Name Yassin Habib, ID 30532 @ 10 a.m

Exam ONE, MTH 205, SPRING 2009

Ayman Badawi

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QUESTION 1. (25 points) 1)Find $\ell\{2^{3x+2}\}$

$$\Rightarrow \ell\left\{e^{(3x+2)\ln 2}\right\} = \left\{e^{3x\ln 2 + 2\ln 2}\right\} = \left\{e^{3x\ln 2}\cdot e^{2\ln 2}\right\}$$

$$= e^{2\ln 2}\,\ell\left\{e^{3\ln 2 x}\right\} = \boxed{\frac{e^{2\ln 2}}{s - 3\ln 2}} = 4\,\frac{1}{s-3\ln(2)}$$

2) Find $\ell\{x^2U(x-2)\}$

$(a=2)$

e^{-2s}

$$\Rightarrow g(x+2) = (x+2)^2$$

$$\Rightarrow \ell\{g(x+2)\} = \ell\{(x+2)^2\} = \ell\{x^2+4x+4\} = \frac{2}{s^3} + \frac{4}{s^2} + \frac{4}{s}$$

$$\Rightarrow e^{-2s}\left(\frac{2}{s^3} + \frac{4}{s^2} + \frac{4}{s}\right)$$

3) USE convolution to find $\ell^{-1}\{\frac{1}{s^2-s}\}$

$$= \ell^{-1}\left\{\frac{1}{s(s-1)}\right\} = \left\{\underbrace{\frac{1}{s}}_{F(s)} \cdot \underbrace{\frac{1}{s-1}}_{G(s)}\right\}$$

$$= 1 * e^{x}$$

$$= \int_0^x e^r\,dr$$

$$\Rightarrow e^r\Big|_{r=0}^{r=x} = e^x - e^0 = \boxed{e^x - 1}$$

4)Find $\ell^{-1}\{\frac{3e^{-2s}}{(s-4)^2+9}\}$, $f(x-2)\,u(x-2)$

$$\Rightarrow \ell^{-1}\left\{e^{-2s}\cdot\underbrace{\frac{3}{(s-4)^2+9}}_{F(s)}\right\}$$

$$\Rightarrow f(x) = \sin 3x\; e^{4x}$$

$$\Rightarrow f(x-2) = \sin(3x-6)\; e^{4x-8}$$

$$\Rightarrow \underline{\underline{u(x-2)\,\sin(3x-6)\,e^{4x-8}}}$$

5) Find $\ell^{-1}\{\frac{s+5}{(2s+6)^3}\}$

$$= \ell^{-1}\left\{\frac{s+5}{[2(s+3)]^3}\right\} = {}^{-1}\left\{\frac{s+5}{8\,(s+3)^3}\right\} = \frac{1}{8}\,\ell^{-1}\left\{\frac{s+5-2+2}{(s+3)^3}\right\} = \frac{1}{8}\,\ell^{-1}\left(\frac{1}{(s+3)^2}+\frac{2}{(s+3}\right.$$

$$= \frac{1}{8}\left(xe^{-3x} + x^2e^{-3x}\right) = \underline{\underline{\frac{xe^{-3x}}{8}\,(1+x)}}$$

QUESTION 2. (10 points) Solve $y^{(2)} + 9y = 3, y(0) = y'(0) = 0$

Apply laplace

$$\Rightarrow s^2Y_{(s)} - s\cancel{y(0)}^{0} - \cancel{y'(0)}^{0} + 9Y_{(s)} = \frac{3}{s}$$

$$\Rightarrow Y_{(s)}(s^2+9) = \frac{3}{s}$$

$$\Rightarrow Y_{(s)} = \frac{3}{s(s^2+9)}$$

$$\Rightarrow y_{(x)} = \ell^{-1}\left\{\frac{3}{s(s^2+9)}\right\}$$

$$= \ell^{-1}\left\{\frac{1}{s}\cdot\frac{3}{s^2+9}\right\}$$

$$= 1 * \sin 3x$$

$$\Rightarrow 1 * \sin 3x = \int_0^x \sin 3r\, dr$$

$$= -\frac{1}{3}\cos 3r\,\Big|_{r=0}^{r=x}$$

$$= -\frac{1}{3}(\cos 3x - 1)$$

$$\Rightarrow y(x) = \underline{\underline{\frac{1}{3} - \frac{\cos 3x}{3}}}$$

QUESTION 3. (15 points) Solve for y(x), $y'(x) = e^{2x} - \int_0^x e^{2x-2r} y(r)\,dr, y(0) = 0$

$\Rightarrow y'(x) = e^{2x} - \underbrace{\int_0^x y(r)\, e^{2(x-r)}\, dr}$ $\rightarrow y(x) * e^{2x}$

$\cdot\ \mathcal{L}\{y(x) * e^{2x}\} = \dfrac{Y_{(s)}}{s-2} \cdot$

Apply laplace

$\Rightarrow sY_{(s)} - \cancel{y(0)}^{\,0} = \dfrac{1}{s-2} - \mathcal{L}\{y(x) * e^{2x}\}$

$\Rightarrow sY_{(s)} = \dfrac{1}{s-2} - \dfrac{Y_{(s)}}{s-2}$

$\Rightarrow sY_{(s)} + \dfrac{Y_{(s)}}{s-2} = \dfrac{1}{s-2}$

$\Rightarrow Y_{(s)}\left(s + \dfrac{1}{s-2}\right) = \dfrac{1}{s-2}$

$\Rightarrow Y_{(s)}\left(\dfrac{s^2-2s+1}{s-2}\right) = \dfrac{1}{s-2}$

$\Rightarrow Y_{(s)}\left(\dfrac{(s-1)^2}{s-2}\right) = \dfrac{1}{s-2}$

$\Rightarrow Y_{(s)} = \dfrac{1}{(s-1)^2}$

$\Rightarrow y(x) = \mathcal{L}^{-1}\left\{\dfrac{1}{(s-1)^2}\right\} = \underline{\underline{xe^x}}$

QUESTION 4. (15 points) Solve for $x(t)$ and $y(t)$ if $x'(t) - 0.5y(t) = t$ and $x(t) + \int_0^t y(r)\,dr = 2t^2, x(0) = y(0) = 0$

① $x'(t) - 0.5\,y(t) = t$ $\qquad x(0) = y(0) = 0.$

② $x(t) + \int_0^t y(r)\,dr = 2t^2$

Apply laplace

$\mathcal{L}\{①\}$

$\Rightarrow sX_{(s)} - \cancel{x(0)}^{\,0} - 0.5\,(Y_{(s)}) = \dfrac{1}{s^2}$

$\Rightarrow \boxed{sX_{(s)} - 0.5\,Y_{(s)} = \dfrac{1}{s^2}}$

$\mathcal{L}\{②\}$

$\Rightarrow \boxed{X_{(s)} + \dfrac{Y_{(s)}}{s} = \dfrac{4}{s^3}}$

$X_{(s)} = \dfrac{\det\begin{bmatrix} 1/s^2 & -0.5 \\ 4/s^3 & 1/s \end{bmatrix}}{\det\begin{bmatrix} s & -0.5 \\ 1 & 1/s \end{bmatrix}}$

$= \dfrac{\dfrac{1}{s^3} + \dfrac{2}{s^3}}{1 + 0.5} = \dfrac{3}{s^3} \times \dfrac{2}{3}$

$= \dfrac{2}{s^3}$

$x_{(t)} = \mathcal{L}^{-1}\left\{\dfrac{2}{s^3}\right\} = \boxed{t^2}$

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$$x'(t) - 0.5\,y(t) = t$$

$x'(t) = 2t$, $\Rightarrow 2t - 0.5\,y(t) = t$

$$\Rightarrow -\frac{1}{2}\,y(t) = t - 2t$$

$$\Rightarrow y(t) = -2(t - 2t)$$

$$= -2t + 4t$$

$$= \underline{\underline{2t}}$$

QUESTION 5. (15 points) Find the general solution to $y^{(3)} + 2y^{(2)} + y' = 0$. USE THE HOMOGENEOUS METHOD.

Solution: $y = e^{mx}$

char(D.E) => $m^3 + 2m^2 + m$

set char(D.E) = 0 , => $m(m^2 + 2m + 1) = 0$

=> $m(m+1)(m+1) = 0$

=> $m = 0$, $m = -1$, $m = -1$

$\downarrow$ $e^0 = 1$ $\quad \downarrow$ e^{-x} $\quad \downarrow$ xe^{-x}

=> $y_g = c_1 + c_2 e^{-x} + c_3 x e^{-x}$

QUESTION 6. (20 points) a) Find the general solution to $y^{(2)} + 16y = 0$

Solution: $y = e^{mx}$

char(D.E): $m^2 + 16$

set char D.E = 0: $m^2 + 16 = 0$

=> $m^2 = -16$

=> $m = \pm\sqrt{-16}$

=> imaginary => use one solution

$+\sqrt{-16} = a + bi$

=> $\sqrt{-16} = a + bi$

=> $a = 0$, $b = \sqrt{16} = 4$

=> $y_1 = e^{ax}\cos bx$

$= \cos 4x$

$y_2 = e^{ax}\sin bx$

$= \sin 4x$

=> $y_G = c_1 \cos 4x + c_2 \sin 4x$

b) If $y(0) = 0$ and $y'(\pi/8) = 0$, what will be the solution of the D.E in part (a)? Does that contradict one of the Theorem in the book? EXPLAIN (Note $sin(\pi/2) = 1, cos(\pi/2) = 0, sin(0) = 0, cos(0) = 1$)

$y(x) = c_1 \cos 4x + c_2 \sin 4x$

=> $y(0) = 0 = c_1$

=> $c_1 = 0$

$y'(\frac{\pi}{8}) = -4c_1 \sin 4x + 4c_2 \cos 4x$

$0 = -4c_1$

=> $c_1 = 0$

$y_a = c_2 \sin 4x$

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c) If $y(0) = 0$ and $y'(\pi/8) = 1$, what will be the solution of the D.E. in part (a)? Does that contradict one of the Theorem in the book? EXPLAIN

* has to be same x value.

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Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: abadawi@aus.edu, www.ayman-badawi.com

b)

$$y'' + 16y = 0$$

Apply laplace $\Rightarrow s^2 Y_{(s)} - s y(0) - y'(0) + 16 Y_{(s)} = 0$

$$\Rightarrow Y_{(s)} \left[s^2 + 16 \right] = s y(0) + y'(0)$$

$$\Rightarrow Y_{(s)} = \frac{s y(0) + y'(0)}{s^2 + 16}$$

$$\Rightarrow Y_{(s)} = \frac{s c_1}{s^2+16} + \frac{c_2}{s^2+16}$$

$$= c_1 \cos 4x + c_2 \sin 4x$$

b) $y(0) = 0$

$y'(\pi/8) = 0$.

$$\Rightarrow Y_{(s)} = \frac{y'(0)}{s^2+16}$$

$$\Rightarrow y(x) = \frac{c_1}{4} \sin 4x$$

$$y'(x) = c_1 \cos 4x$$

$$y'\left(\frac{\pi}{8}\right) = c_1 (0)$$

$$= 0$$

$\Rightarrow$ no contradiction

c) $y'\left(\frac{\pi}{8}\right) = \underline{\underline{0}}$, can't be $= 1$

there is a contradiction since $1 = c(0)$

$\Rightarrow c = \frac{1}{0}$ $\Rightarrow$ no constant can be multiplied to satisfy the theorm

$\Rightarrow$ contradiction.

ESS...

OK

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