langle -7, -0, 7\rangle \rightarrow D$

let $z=0$

$x+4y=10$

$-x+3y=11$

$x=-2$ $y=3$

point: $(-2,3,0)$

line equation

$L: x = -7t-2$

$y = 3$

$z = 7t$

vi) $V = \langle 1, 4, 11 \rangle$ $\quad$ $P: 5x + 7y - 3z = 19.$

$N = \langle 5, 7, -3 \rangle$

$V \cdot N = 0$ then yes $\quad$ $V \cdot N \neq 0$ then No.

$(1\times5) + (4\times7) + (-3\times11)$

$5 + 28 - 33 = 0$ $\quad$ $V \cdot N = 0$

So the vector V ~~yes~~ can be drawn in Plane P.

vii). $Q = (1, 2, 4)$ $\quad$ $L: x = t + 3.$

$y = -2t + 1$

$z = 2t + 4$

$D = \langle 1, -2, 2 \rangle$

let $t = 0$

$x = 3 \quad y = 1 \quad z = 4$ $\quad$ $I = (3, 1, 4).$

$\overrightarrow{QI} = V$

$V = \langle -2, 1, 0 \rangle$

$$\begin{vmatrix} i & j & k \\ -2 & 1 & 0 \\ 1 & -2 & 2 \end{vmatrix}$$

$$\frac{|V \times D|}{|D|} = |QL|$$

$\big((1\times2) - (-2\times0)\big), -\big|(-2\times2) - (0\times1)\big|, (-2\times-2) - (1\times1)$

$$\frac{\sqrt{2^2 + 4^2 + 3^2}}{\sqrt{1^2 + (-2)^2 + 2^2}} = \boxed{\frac{\sqrt{29}}{3} \text{ units}}$$

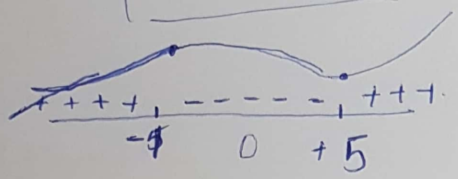
$\langle 2, 4, 3 \rangle$

Q2 i) Let $f(x) = x^3 - 6x^2 - 15x + 10$

i) $f'(x) = 3x^2 - 12x - 15$

$3x^2 - 12x - 15 = 0$

$\boxed{x = 5 \quad x = -1}$



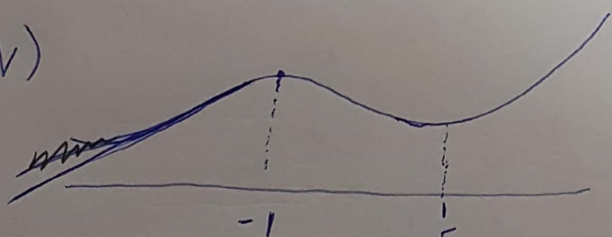
i) $f(x)$ increases ~~for~~ at ~~$(x > 5)$~~ / $(5, +\infty)$. $\cup$ $(-\infty, -1)$

ii) $f(x)$ decreases for $(-1, 5)$

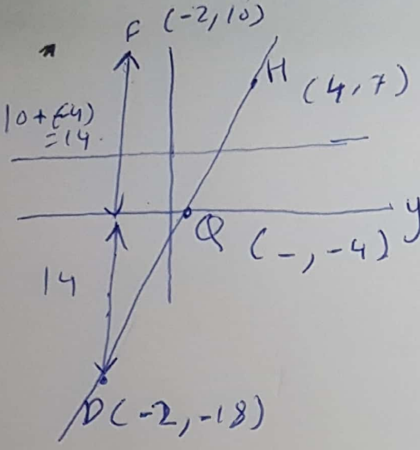
iii) Local max is at $x = -1$

Local min is at $x = +5$

iv)



Q3) $H = (4,7) \cdot F = (-2,10) \cdot$ Find Q $y = -4$ $|HQ| + |FQ|$ min.



$y = mx + c$

$m = \frac{\Delta y}{\Delta x} = \frac{7-(-18)}{4-(-2)} = \frac{25}{6}$

$\cancel{y} = \frac{25}{6}(4) + c$

$7 = \frac{50}{3} + c$

$c = -\frac{29}{3}$

$\boxed{y = \frac{25}{6}x - \frac{29}{3}}$

$\cancel{y} = -4 = \frac{25}{6}(x) - \frac{29}{3}$

$\frac{17}{3} = \frac{25}{6}(x)$

$\frac{102}{75} = x$

$x = \frac{34}{25}$

$Q = \left(\frac{34}{25}, -4\right)$

$\boxed{Q = (1.36, -4)}$

Q4 i) $y = \sqrt{3x+2} + \frac{4}{x^7} + 10$

$y = (3x+2)^{1/2} + 4x^{-7} + 10$

$\boxed{y' = \frac{1}{2}(3x+2)^{-1/2} - 28x^{-8}}$

ii) $y = e^{(7x+2)} + 10x^2 + 5$

$y' = e^{(7x+2)} \cdot (7) + 20x$

$\boxed{y' = 7e^{(7x+2)} + 20x}$

iii) $y = \ln[(2x^5 + 8x^2 - 3x)/(2x+7)^3]$

$y = \ln(2x^5 + 8x^2 - 3x) - 3\ln(2x+7)$

$y' = \frac{10x^4 + 16x - 3}{2x^5 + 8x^2 - 3x} - \frac{3(2)}{2x+7}$

$\boxed{y' = \frac{10x^4 + 16x - 3}{2x^5 + 8^2 - 3x} - \frac{6}{2x+7}}$

iv) $y = 10(3x^6 + 5x^3 + 2)^7$

$\boxed{y' = 70(3x^6 + 5x^3 + 2)^6 \cdot (18x^5 + 15x^2)}$

4.9 Solution for Final Exam

Final-
Solution

Final Exam MTH 111 Fall 2020

1.a) Area bounded $= \int_0^{\pi/4} \cos(x) - \sin(x)\,dx + \int_{\pi/4}^{\pi/2} \sin(x) - \cos(x)\,dx$

$= \left[\sin(x) + \cos(x)\right]_0^{\pi/4} + \left[-\cos(x) - \sin(x)\right]_{\pi/4}^{\pi/2}$

$= \left[\sin\left(\frac{\pi}{4}\right) + \cos\left(\frac{\pi}{4}\right)\right] - \left[\sin(0) + \cos(0)\right] + \left[-\cos\left(\frac{\pi}{2}\right) - \sin\left(\frac{\pi}{2}\right)\right]$

$- \left[-\cos\left(\frac{\pi}{4}\right) - \sin\left(\frac{\pi}{4}\right)\right]$

4/4

$= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \cancel{-0} - 1 \cancel{-0} - 1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}$

$= \frac{4}{\sqrt{2}} - 2 = 2\sqrt{2} - 2 =$ ~~0.82 sq. (approx.)~~ 0.82 sq. units (approx.)

b) Volume $= \pi \int_0^{\pi/2} \left[2 + \cos(x) - (-1)\right]^2 dx$

$= \pi \int_0^{\pi/2} \left[2 + \cos(x) + 1\right]^2 dx$

$= \pi \int_0^{\pi/2} \left[3 + \cos(x)\right]^2 dx = \pi \int_0^{\pi/2} 9 + 6\cos(x) + \cos^2 x \, dx$

$= \pi \int_0^{\pi/2} 9 + 6\cos(x) + \frac{1}{2} + \frac{1}{2}\cos(2x)\, dx$

$= \pi \left[9x + 6\sin(x) + \frac{1}{2}\cdot x + \frac{1}{4}\cdot \sin(2x)\right]_0^{\pi/2}$

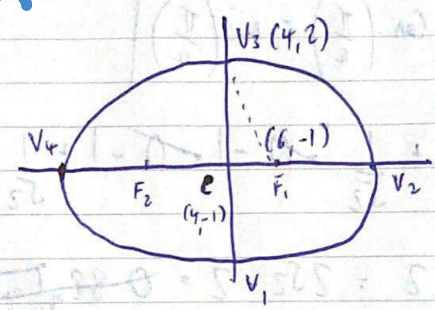
$$= \pi\left\{\left[9\cdot\frac{\pi}{2} + 6\sin\left(\frac{\pi}{2}\right) + \frac{1}{2}\cdot\frac{\pi}{2} + \frac{1}{4}\sin\left(8\cdot\frac{\pi}{8}\right)\right] - \left[0 + 6\sin(0) + 0 + \frac{1}{4}\sin(0)\right]\right\}$$

$$= \pi\left[\frac{9\pi}{2} + 6 + \frac{\pi}{4} + 0\right] - 0 = \frac{9\pi^2}{2} + 6\pi + \frac{\pi^2}{4}$$

cube units

$$= \frac{18\pi^2 + 24\pi + \pi^2}{4} = \frac{19\pi^2 + 24\pi}{4}$$

2.



a) i) From observation, $C = (4,-1)$

$B = |CV_3| = 3$ units

$|V_3F_1|^2 = |V_3C|^2 + |CF_1|^2$

$\left(\frac{k}{2}\right)^2 = 3^2 + 2^2 = 9 + 4 = 13$

$\left(\frac{k}{2}\right)^2 = 13 \Rightarrow \frac{k}{2} = \sqrt{13}$

$K = 2\sqrt{13}$

ii) $C = (4, -1)$

$V_1 = (4, -4)$

~~$V_2 = (4 + \sqrt{13}(-1))$~~ $V_2 = (4 + \sqrt{13}, -1)$

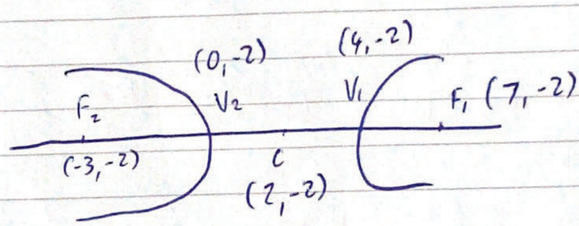
~~$V_4 =$~~ $V_4 = (4 - \sqrt{13}, -1)$

iii) $F_2 = (2, -1)$

iv) Eq. of ellipse $= \dfrac{(x-4)^2}{13} + \dfrac{(y+1)^2}{9} = 1$

6/6

3. i)



$|V_1V_2| = K$

$K = 4$

ii) $C = (2, -2)$

$F_2 = (-3, -2)$

iii) $\left(\dfrac{K}{2}\right)^2 = |CF_1|^2 + B^2$

$4 = 25 + B^2$

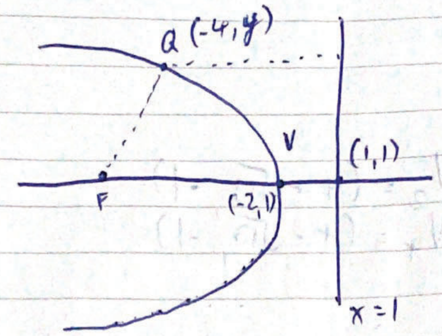
$B^2 = 21$

$B = \sqrt{21}$

$\therefore$ Eq. of hyperbola $= \dfrac{(x-2)^2}{4} - \dfrac{(y+2)^2}{21} = 1$

5/5

4.



i) $|VL| = -2 - 1 = 3$ units

$F = (-5, 1)$

ii) $|FQ| = |QL|$

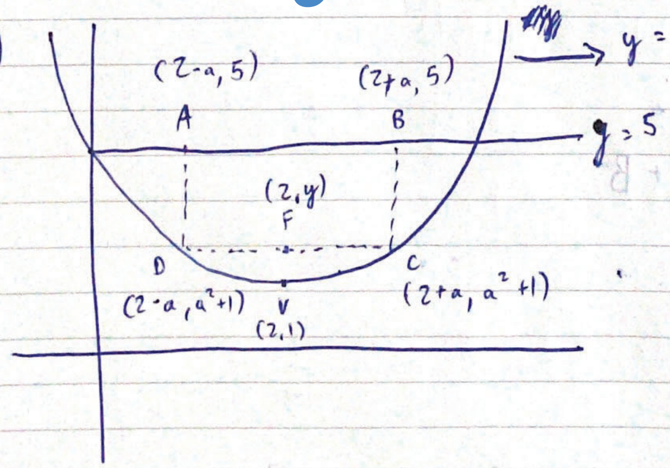
$= 5$ units

iii) $4d(x - x_0) = (y - y_0)^2$

$4(-3)(x+2) = (y-1)^2$ $(\because d = -3) \rightarrow x$ -ve

5/5

5. i)



$|AD| = 5 - (a^2+1)$

$|AB| = (2+a) - (2-a) = \cancel{2}+a - \cancel{2}+a$
$= 2a$

Rectangle of max area $\Rightarrow |AD| \times |AB|$

$y \Rightarrow [5-(a^2+1)](2a)$

$y = 10a - 2a^3 - 2a$
$= 8a - 2a^3$

$y' = 8 - 6a^2 = 0$
$y' \Rightarrow 8 = 6a^2$
$a^2 = \frac{8}{6} = \frac{4}{3}$

$a = \frac{2}{\sqrt{3}}$

$y'' = -12a \rightarrow$ Max area.

$\therefore A = \left(2 - \frac{2}{\sqrt{3}}, 5\right)$

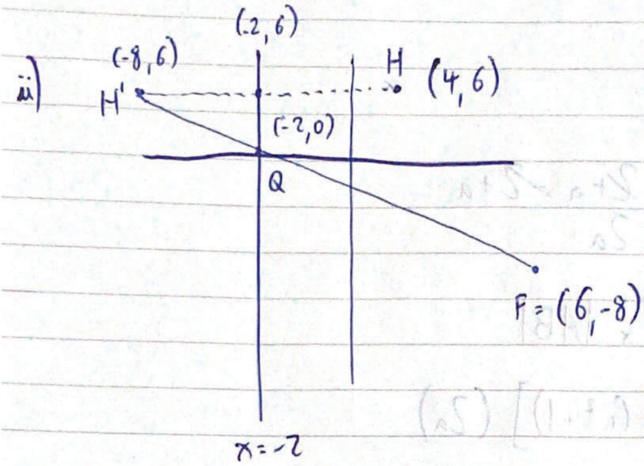
$B = \left(2 + \frac{2}{\sqrt{3}}, 5\right)$

$C = \left(2 - \frac{2}{\sqrt{3}}, \frac{4}{3}+1\right) = \left(2 - \frac{2}{\sqrt{3}}, \frac{7}{3}\right)$

$D = \left(2 + \frac{2}{\sqrt{3}}, \frac{7}{3}\right)$

5/5

ii)



$y = mx + b$

$m = \frac{-8-6}{6+8} = \frac{-14}{14} = -1$

$-8 = -1(6) + b$
$-8 = -6 + b$
$b = -8 + 6$
$= -2$

$\therefore y = -1(-2) - 2$
$= 2 - 2 = 0$

$\therefore Q = (-2, 0)$

5/5

6. i) $y = [\sin(3x) + 2x + 1]^5$

$y' = 5[\sin(3x) + 2x + 1]^4 \cdot [3\cos(3x) + 2]$

ii) $y = \ln\left[\frac{(5x+2)^4}{(3x+7)^3}\right]$

$= \ln\left[(5x+2)^4\right] - \ln\left[(3x+7)^3\right]$

$y' = \frac{4(5x+2)^3 \cdot 5}{(5x+2)^4} - \left[\frac{3(3x+7)^2 \cdot 3}{(3x+7)^3}\right]$

iii) $y = \cos(2x)\, e^{(x^2+1)}$

$y' = \left[-2(\sin(2x)) \cdot e^{(x^2+1)}\right] + \left[\cos(2x) \cdot e^{(x^2+1)} \cdot (2x)\right]$

iv) $y = \sqrt{3x+1} + \frac{4}{x^3}$

$= (3x+1)^{1/2} + 4(x)^{-3}$

$y' = \left[\frac{1}{2} \cdot (3x+1)^{-1/2} \cdot (3)\right] - \left[12\,(x)^{-4}\right]$

6/6

7. i) $\int (e^{2x} + x)\,(e^{2x} + x^2 + 1)^8\, dx$

$u = e^{2x} + x^2 + 1$

$u' = 2e^{2x} + 2x$

$\Rightarrow \frac{1}{2} \int 2(e^{2x} + x)(e^{2x} + x^2 + 1)^8\, dx$

$\Rightarrow \frac{1}{2} \cdot \frac{1}{9} \cdot (e^{2x} + x^2 + 1)^9 + C$

ii) $\int \frac{\sin(x) - 2x}{\cos(x) + x^2 + 3}\, dx$

$u = \cos(x) + x^2 + 3$

$u' = -\sin(x) + 2x$

$$= \frac{1}{-1} \int \frac{-1(\sin(x) - 2x)}{\cos(x) + x^2 + 3} dx$$

$$= \frac{1}{-1} \int -1(\sin(x) - 2x)\left[\cos(x) + x^2 + 3\right]^{-1} dx$$

$$= -\ln\left|\cos(x) + x^2 + 3\right| + C$$

4/4

8. i) $Q_1 = (2, 1, 0)$
$Q_2 = (4, 2, 0)$
$Q_3 = (-8, 3, 10)$

$$\overrightarrow{Q_1Q_2} = \langle 2, 1, 0 \rangle$$

$$\overrightarrow{Q_1Q_3} = \langle -10, 2, 10 \rangle$$

$$|Q_1Q_2 \times Q_1Q_3| \Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 0 \\ -10 & 2 & 10 \end{vmatrix}$$

$$= \hat{i}(10) - \hat{j}(20) + \hat{k}(4 + 10)$$

$$= 10\hat{i} - 20\hat{j} + 14\hat{k}$$

$$\Rightarrow \sqrt{100 + 400 + 196} = \sqrt{696}$$

Area of $\Delta = \frac{1}{2}\sqrt{696} = 13.19$ sq units. (aprox.)

ii) From prev.

$\vec{N} = Q_1Q_2 \times Q_1Q_3$

$= \langle 10, -20, 14 \rangle$

Eq. of plane $= 10(x+8) - 20(y-3) + 14(z-10)$

6/6

9. i) $L \Rightarrow x = \cancel{at+2}$ $at + 2$
$y = 4t + a$
$z = -t + b$

$N_1 = \langle a, 4, -1 \rangle$

Plane $\Rightarrow x + 2y + 3z = 13$ $\quad N_2 = \langle 1, 2, 3 \rangle$

Since line lies entirely inside the plan:

$N_1 \cdot N_2 = 0$
$a + 8 - 3 = 0$
$a = 3 - 8$
$= -5$

Sub. L in plane eq.:

$at + 2 + 2(4t + a) + 3(-t + b) = 13$
$\cancel{-5t} + 2 + \cancel{8t} - 10 \cancel{-3t} + b = 13$
$-8 + b = 13$
$b = 13 + 8$
$= 21$

$\therefore a = -5, \ b = 21$

3/3

ii) $P_1: x + 2y - z = 10 \Rightarrow N_1 = \langle 1, 2, -1 \rangle$

$P_2: -x - y + z = -7 \Rightarrow N_2 = \langle -1, -1, 1 \rangle$

$$N_1 \times N_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ -1 & -1 & 1 \end{vmatrix}$$

$$= \hat{i}(2-1) - \hat{j}(1-1) + \hat{k}(-1+2)$$

$$= \hat{i} + \hat{k} \Rightarrow \langle 1, 0, 1 \rangle$$

Let $z = 0$

$$x + 2y = 10$$
$$-x - y = -7$$

$\Rightarrow y = 3$

$\Rightarrow -x - 3 = -7$

$-x = -7 + 3$

$= -4$

$x = 4$

$\therefore$ Parametric eq. of $L \Rightarrow x = t + 4$

$y = 3$

$z = t$

5/5

iii) $P_2 = -x - y + z = -7$

$Q = (1, 4, -20)$

Dist. $\Rightarrow \dfrac{|-1-4-20+7|}{\text{Sqr}(3)} = \dfrac{|-18|}{} = 18/\text{sqrt}(3)$ units

1/2

10. Critical values $\Rightarrow -5, -3, -1, 4, 8$

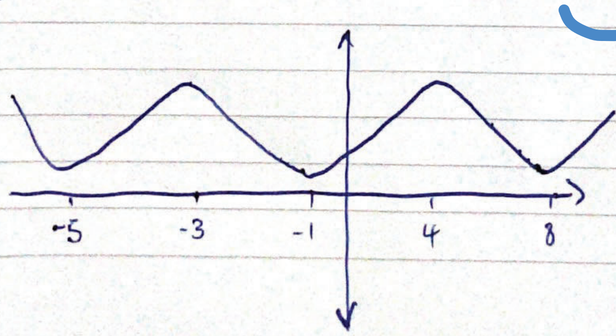
Intervals $\Rightarrow (\infty, -5); (-5, -3); (-3, -1); (-1, 4); (4, 8); (8, \infty)$
$\qquad -\qquad +\qquad -\qquad +\qquad -\qquad +$

i) Values of x where $f(x)$ have local min. $\Rightarrow -5, -1, 8$

ii) Values of x where $f(x)$ has local max. $\Rightarrow -3, 4$

iii) Values of x where $f(x)$ increases $\Rightarrow (-5, -3) \cup (-1, 4) \cup (8, \infty)$

iv)



Curve of $f(x)$

6/6

5 Section : Assessment Tools-Quizzes (unanswered)

5.1 Quiz I

Name————————————, ID ——————

Quiz One, MTH 111 , Fall 2020

Ayman Badawi

QUESTION 1. consider the ellipse $\frac{(y-2)^2}{9} + \frac{(x+3)^2}{4} = 1$

(i) Roughly, sketch such ellipse.

(ii) Find the center c

(iii) Find the ellipse constant k.

(iv) Find the foci, F_1, F_2

(v) Find all vertices.

(vi) Given that $Q = (x_1, y_1)$ is a point on the ellipse and $|QF_1| = 2$. Find $|QF_2|$.

Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: abadawi@aus.edu, www.ayman-badawi.com

5.2 Quiz II

Quiz Two, MTH 111 , Fall 2020

Ayman Badawi

QUESTION 1. consider the parabola $-12(x+2) = (y-4)^2$

(i) Roughly, sketch such Parabola.

(ii) Find the vertex, V

(iii) Find the focus, F.

(iv) Find the equation of the directrix line.

(v) Given that $Q = (-6, y_1)$ is a point on the parabola. Find $|QF|$. (Think: it is not difficult!!)

QUESTION 2. Given $y = x^2 - 10x + 20$

(i) Rewrite the equation of the parabola in the standard form and sketch (roughly).

(ii) Find the Focus F.

(iii) Find the equation of the directrix line.

Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: `abadawi@aus.edu, www.ayman-badawi.com`

5.3 Quiz III

Name——————————, ID ————

Quiz three, MTH 111 , Fall 2020

Ayman Badawi

QUESTION 1. (**SHOW THE WORK**)
consider the hyperbola $\frac{(x-2)^2}{4} - \frac{(y-1)^2}{12} = 1$

(i) Roughly, sketch such hyperbola.

(ii) Find the center c

(iii) Find the hyper-constant k.

(iv) Find the foci, F_1, F_2

(v) Find all vertices.

(vi) Given that $Q = (x_1, y_1)$ is a point on the hyperbola and $|QF_1| = 3$. Find $|QF_2|$.

Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: `abadawi@aus.edu, www.ayman-badawi.com`

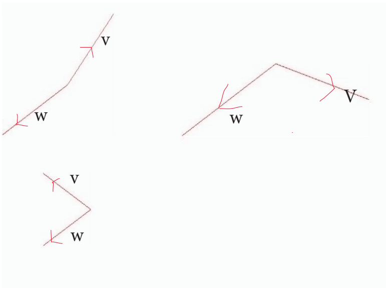
5.4 Quiz IV

Name————————————————, ID ————————

Quiz Four, MTH 111 , Fall 2020

Ayman Badawi

QUESTION 1. For each of the below figures, draw $Proj_W^V$.



QUESTION 2. Let $V =< 3, 4 >$ and $W =< -1, 4 >$. Find $|Proj_V^w|$ (i.e., find the length of the projection vector (W over V)).

QUESTION 3. Find a parametric equations of the line that passes through the points $(1, 2, 4) and (-3, 6, -8)$

QUESTION 4. Let $L_1 : x = t+2; y = -2t+1; z = -t+4; t \in R$ and $L_2 : x = -3w+13; y = 2w-9; z = 3w-7; w \in R$. If L_1 intersects L_2, then find the intersection point.

Is L_1 perpendicular to L_2?

Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: `abadawi@aus.edu, www.ayman-badawi.com`

5.5 Quiz V

Name——————————, ID ——————

Quiz Five, MTH 111 , Fall 2020

Ayman Badawi

QUESTION 1. Find an equation of the plane that passes through the points $Q_1 = (-2, 1, 3), Q_2 = (4, 2, 4), Q_3 = (0, 5, 5)$.

QUESTION 2. Given P : $-2x + 6y + z = 2$ is an equation of a plane.

i) Can we draw the vector $V =< 8, 4, -8 >$ inside the plane? explain.

ii) Does the point (8, -4, -8) lie on the plane?

iii) Does the line $L : x = t - 3, y = 2t - 1, z = -9t - 2, (t \in R)$ lie entirely inside the plane P (above)? If not, does L intersect P? If yes, find the intersection point

Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: `abadawi@aus.edu, www.ayman-badawi.com`

5.6 Quiz VI

Name————————————, ID ——————

Quiz Six, MTH 111 , Fall 2020

Ayman Badawi

QUESTION 1. $P_1 : -2x + 4y + 3z = 6$ and $P_2 : x - y + 2z = 4$ intersect at a line L find a parametric equations of L.

Is P_1 perpendicular to P_2?

QUESTION 2. Given $Q = (1, 2, 4)$ is not on the line $L : x = t + 1, y = -2t + 3, z = 2t + 5$ and Q is not on the plane $P : 2x + y + z = 42$

i) Find $|QL|$

ii) Find $|QP|$

QUESTION 3. i) Let $y = 2x^3 + \sqrt[5]{x^3} + 10$. Find y'.

ii) Let $\dfrac{2x+4}{x^5}$. Find y'.

Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: `abadawi@aus.edu, www.ayman-badawi.com`

6 Section: Assessment Tools-EXAMS (unanswered)

6.1 Exam I

Name————————————, ID ————————

Exam One, MTH 111 , Fall 2020

Ayman Badawi

QUESTION 1. consider the parabola $-8(y-2)=(x+3)^2$

(i) Roughly, sketch such Parabola.

(ii) Find the vertex, V

(iii) Find the focus, F.

(iv) Find the equation of the directrix line.

(v) Given that $Q=(x_1,-16)$ is a point on the parabola. Find $|QF|$. (Think: it is not difficult!!)

QUESTION 2. Given $2y = x^2 + 6x + 13$

(i) Rewrite the equation of the parabola in the standard form and sketch (roughly).

(ii) Find the Focus F.

(iii) Find the equation of the directrix line.

QUESTION 3. consider the ellipse $\frac{(x+3)^2}{16} + \frac{(y-1)^2}{25} = 1$

(i) Roughly, sketch such ellipse.

(ii) Find the center c

(iii) Find the ellipse constant k.

(iv) Find the foci, F_1, F_2

(v) Find all vertices.

(vi) Given that $Q = (x_1, y_1)$ is a point on the ellipse and $|QF_1| = 7$. Find $|QF_2|$.

QUESTION 4. An ellipse is centralized at (-2, 1) such that (10, 1) and (-2, 6) are two vertices of such ellipse.

(i) Find the foci (i.e., F_1, F_2) of the ellipse

(ii) Find the equation of the ellipse.

QUESTION 5. consider the hyperbola $\frac{(y+1)^2}{9} - \frac{(x-3)^2}{16} = 1$

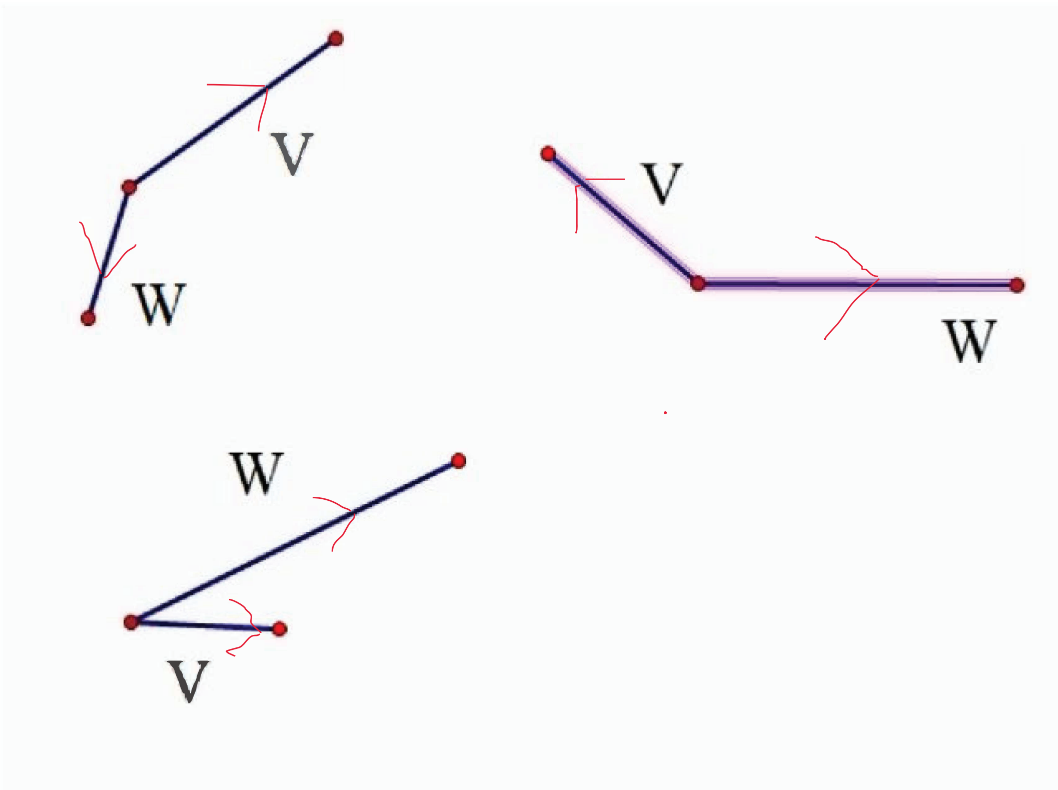
(i) Roughly, sketch such hyperbola.

(ii) Find the hyper-constant k.

(iii) Find the foci, F_1, F_2

(iv) Find all vertices.

QUESTION 6. For each of the below figures, draw $Proj_V^W$.



QUESTION 7. Let $L_1 : x = 4t + 2; y = -2t + 1; z = t + 4; t \in R$ and $L_2 : x = -2w + 12; y = -2w - 1; z = 4w + 2; w \in R$. If L_1 intersects L_2, then find the intersection point.

Is L_1 perpendicular to L_2? (explain)

Find the symmetric equation of the line L_1 (above).

QUESTION 8. Use the concept of cross product in order to find the area of the triangle that have the vertices $a = (-4, 2), b = (1, -1), c = (4, 5)$

QUESTION 9. Let $L_1 : x = 3t + 2; y = 2t + 1; z = 3t + 4; t \in R$ and $L_2 : x = -6w + 14; y = -4w + 9; z = -6w + 16; w \in R$. Is $L_1 \parallel L_2$?

Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: `abadawi@aus.edu, www.ayman-badawi.com`

6.2 Exam II

Name————————————, ID ——————

Exam II: MTH 111, Fall 2020

Ayman Badawi

$$\text{Points} = \frac{\qquad}{56}$$

QUESTION 1. (i) **(4 points)** Does the line $L : x = -2t + 8, y = t - 1, z = t + 3$ lie entirely inside the plane $x + 2y + z = 12$? If not, does it intersect the plane? If yes, then find the intersection point.

(ii) **(3 points)** Find the value a so that the plane $P_1 : 2x+y+3z = 4$ is perpendicular to the plane $P_2 : 4x+6y+az = 8$.

(iii) **(4 points)** For what values of a, b is the plane $P_1 : 2x + y + 3z = 4$ parallel to the plane $P_2 : 4x + 2y + az = b$? (i.e., P_1 does not intersect P_2).

(iv) **(4 points)** Find the distance between $Q = (4, 4, -15)$ and the plane $P : -2x + 2y - z = 21$.

(v) **(6 points)** The two planes $P_1 : x + 4y + z = 10$ and $P_2 : -x + 3y - z = 11$ intersects in a line L. Find a parametric equations of L.

(vi) **(2 points)** Can we draw the vector $V =< 1, 4, 11 >$ inside $P : 5x + 7y - 3z = 19$? explain

(vii) **(4 points)** Find the distance between the point $Q = (1, 2, 4)$ and the line $L : x = t+3, y = -2t+1, z = 2t+4 (t >= 0)$

QUESTION 2. (10 points) Let $f(x) = x^3 - 6x^2 - 15x + 10$.

(i) For what values of x does $f(x)$ increase?

(ii) For what values of x does $f(x)$ decrease?

(iii) Find all local minimum, maximum points of $f(x)$ (just find the x-values where local min. and local max exist).

(iv) Roughly, sketch the graph of $f(x)$.

QUESTION 3. (7 points) Given $H = (4, 7)$ and $F = (-2, 10)$. Find a point Q on the line $y = -4$ such that $|HQ|+|FQ|$ is minimum.

QUESTION 4. (12 points) Find y' and DO NOT SIMPLIFY

(i) $y = \sqrt{3x + 2} + \frac{4}{x^7} + 10$

(ii) $y = e^{(7x+2)} + 10x^2 + 5$

(iii) $y = ln[(2x^5 + 8x^2 - 3x)/(2x + 7)^3]$

(iv) $y = 10(3x^6 + 5x^3 + 2)^7$

Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: abadawi@aus.edu, www.ayman-badawi.com

6.3 Final Exam

Name——————————, ID ——————

Final Exam, MTH 111 , Fall 2020

Ayman Badawi

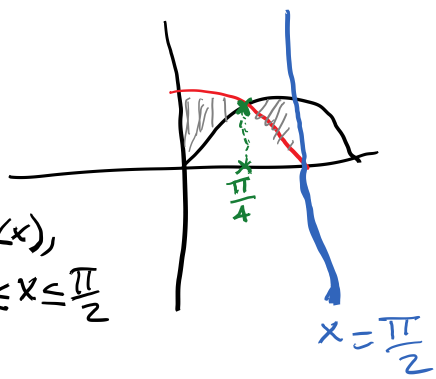
Score = $\overline{66}$

QUESTION 1. (8 points) Stare at the following graphs

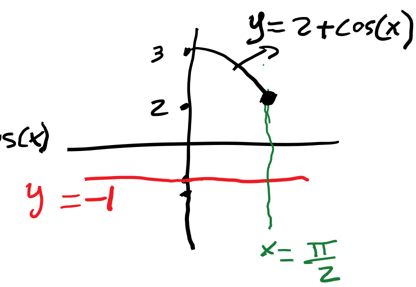
Red : $y_1 = \cos(x)$

Black: $y_2 = \sin(x)$

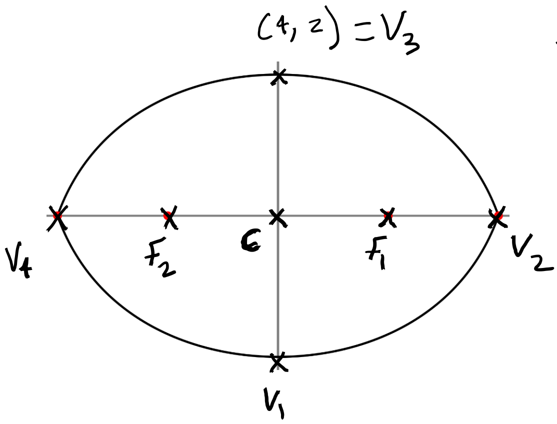
Find the area bounded by $y_1 = \cos(x)$, $y_2 = \sin(x)$, and $0 \le x \le \frac{\pi}{2}$



Find the Volume of the object when we rotate $y = 2 + \cos(x)$ about $y = -1$, where $0 \le x \le \frac{\pi}{2}$



QUESTION 2. (6 points) Stare at the below ellipse. Then



$F_1 = (6, -1)$

$V_3 = (4, 2)$

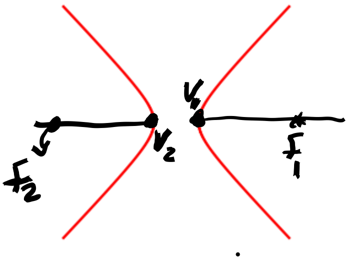
(i) Find the ellipse-constant k.

(ii) Find $c(the\ center), v_1, v_2, v_4$.

(iii) Find F_2

(iv) Find the equation of the ellipse.

QUESTION 3. (5 points) Stare at the below hyperbola. Then



$V_1 = (4, -2)$

$F_1 = (7, -2)$

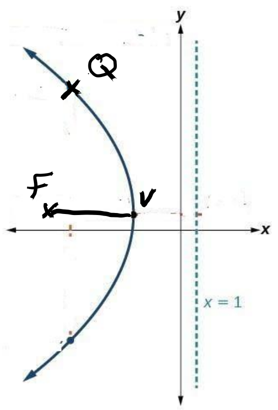
$V_2 = (0, -2)$

(i) Find the hyperbola-constant k.

(ii) Find $c(the\ center), F_2$.

(iii) Find the equation of the hyperbola.

QUESTION 4. (5 points) Stare at the below parabola. Then



X=1 is the directrix

V = (-2,1) is the Vertex.

Q = (-4, y) on the curve.

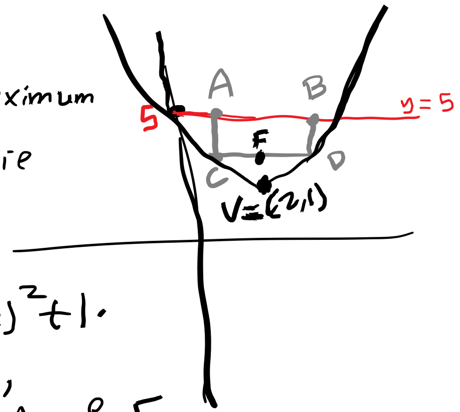
(i) Find the focus F.

(ii) Find $|FQ|$.

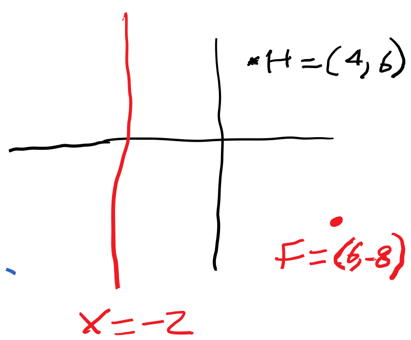
(iii) Find the equation of the parabola.

QUESTION 5. (10 points) Stare at the following pictures.

ABDC is a rectangle of maximum area, where A, B lie on the line $y=5$, C, D lie on the parabola $y=(x-2)^2+1$.
Note $|FD|=|FC|$, and x-coordinate of F is 2.
$V=(2,1)$ is the vertex.
Find the points A, B, C, D, i.e, write each point as (—,—).



H = (4, 6)
F = (6, −8)
Find a point Q on the line $x=-2$ s.t. $|FQ|+|QH|$ is minimum.



QUESTION 6. (6 points) Find y'. Do not simplify.

(i) $y = (sin(3x) + 2x + 1)^5$

(ii) $y = ln[\frac{(5x+2)^4}{(3x+7)^3}]$

(iii) $y = cos(2x)e^{(x^2+1)}$

(iv) $y = \sqrt{3x+1} + \frac{4}{x^3}$

QUESTION 7. (4 points)

i) Find $\int (e^{2x} + x)(e^{2x} + x^2 + 1)^8 \, dx$

ii) Find $\int \frac{sin(x) - 2x}{cos(x) + x^2 + 3} \, dx$

QUESTION 8. (6 points) Consider the points: $Q_1 = (2, 1, 0), Q_2(4, 2, 0), Q_3 = (-8, 3, 10)$.

(i) Find the area of the triangle $Q_1Q_2Q_3$.

(ii) Find the equation of the plane that passes through Q_1, Q_2, and Q_3.

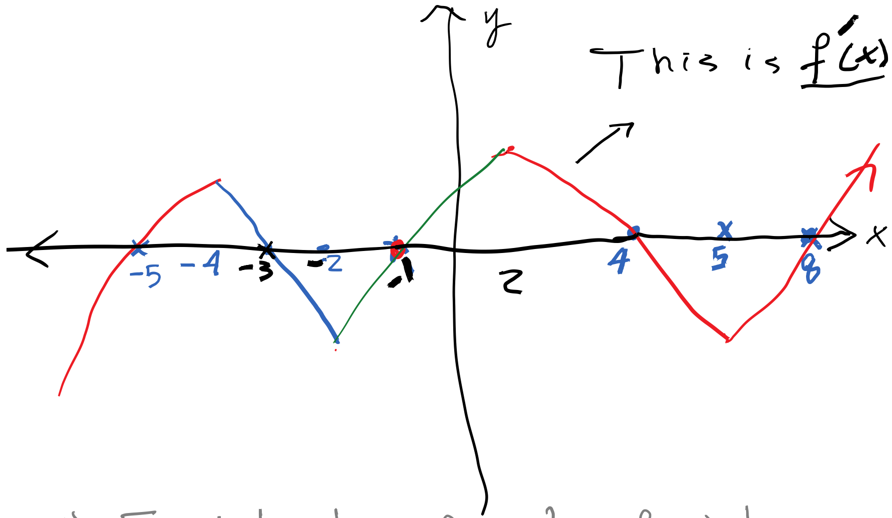
QUESTION 9. (10 points)

i) If the line $L : x = at + 2, y = 4t + a, z = -t + b$ lies entirely inside the plane $x + 2y + 3z = 13$, then find the values of a and b.

ii) The Plane $P_1 : x + 2y - z = 10$ intersects the plane $P_2 : -x - y + z = -7$ in a line L. Find a parametric equations of L.

iii) Let P_2 as in (ii). Find the distance between $Q = (1, 4, -20)$ and P_2.

QUESTION 10. (6 points) Stare at the following graph of $f'(x)$. Then answer the following.



1) For what values of x does f(x) have local minimum?

2) For what values of x does f(x) have local maximum?

3) For what values of x does f(x) increase?

4) Roughly, sketch the curve of f(x)

Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: `abadawi@aus.edu`, `www.ayman-badawi.com`

Faculty information

Ayman Badawi, American University of Sharjah, UAE.
E-mail: abadawi@aus.edu