Q1. $\frac{dy}{dx} = \frac{y}{x} - \frac{x}{y}$ , $y(1) = -\sqrt{2}$ $\qquad x>0$

let $u = \frac{y}{x} \Rightarrow y' = xu' + u$

$xu' + u = u - \frac{1}{u}$

(3) $\boxed{xu' = \frac{-1}{u}}$

$\int u\, du = \int \frac{-1}{x} dx \Rightarrow \frac{u^2}{2} = -\ln x + C$

(2) $\boxed{\frac{y^2}{2x^2} = -\ln x + C}$

$y(1) = -\sqrt{2} \Rightarrow \frac{2}{2} = -\ln 1 + C \Rightarrow C = 1$

(2) $\boxed{y^2 = 2x^2(-\ln x + 1)}$

(1) $\boxed{y = \mp \sqrt{2}\, x \sqrt{1 - \ln x}}$

Since $y(1) = -\sqrt{2}$ , the solution is

(1) $y = -x\sqrt{2(1-\ln x)}$

22. $x \ln y \dfrac{dy}{dx} = 3x^2 y \quad , \; y(2) = e^3$

$x > 0$

③ $\displaystyle\int \frac{\ln y}{y}\, dy = \int 3x\, dx$

③ $\dfrac{(\ln y)^2}{2} = 3\dfrac{x^2}{2} + C$

$y(2) = e^3 \Rightarrow \dfrac{(\ln e^3)^2}{2} = 3\,\dfrac{2^2}{2} + C \Rightarrow C = \dfrac{9}{2} - \dfrac{12}{2}$

$= \dfrac{-3}{2}$

③ $(\ln y)^2 = 3x^2 - 3$

$\ln y = \mp\sqrt{3x^2 - 3}$

or $y = e^{\pm\sqrt{3x^2-3}}$

$y(2) = e^3 \Rightarrow \boxed{y = e^{\sqrt{3x^2-3}}}$

Q3. $\frac{dx}{dy} = x + 3y$

(2) $\frac{dx}{dy} - x = 3y$     linear with $p(y) = -1$, $f(y) = 3y$

(2) $\mu = e^{\int - dy} = e^{-y}$

(2) $e^{-y} x = \int e^{-y}\, 3y \, dy$     $u = 3y$ $\quad dv = e^{-y}$, $\quad du = 3dy$ $\quad v = -e^{-y}$

$= -3ye^{-y} + \int 3e^{-y} dy$

(2) $= -3ye^{-y} - 3e^{-y} + C$

(1) $x = -3y - 3 + Ce^{y}$

---

or

(2) let $u = x + 3y \Rightarrow u' = 1 + 3y'$

(2) $\frac{u' - 1}{3} = \frac{1}{u} \Rightarrow u' = \frac{3}{u} + 1 = \frac{3 + u}{u}$

(2) $\int \frac{u}{3+u} du = \int dx \Rightarrow \int \left(1 - \frac{3}{1+u}\right) du = x + C$

$u - 3 \ln|3 + u| = x + C$

(3) $\boxed{x + 3y - 3 \ln|3 + x + 3y| = x + C}$

$y - \ln|3 + x + 3y| = C_1$

$3 + x + 3y = e^{y - C_1} = ce^{y}$

Q4. $(y e^{2xy} + x)\,dx + (b x e^{2xy})\,dy = 0$

③ $M_y = N_x \Rightarrow 2xy\,e^{2xy} + e^{2xy} = b(2yx\,e^{2xy} + e^{2xy})$

$\Rightarrow b = 1$

$\int (y e^{2xy} + x)\,dx + \int x e^{2xy}\,dy = c$

consider y as cons. delete terms with x

⑥ $y \dfrac{e^{2xy}}{2y} + \dfrac{x^2}{2} = c$

or

$\exists f$ s.t. $df = M dx + N dy = f_x\,dx + f_y\,dy$

② $f_x = M \Rightarrow f_x = y e^{2xy} + x \Rightarrow f = \dfrac{y e^{2xy}}{2y} + \dfrac{x^2}{2} + g(y)$

$f_y = \frac{1}{2} e^{2xy} \cdot 2x + g'(y) = N$

$e^{2xy}\, x + g'(y) = x e^{2x}$

$g'(y) = 0 \Rightarrow g(y) = C_1$

② $\Rightarrow f = \dfrac{e^{2xy}}{2} + \dfrac{x^2}{2} + C_1$

Solution is $f = C_2$

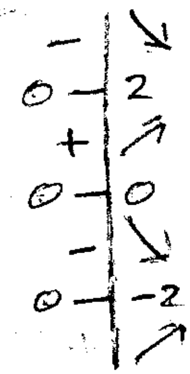
$\Rightarrow$

② $e^{2xy} + x^2 = c$

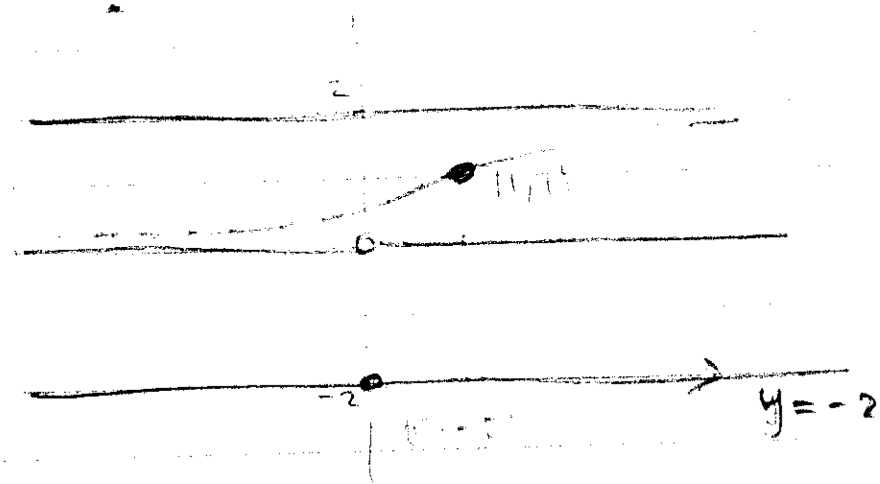
Q5. $y' = y(2-y)(2+y)$

(3) a) $y' = 0 \Rightarrow$ critical pts: $y=0$, $y=2$, $y=-2$

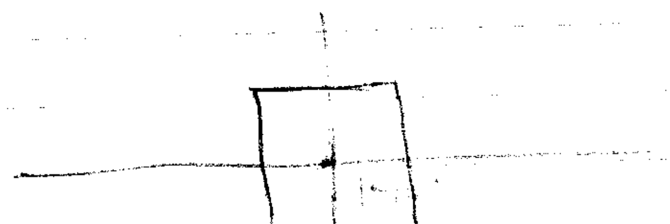
(3) b) $y=0$ is unstable sol.
$y=2$ is stable sol.
$y=-2$ is stable sol.




(3) c)




(3) d)



Since $\exists$ a rectangular region $R \ni (0,0)$
s.t. $f = y(4-y^2)$ & $\frac{\partial f}{\partial y} = y(-2y) + (4-y^2)$
are continuous on $R$,
the existence th. guarantees a unique sol. through $(0,0)$

Q5. max $V = 50$ gal $\quad V_0 = 10$ gal

$A(0) = 0 \quad C_{in} = 1$ lb/gal $\quad r_{in} = 4$ gal/min

$r_{out} = 2$ gal/min

(2) a) $V(t) = V_0 + (r_{in} - r_{out})t$

$50 = 10 + (4 - 2)t \Rightarrow t = 20$ min

b) $$C_{out} = \frac{A(t)}{V_0 + (r_{in} - r_{out})t} = \frac{A}{10 + 2t}$$

$$\frac{dA}{dt} = r_{in} C_{in} - r_{out} C_{out}$$

(3) $$\frac{dA}{dt} = 4 \times 1 - 2 \times \frac{A}{10+2t}, \quad 0 < t \le 20$$

$$\frac{dA}{dt} + \frac{1}{5+t} A = 4 \quad \text{linear} \quad p(t) = \frac{2}{10+2t}, \; f = 4$$

$$\mu = e^{\int \frac{1}{5+t} dt} = e^{\ln(5+t)} = 5 + t$$

$$(5+t)A = \int 4(5+t)\,dt = 20t + 2t^2 + C$$

$$A(0) = 0 \rightarrow 0 = 0 + C \Rightarrow C = 0$$

(5) $$A(t) = \frac{20t + 2t^2}{5+t} = \frac{2t(10+t)}{5+t}$$

(2) c) $$t = 3 \Rightarrow C_{out} = \frac{A(3)}{10 + 2(3)} = \frac{\frac{(6)(13)}{5+3}}{16} = \frac{39}{64}$$ lb/gal.