Exam II MTH 111, Fall 2016

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75/75 Excellent

Total points = $\frac{?}{80}$

QUESTION 1. **(8 points).** See the below picture and read the question.

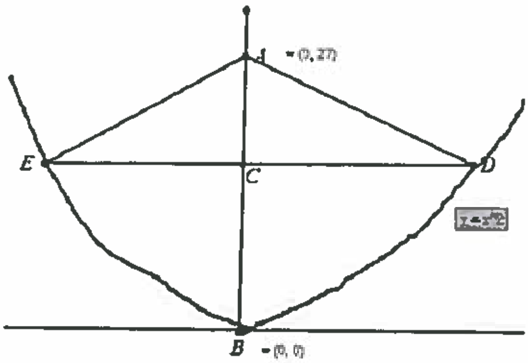

Figure 1. Question: Find the length of the height AC (note $A = (0, 27)$) and the length of of the base ED of the triangle ADE that has maximum area. Note that $|CD| = |CE|$, and the points D and E are on the curve $y = x^2$. Area of a rectangle $= \frac{1}{2} \times base \times height$

let $|CD| = |CE| = a$

$\therefore$ $D = (a, a^2)$

$E = (-a, a^2)$

$C = \left(\frac{a-a}{2}, \frac{a^2+a^2}{2}\right)$

$C = (0, a^2)$

$AC = 27 - a^2$

$ED = 2a$

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A = Area of Δ = $\frac{1}{2} b \times h$

$= \frac{1}{2}(2a)(27-a^2)$

$= 27a - a^3$

$A' = 27 - 3a^2$

$A' = 0$

$27 = 3a^2$

$a^2 = 9$

$a = 3$

$\therefore$ $AC = 27 - 9 = 18$ units

$ED = 6$ units

$A'' = -6a = -ve$

Hence maximum

$\therefore$ Area is maximum when $AC = 18$ units and $ED = 6$ units

QUESTION 2. (8 points). See the below picture and read the question.

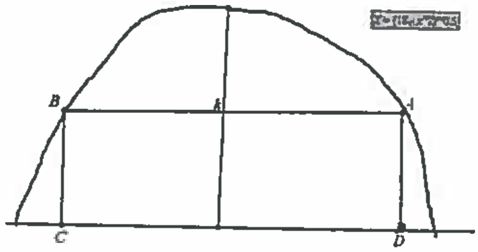


Figure 2. Question: Find the length AD and the the width AB of the rectangle $ADCB$ that has maximum area and it can be drawn inside $y = \sqrt{18 - x^2}$. Note that $|AK| = |BK|$, and the points A and B are on the curve $y = \sqrt{18 - x^2}$.

let $|AK| = |BK| = a$

$\therefore \quad A = (a, \sqrt{18-a^2})$

$D = (a, 0)$

Hence length, $l = 2a$

$w = \sqrt{18-a^2}$

Area of Rectangle $= A = 2a(\sqrt{18-a^2})$

$A' = 2\sqrt{18-a^2} + \cancel{2}a \cdot \dfrac{1}{\cancel{2}\sqrt{18-a^2}}(-2a)$

8/8

$A' = 2\sqrt{18-a^2} - \dfrac{2a^2}{\sqrt{18-a^2}}$

$A' = \dfrac{2(18-a^2) - 2a^2}{\sqrt{18-a^2}}$

$= \dfrac{36 - 2a^2 - 2a^2}{\sqrt{18-a^2}} = \dfrac{36 - 4a^2}{\sqrt{18-a^2}}$

$A' = 0$

$36 - 4a^2 = 0$ AB $l = 2 \times 3 = 6$ units

$36 = 4a^2$ AD$= w = \sqrt{18-9} = 3$ units

$9 = a^2$

$\therefore a = 3$

$A'' = \dfrac{-8a(\sqrt{18-a^2}) - (36-4a^2)\left[\frac{1}{\cancel{2}\sqrt{18-a^2}}(-\cancel{2}a)\right]}{18-a^2}$

$= \dfrac{-72+24}{9} = -ve =$ maximum

$\therefore$ Area is maximum when AD $= 3$ units and AB $= 6$ units

QUESTION 3. (8 points). Let $Q = (4, 8)$, $A = (1, -6)$. Find a point B on the line $x = -10$ such that $|QB| + |AB|$ is minimum.

Reflect A on line $x = -10$

$A' \to (-21, -6)$

Equation of line $A'Q$.

slope, $m = \frac{8+6}{4+21} = \frac{14}{25}$

$y = mx + c$

$8 = \frac{14}{25} \times 4 + c$

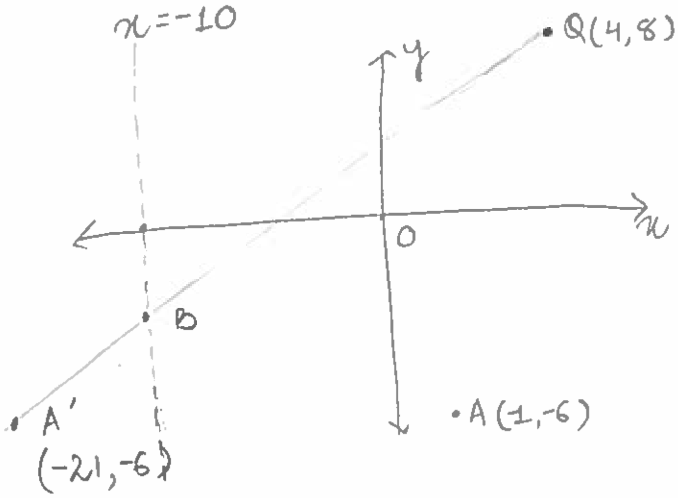
$c = 8 - \frac{14}{25} \times 4$

$= 8 - \frac{56}{25} = \frac{144}{25}$

$\therefore$ equation: $y = \frac{14}{25}x + \frac{144}{25}$.

Now point B lies on $A'Q$.

$\therefore y = \frac{14}{25}(-10) + \frac{144}{25}$.



$y = \frac{144}{25} - \frac{140}{25} = \frac{4}{25}$

Hence point B is $(-10, \frac{4}{25})$

8/8

QUESTION 4. (8 points). Let $y = e^{(x-1)} + ln(x) + x + 1$

(i) Find the equation of the tangent line to the curve at $(1, 3)$

$y = e^{(x-1)} + \ln x + x + 1$

$y' = e^{(x-1)} + \frac{1}{x} + 1$

At $(1, 3)$; $y' = e^0 + 1 + 1 = 3$ (slope of tangent)

$y = mx + c$

$3 = 3(1) + c$

$c = 0$

$\therefore$ equation of tangent is $y = 3x$.

8/8

(ii) Find the equation of the normal line to the curve at $(1, 3)$.

slope of normal $= \frac{-1}{3}$

$y = mx + c$

$3 = -\frac{1}{3}(1) + c$

$c = 3 + \frac{1}{3} = \frac{10}{3}$

$\therefore$ equation of normal is $y = -\frac{1}{3}x + \frac{10}{3}$

Ayushi

QUESTION 5. (9 points). Let $f(x) = e^{(x-1)} - 8x + 2$

(i) For what values of x is the tangent line to the curve horizontal?

$f'(x) = e^{(x-1)} - 8$

for tangent line to be horizontal —

$f'(x) = 0$

$e^{(x-1)} - 8 = 0$

$e^{(x-1)} = 8$

$x - 1 = \ln 8$

$x = \ln 8 + 1 = 3.08$

4/4

$\therefore$ for $x = 3.08$, tangent line is horizontal.

(ii) For what ~~values~~ of x does $f(x)$ increase?

For, $x > 3.08$,

$f'(x)$ is positive

Hence $f(x)$ increase from $x \in (3.08, \infty)$

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3.08

4/4

(iii) For what values of x does $f(x)$ decrease?

For, $x < 3.08$

$f'(x)$ is negative

Hence $f(x)$ decrease from $x \in (-\infty, 3.08)$

4/4

QUESTION 6. (9 points). Find y' and do not simplify

(i) $y = e^{(3x^2+x+1)} + 10x - 1$

$y' = (6x+1)e^{(3x^2+x+1)} + 10$

3/3

(ii) $y = 3(2 - 2x^3)^7$

$y' = 3 \times 7 (2-2x^3)^6 (-6x^2)$

$= -126\, x^2 (2-2x^3)^6$

3/3

(iii) $y = ln[(2x+3)^{10}(-5x+30)^9]$

$y = \ln[(2x+3)^{10}] + \ln[(-5x+30)^9]$

$y = 10 \ln(2x+3) + 9 \ln(-5x+30)$

$y' = \frac{10}{2x+3} \times (2) + \frac{9}{(-5x+30)} (-5)$

$y' = \frac{20}{2x+3} - \frac{45}{30-5x}$

3/3

QUESTION 7. (8 points). Let $L_1 : x = 2t + 1, y = -t + 3, z = t$ and $L_2 : x = 4t - 7, y = t - 5, z = -2t + 12$. If L_1 interests L_2, then find the intersection point.

L_1: $x = 2t+1$, $y = -t+3$, $z = t$

L_2: $x = 4s-7$, $y = s-5$, $z = -2s+12$

At Pt. of intersection —

$L_1(x) = L_2(x)$

$2t+1 = 4s-7$

$2t-4s = -8$ —(1)

$t+s = 8$ —(2)

$L_1(y) = L_2(y)$

$-t+3 = s-5$

$-t-s = -8$ —(2)

Elimination Method —

$2t - 4s = -8$

$-2t - 2s = -16$

$-6s = -24$

$s = 4$

$\therefore t = 8-4 = 4$

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$\therefore x = 9$, $y = -1$, $z = 4$

At intersection; $L_1(z) = L_2(z)$

$t = -2s+12$

$4 = -2(4)+12$

$4 = 4$

(Hence verified)

$\therefore$ Point of Intersection is $(9, -1, 4)$

QUESTION 8. (5 points). Given $y^2 + ln(x)e^y + xy - x^2 + ye^{3x} - 20 = 0$. Find y' and do not simplify.

$y^2 + \ln(x)e^y + xy - x^2 + ye^{3x} - 20 = 0$

$f_x = \frac{e^y}{x} + y - 2x + 3ye^{3x}$

$f_y = 2y + \ln(x)e^y + x + e^{3x}$

$y' = -\frac{f_x}{f_y} = \frac{-\left(\frac{e^y}{x} + y - 2x + 3ye^{3x}\right)}{2y + \ln(x)e^y + x + e^{3x}}$

5/5

QUESTION 9. (5 points). Without sketching, convince me that $f(x) = 3x^5 + 7x + 20$ intersects the x-axis only in one point.

If ~~the line~~ f(x) intersects at x-axis, $\therefore$ y value should be zero at that point.

Hence, $f(x) = y = 0$

$3x^5 + 7x + 20 = 0$

Solving the equation;

$x = -1.295$ (approx)

Hence f(x) intersects x axis at only one point

Ayusha

QUESTION 10. (12 points). See the below picture and read the question.

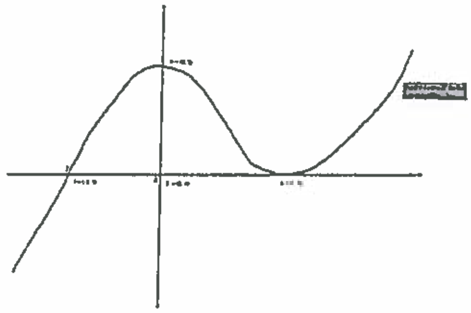

Figure 3. Question: You are looking at the curve of $f'(x)$. The curve of $f'(x)$ intersects the x-axis at the points $A = (6,0)$ and $J = (-3,0)$; also it intersects the y axis at $B = (0,5)$.

(i) Find all x values where $f(x)$ is maximum.

~~f(x)~~ There is no maxima

(ii) Find all x values where $f(x)$ is minimum.

$f(x)$ is minimum at $x=-3$

(iii) For what values of x does $f(x)$ increase?

$f(x)$ increases for $x \in (-3, \infty)$

(iv) For what values of x does $f(x)$ decrease?

$f(x)$ decreases for $x \in (-\infty, -3)$

(v) For what values of x do the slopes of tangent lines are negative?

for $x \in (-\infty, -3)$, slope of tangent line is negative

(vi) For what values of x do the slopes of normal lines are positive?

$x \in (-3, \infty)$ slope of tangent is positive

Faculty information

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