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Excellent

EXAM ONE, MTH 205, SPRING 008
THIS IS THE REAL TEST

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QUESTION 1. 10 points *Solve for $x(t)$ only if $x(t)' + y(t) = 1$ and $-x(t) - y(t)' = e^{-2t}, x(0) = y(0) = 0$.*

$\ell\{x(t)\} = X(s)$

$sX(s) + Y(s) = \frac{1}{s} \Rightarrow s^2X(s) + sY(s) = 1$

$-X(s) - sY(s) = \frac{1}{s+2}$ $\quad -X(s) - sY(s) = \frac{1}{s+2}$ $(+)$

$(s^2-1)X(s) + 0 = 1 + \frac{1}{s+2}$.

$X(s) = \frac{1}{s^2-1} + \frac{1}{(s+2)(s^2-1)}$

$= \frac{1}{(s-1)(s+1)} + \frac{1}{(s+2)(s-1)(s+1)}$

$\frac{A}{s-1} + \frac{B}{s+1} = \frac{1}{(s-1)(s+1)}$

$A = \frac{1}{2} \quad B = \frac{-1}{2}$

$= \frac{1}{2(s-1)} - \frac{1}{2(s+1)} + \frac{1}{3(s+2)} + \frac{1}{5(s-1)} - \frac{1}{2(s+1)}$

$\frac{C}{(s+2)} + \frac{D}{(s-1)} + \frac{E}{(s+1)} = \frac{1}{(s+2)(s-1)(s+1)}$

$C = \frac{1}{(-3)(-1)} = \frac{1}{3}$

$D = \frac{1}{3(2)} = \frac{1}{5}$

$E = \frac{1}{(1)(-2)} = -\frac{1}{2}$

$x(t) = \frac{e^t}{2} - e^{-t} + \frac{e^{-2t}}{3} + \frac{e^t}{5}$

$= \frac{7e^t}{10} - e^{-t} + \frac{e^{-2t}}{3}$.

QUESTION 2. 5 points *Is there a function $f(x)$ such that $\ell\{f(x)\} = \frac{3s}{s+1}$? If yes, then find $f(x)$, if no, then explain.*

$\lim_{x \to \infty} f(x) = \lim_{x \to \infty} \frac{3\cancel{x}}{\cancel{x}+1} = 3 \neq 0 \Rightarrow$ There is no la place transformation for $f(x)$.

QUESTION 3. 10 points *Find the general solution to $y(x)$ if $y^{(2)} - 3y' = 15$*

$y = e^{mx}$

y_c: $e^{mx}(m^2 - 3m) = 0 \Rightarrow m(m-3) = 0$

$m = 0$ $\quad$ $m = 3$

$y_c = c_1 e^0 + c_2 e^{3x} = c_1 + c_2 e^{3x}$

15 is a solution to y_c

$\Rightarrow$ $y_p = \cdot x \cdot a_0. \quad y' = a_0, \quad y'' = 0$

sub: $0 - 3(a_0) = 15 \Rightarrow a_0 = -5$ $\qquad y_p = -5x.$

$y_g = c_1 + c_2 e^{3x} - 5x$

QUESTION 4. 10 points *Solve for $f(x)$ if $f(x) = 1 + x + \int_0^x f(z)\,dz$*

$\mathcal{L}\{f(x)\} = F(s).$

$\int_0^x f(z)\, k(x-z)\, dz$

$k(x-z) = 1$

$F(s) = \frac{1}{s} + \frac{1}{s^2} + \frac{F(s)}{s}$

$F(s)\left(1 - \frac{1}{s}\right) = \frac{1}{s} + \frac{1}{s^2}$

$F(s)\left(\frac{s-1}{s}\right) = \frac{1}{s} + \frac{1}{s^2}$

$F(s) = \frac{1}{s-1} + \frac{1}{s(s-1)}$

$= \frac{1}{s-1} - \frac{1}{s} + \frac{1}{s-1}$

$= \frac{2}{s-1} - \frac{1}{s}$

$\Rightarrow f(x) = 2e^x - 1$

$\frac{A}{s} + \frac{B}{s-1} = \frac{1}{s(s-1)}$

$As - A + Bs = 1$

$A + B = 0$

$-A = 1 \quad A = -1$

$B = 1$

QUESTION 5. 10 points *Find the general solution to* $y(x)$ *if* $y^{(3)} + 3y^{(2)} + 7y' + 5y = 0$ *and* $y(x) = 3e^{-x}$ *is a solution to the D.E.*

$y = e^{mx}$

$3e^{-x}$ is a solution

$e^{mx}(m^3 + 3m^2 + 7m + 5) = 0$

$\Rightarrow -1 = m$ is a solution.

-1	1	3	7	5
		-1	-2	-5
	1	2	5	0

$m^2 + 2m + 5$

we are left with.

$m^2 + 2m + 5 = 0$

$m = \dfrac{-2 \pm \sqrt{4-20}}{2}$

$= \dfrac{-2 \pm 4i}{2} = -1 \pm 2i$

$y_g = c_1 e^{-x} + e^{-x}(c_2 \cos 2x + c_3 \sin 2x)$

QUESTION 6. 10 points *Use LAPLACE to solve* $2y' - 2y = U(x-5)e^{4x}, y(0) = 1$

$2sY(s) - 2 - 2Y(s) = \left[\dfrac{e^{-5s}}{s}\right]_{s \to s-4}$

$Y(s)[2(s-1)] = \dfrac{e^{-5(s-4)}}{s-4} + 2 = \dfrac{e^{-5s} \cdot e^{20}}{s-4} + 2$

$Y(s) = \dfrac{e^{20}}{2}\left(\dfrac{e^{-5s}}{(s-4)(s-1)}\right) + \dfrac{2}{2(s-1)}$

$= \dfrac{e^{20}}{2}\left(\dfrac{e^{-5s}}{3(s-4)} - \dfrac{e^{-5s}}{3(s-1)}\right) + \dfrac{2}{2(s-1)}$

$\dfrac{A}{s-4} + \dfrac{B}{s-1} = \dfrac{1}{(s-4)(s-1)}$

$A = \dfrac{1}{3}$

$B = -\dfrac{1}{3}$

$y(x) = \dfrac{e^{20}}{6}\left[e^{4(x-5)}U(x-5) - e^{(x-5)}U(x-5)\right] + e^x$

$= \dfrac{e^{20}\,U(x-5)}{6}\left[e^{4(x-5)} - e^{x-5}\right] + e^x.$

$\mathcal{L}\{U(x-5)e^{4x}\}$

$= \dfrac{e^{-5(s-4)}}{s-4}$

$\dfrac{1}{s-4}$ e^{at}

$e^{4(t-5)}$

QUESTION 7. *1.5 points Find* $\ell\{3^{x+1}sin(2x)\}$

$= \ell\{3e^{\ln 3x}\sin 2x\}$

$= 3\ell\{e^{\ln 3x}\sin 2x\}$

$= 3\left(\frac{2}{(s-\ln 3)^2+4}\right)$

$3^{x+1} = e^{\ln 3^{x+1}}$

$= e^{\ln 3\, x+\ln 3}$

$= e^{\ln 3x}\cdot e^{\ln 3}$

$= e^{\ln 3x}\cdot 3$

2. **5 points** *Find* $\ell\{xe^xU(x-2)\}$

$\ell\{e^x[xU(x-2)]\}$

$\ell\{xU(x-2)\} = \left[e^{-2s}\ell\{x+2\}\right]_{s\to s-1}$

$= \left[e^{-2s}\left(\frac{1}{s^2}+\frac{2}{s}\right)\right]_{s\to s-1}$

$\ell\{\cancel{e}\, xU(x-2)\} = e^{-2(s-1)}\left[\frac{1}{(s-1)^2}+\frac{2}{s-1}\right]$

$e^{-2s}\ell\{x+2\}$

$e^{-2s}\left[\frac{1}{s^2}+\frac{2}{s}\right]$

$e^{-2(s-1)}\left[\frac{1}{(s+1)^2}+\frac{2}{s+1}\right]$

3. **5 points** *Use convolution to find* $\ell^{-1}\{\frac{5}{s(s^2+25)}\}$

$F(s) = \frac{1}{s}$ $G(s) = \frac{5}{s^2+25}$

$f(t) = 1$ $g(t) = \sin 5t$

$f(t-r) = 1$

$\ell^{-1}\{F(s) \cdot G(s)\} = f(t) * g(t) = 1 * \sin 5t.$

$= \int_0^t f(t-r) g(r) dr = \int_0^t \sin 5r \, dr$

$= -\frac{1}{5} [\cos 5r]_0^t = -\frac{1}{5}(\cos 5t - 1) = \frac{1 - \cos 5t}{5}.$

4. **5 points** *Find* $\ell^{-1}\{\frac{e^{-3s}}{(s-5)^2}\}$

$F(s) = \frac{1}{(s-5)^2}$ $\left(\frac{-1}{s-5}\right)'$

$\ell^{-1}\{e^{-3s}/(s-5)^2\} = (t-3)e^{5(t-3)} u(t-3)$

$(-1)^1 \cdot t \cdot -e^{5t} =$

$\ell\{te^{5t}\}$

$= (-1)^1 \left(\frac{1}{s-5}\right)'$

$= \frac{-1(-1)}{(s-5)^2}$

$f(t) = te^{5t}$

$f(t-3) =$

5. **5 points** *Find* $\ell\{\int_0^t \sin(3r)e^{5t+r} \, dr\}$

$= \ell\{\int_0^t \sin 3r \, e^{5t+r} \cdot e^{-6r} \cdot e^{6r} dr\}$

$= \ell\{\int_0^t \sin 3r \cdot e^{6r} \, e^{5t-5r} dr\}$

$= F(s) \cdot G(s)$

$= \frac{3}{(s-6)^2+9} \times \frac{1}{s-5}$

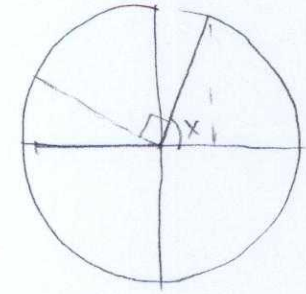
$f(t) = e^{6t} \sin 3t$

$k(t) = e^{5t}$

$F(s) = \frac{3}{(s-6)^2+9}$

$k(s) = \frac{1}{s-5}$

Hasnaa Rabbat



QUESTION 8. *A mass weighted 32 pounds is attached to a 7-foot spring. At the equilibrium the spring measures 10.2 feet. If the mass is initially released from rest at a point 2 feet above the equilibrium and there is a resistance numerically equals 2 times the velocity of the mass, then :*

$s = 10.2 - 7 = 3.2$

1) **10 points** *Find the motion-equation $x(t)$.* $x(0) = -2$ $\quad x'(0) = 0$

$m = \frac{32}{32} = 1$ slug. $\quad \beta = 2$. $\quad W = k \cdot s \Rightarrow k = W/s = \frac{32}{3.2} = 10$ pound/foot.

$m x''(t) + \beta x'(t) + k x(t) = 0$

$x''(t) + 2x'(t) + 10x(t) = 0 \qquad x = e^{mt}$

$m^2 + 2m + 10 = 0$

$m = \frac{-2 \mp \sqrt{4-40}}{2} = -1 \pm 3i$

$x_g = x(t) = e^{-t}(c_1 \cos 3t + c_2 \sin 3t)$

$x(t) = e^{-t}(-2\cos 3t - \frac{2}{3}\sin 3t)$

2) **5 points** *Find the phase angle Φ, then rewrite $x(t)$ in the form $x(t) = Af(t)sin(something + \phi)$.*

$A = \sqrt{c_1^2 + c_2^2} = \sqrt{4 + \frac{4}{9}} = \frac{\sqrt{40}}{3}$.

$x(t) = \frac{\sqrt{40}}{3} e^{-x}\left(\frac{-6}{\sqrt{40}} \cos 3x - \frac{2}{\sqrt{40}} \sin 3x\right)$

$\frac{-\sin \quad -\cos}{3rd.}$

$\sin\phi = \frac{-6}{\sqrt{40}} : -1.25 \text{ rad} = -\pi + 1.25$ (in 3rd quart)

$-1.89 = \phi$

NO NO

$x(t) = \frac{\sqrt{40}}{3} e^{-t} \sin(3t - 1.89)$

3) **2 points** *Find the first time at which the mass passes through the equilibrium position.*

equilibrium position $x(t) = 0$.

$x(t) = 0 = \frac{\sqrt{40}}{3} e^{-t} \sin(3t - 1.89)$

$\Rightarrow \sin(3t - 1.89) = 0 \Rightarrow 3t - 1.89 = 0 \Rightarrow t = 0.63$ s.

OK on wrong

4) **3 points** *Find the first time at which the mass has a velocity equals zero.*

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$x'(t) = 0 = -e^{-t}(-2\cos 3t - \frac{2}{3}\sin 3t) + e^{-t}(6\sin 3t - 2\cos 3t)$

$0 = e^{-t}(2\cos 3t + \frac{2}{3}\sin 3t + 6\sin 3t - 2\cos 3t)$

$\Rightarrow \frac{2}{3}\sin 3t + 6\sin 3t = 0$

$\frac{20}{3}\sin 3t = 0$

$x'(t) = -\frac{\sqrt{40}}{3} e^{-t} \sin(3t - 1.89) + \sqrt{40} e^{-t} \cos(3t - 1.89) = 0$

$\tan(3t - 1.89) = 3$

$1.25 = 3t - 1.89 \Rightarrow t = \pi/3$ s.

OK on wrong