FIRST EXAM FOR MTH 221

Name——————————, Id. Num.————————————————, Score $\frac{}{100}$

QUESTION 1. *Let* $D = \begin{bmatrix} 1 & -2 & 0 & 1 \\ -3 & 6 & 1 & 6 \\ 4 & -8 & -2 & 4 \end{bmatrix}$

*a)(***7 points***) Solve* $DX = \begin{bmatrix} -3 \\ 2 \\ -16 \end{bmatrix}$.

*b) (***5 points***) Use part (a) to solve* $DX = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

QUESTION 2. *Let* $A = \begin{bmatrix} 3 & 6 & -6 \\ 2 & 5 & 1 \\ 1 & 2 & -1 \end{bmatrix}$

*a)(**8 points**) Find the LU-factorization of A.*

*b)(**8 points**) Find* A^{-1}.

c) (Continue Question 2) (**6 points**) *Solve* $A^T X = \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}$

d) (**6 points**) *Write* A *as a product of elementary Matrices.*

e)(**6 points**) *Find* $(A^2)^{-1}$.

QUESTION 3. *(**9 points**) Let* N *be a* 2×2 *matrix such that* $\left(\begin{bmatrix} 4 & -7 \\ -3 & 5 \end{bmatrix} N^T + 3I_2\right)^T = 2N$. *Find* N.

QUESTION 4. *Let* $A = \begin{bmatrix} 3 & a & 6 \\ -3 & -4 & -2 \\ -3 & -a & b \end{bmatrix}$

*1) (**6 points**) For what values of* a, b, A *is nonsingular.*

*2) (**6 points**) Consider the system* $AX = \begin{bmatrix} 6 \\ 4 \\ c \end{bmatrix}$ *For what values of* a, b, c *will the system have infinitely many solutions?*

QUESTION 5. *(***4 points***) Given A is a 2×2 matrix such that* $\begin{bmatrix} 1 & 0 \\ -5 & 1 \end{bmatrix} A = \begin{bmatrix} 6 & 2 \\ -1 & 4 \end{bmatrix}$. *Find* $A \begin{bmatrix} 1 & 0 \\ -5 & 1 \end{bmatrix}$

QUESTION 6. *(***9 points***) Given A, B are 3×3 matrices such that* $det(A) = -2$, $det(B) = 4$. *Find*

1) $det(2A^{-1}B^T)$

2) $det(A^{-1} + adj(A))$

3) $det(adj(B^{-1})A)$

QUESTION 7. *Let* $A = \begin{bmatrix} 1 & 1 & -4 & 2 \\ -1 & 0 & 3 & 4 \\ 4 & 4 & 14 & -2 \\ -1 & -1 & 4 & -4 \end{bmatrix}$

*1) (***6 points***) Use Cramer rule to solve for* x_3 *in the system* $AX = \begin{bmatrix} 2 \\ -4 \\ 1 \\ -1 \end{bmatrix}$

*2) (***4 points***) Without finding* A^{-1} *find the (2, 4)-entry of* A^{-1}.

QUESTION 8. *a) (***5 points***) Explain in few words why an* $n \times n$ *matrix with two identical rows is singular.*

*b) (***5 points***) Let* A, B *be NONZERO* $n \times n$ *matrices such that* AB *is a zero matrix. Show that* A *AND* B *are both singular matrices .*

Department of Mathematics & Statistics, American University Of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates