بسم الله الرحمن الرحيم

99 +1 = 100

TEST NUMBER TWO FOR MTH102 SPRING007

AYMAN BADAWI

Name Amna Al Shirawi; Id. Num. 23558, Score $\frac{}{100}$

QUESTION 1. **(25 points)** *Let $f(x) = (x - 2)e^x$ defined on the closed interval [0, 4].*

(1) *For what values of x in the interval [0, 4] does $f(x)$ increase?*

$f'(x) = e^x + (x-2)e^x = 0$

$\boxed{f(x) \text{ increases over } [1,4]}$

$e^x(1 + x - 2) = 0$

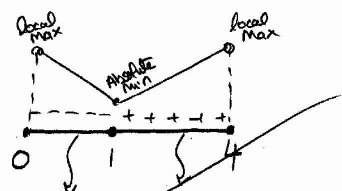
$e^x(-1 + x) = 0$

$-1 + x = 0$

$\boxed{x = 1}$

included in [0,4]

$f(0) = -2$



$f'(0.5)$ $\quad$ $f'(2)$

$e^{0.5}(-1+0.5)$ $\quad$ $e^{2}(-1+2)$

$= (-)$ $\quad$ $= (+)$

(2) *For what values of x in the interval [0, 4] does $f(x)$ decrease?*

$f(x)$ decreases over $x = [0, 1]$

~~on -2.8 -2.72~~

(3) *Find the absolute max. of $f(x)$ and the absolute min of $f(x)$ on [0, 4].*

x	$f(x)$	
0	-2	
1	-2.72	← Absolute Min
4	109.19	← Absolute Max

* Absolute Max of $f(x)$ is 109.2 at $x = 4$

* Absolute Min of $f(x)$ is -2.72 at $x = 1$

QUESTION 2. (**15 points**) *Find $f'(x)$. DO NOT SIMPLIFY*

(1) $f(x) = 3xe^{x+7} + 7x^3 - 10$

$u = 3x \quad u' = 3$

$v = e^{x+7} \quad v' = e^{x+7}$

$f'(x) = 3e^{x+7} + 3xe^{x+7} + 21x^2$

(2) $f(x) = ln(x^2 - 7x)$

$\ln u = \frac{u'}{u}$

$f'(x) = \frac{2x-7}{x^2-7x}$

(3) $f(x) = 3^{2x-1} + log_5(7x-5)$

$\log_5(7x-5) = \frac{1}{\ln 5} \cdot \frac{7}{7x-5}$

$a^u = \ln a \cdot a^u \cdot u'$

$f'(x) = \ln 3 \cdot 3^{2x-1} \cdot 2 + \frac{1}{\ln 5} \cdot \frac{7}{7x-5}$

QUESTION 3. (**15 points**) *a)Find y' for the relation $x^2y - ln(y) + e^x - x^2 = 0$*

$2xy + x^2y' - \frac{y'}{y} + e^x - 2x = 0$

$x^2y' - \frac{y'}{y} = 2x - e^x - 2xy$

$y'(x^2 - 1/y) = 2x - e^x - 2xy$

$\boxed{y' = \frac{2x - e^x - 2xy}{x^2 - 1/y}}$

b) Find the equation of the tangent line to the curve $x^2 - y^2 = 5$ at the point (3, 2)

$2x - 2yy' = 0$

$2x = 2yy'$

$2yy' = 2x$

$y' = \frac{\cancel{2}x}{\cancel{2}y}$

$\boxed{y' = \frac{x}{y}}$

$y'\big|_{(3,2)} = \frac{3}{2} = 1.5 = m$

$(3,2)$

$y - 2 = 1.5(x-3)$

$y = 1.5x - 4.5 + 2$

$\boxed{y = 1.5x - 2.5}$

$2 = 1.5(3) + b$

$\boxed{b = -2.5}$

QUESTION 4. (**20 points**) *work the following integrals:*

(1) $\int 6x^2\underbrace{(x^3+1)}_{3x^2}{}^{12}\ dx$

$= 2\int 3x^2 (x^3+1)^{12} dx$

$= 2 \cdot \frac{(x^3+1)^{13}}{13} + C = \boxed{(2/13)\cdot (x^3+1)^{13} + C}$

$\int \frac{u'}{u} du$

$= \ln|u| + C$

(2) $\int \frac{2x+3}{\underbrace{x^2+3x-6}_{2x+3}}\ dx$

$= \boxed{\ln|x^2+3x-6| + C}$

D	$\int$
x^3 +	e^x
$3x^2$ −	e^x
$6x$ +	e^x
6	e^x
0	e^x

← (3) $\int x^3 e^x\ dx = \boxed{x^3e^x - 3x^2e^x + 6xe^x + \cancel{6e^x}} + C$

−1

$\ln u = \frac{u'}{u}$

$\frac{1}{2} - \frac{2}{2} = -\frac{1}{2}$

$x^{1/2} = \frac{1}{2}x^{-1/2}$

$= \frac{1}{2\sqrt{x}}$

$\frac{1}{2\sqrt{x}} \cdot \frac{1}{\sqrt{x}}$

(4) $\int ln(\sqrt{x})\ dx = uv - \int v du$

$\int 1/2 dx \to \frac{1}{2}x$

$u = \ln(\sqrt{x})$ $\quad dv = dx$ $\quad = x\ln(\sqrt{x}) - \int \cancel{x} \cdot \frac{1}{2\cancel{x}} dx$

$du = \frac{1/2\sqrt{x}}{\sqrt{x}} dx$ $\quad v = x$ $\quad = x\ln(\sqrt{x}) - \frac{x}{2} + C$

$du = \frac{1}{2x} dx$

(5) $\int \underbrace{ln(x+3)}_{u}\underbrace{dx}_{dv}$ (**EXTRA 5 POINTS**) $= uv - \int v du$

$\frac{1}{(x+3)} = (x+3)^{-1}$

$-(x+3)^{-2}$

$u = \ln(x+3)$ $\quad dv = dx$ $\quad = x\ln(x+3) - \int x \cdot \frac{1}{x+3} dx$

$du = \frac{1}{x+3} dx$ $\quad v = x$

$\cancel{\int \ln(x+3) dx} = x\ln(x+3) - x\ln|x+3| + \int \ln|x+3| dx$

$\int \frac{x}{x+3} dx$

D	I $\int$
x	$\frac{1}{x+3}$
1	$\ln\lvert x+3\rvert$
0	

$u = x$ $\quad dv = \frac{1}{x+3} dx$

$du = 1dx$ $\quad v = \ln|x+3|$

Solution behind second page.

$1/3 + 3/3 = 4/3$

QUESTION 5. (**10 points**) *1. Evaluate* $\int_0^2 (3x^2+1)\sqrt[3]{\underbrace{x^3+x-2}_{3x^2+1}}\,dx$ $\quad (x^3+x-2)^{1/3}$

$$= \frac{3(x^3+x-2)^{4/3}}{4}\Big]_0^2 = \frac{3}{4}(2^3+2-2)^{4/3} - \frac{3}{4}(-2)^{4/3} = \frac{3}{4}\left(\sqrt[3]{8^4}\right) - \frac{3}{4}\left(\sqrt[3]{(-2)^4}\right)$$

$$= \frac{3}{4}(16) - \frac{3}{4}(2.5) = 10.11$$

2. Evaluate $\int_0^1 e^{x+1}\,dx$

$$= e^{x+1}\Big]_0^1 = e^2 - e^1 = 4.67$$

$10p^{-1} = -10p^{-2}$

QUESTION 6. (**15 points**) *Let* x *be number of units from a Product A, and* p *be the selling price per unit. Given* $\boxed{x = 20 + \frac{10}{p} - p}$.

(1) *Find the elasticity when* $p = 8$.

$$f'(p) = \frac{-10}{p^2} - 1$$

$$E = \frac{-Pf'(p)}{f(p)} \qquad f'(8) = \frac{-10}{8^2} - 1 = -1.16$$

$$f(8) = 20 + \frac{10}{8} - 8 = 13.25$$

$$E(8) = \frac{-8(-1.16)}{13.25} = 0.7 < 1 \rightarrow \therefore \text{Demand is inelastic.}$$

(2) *Write the revenue in terms of* p, *i.e. find* $R(p)$.

$$R(p) = p\left(20 + \frac{10}{p} - p\right) = 20p + 10 - p^2 \qquad \boxed{R(p) = 20p + 10 - p^2}$$

(3) *For what value of* p *does the maximum revenue occur?*

$$R'(p) = 0 = 20 - 2p \Rightarrow 20 = 2p \Rightarrow \boxed{p = 10}$$

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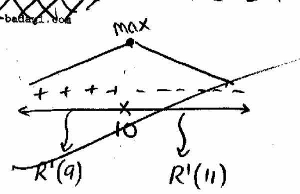
$$R''(p) = -2 < 0 \rightarrow \text{Max}$$



$$R(10) = 200 + 10 - 100 = 110$$

Maximum Revenue occurs at (10, 110) of 110 at x=10