REVIEW FOR THE FIRST EXAM , MTH 221, FALL 008, THIS IS NOT THE TEST BUT TO TEST A TEST

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QUESTION 1. *In at most two lines, write down what do you understand from each of the following statements?*

(1) v_1, v_2, v_3, v_4 *are independent elements of* R^4.
(2) *Let A be a* 3×7. *Find N(A) (i.e., Nul(A)), then find Nullity of A.*
(3) f_1, f_2, f_3 *are dependent polynomials in* P_7.
(4) *Let* $M = span\{x^2 + x + 1, x^2 - x + 1\}$. *Is* $1 \in M$*?*
(5) *S is a subspace of* $R^{2\times 3}$ *and dim(S) = 4.*
(6) *Find a basis for* P_{11}.
(7) *Write a given system in the matrix-standard form.*
(8) *The given system is consistent.*
(9) *The given system is inconsistent.*
(10) $f_1, f_2, f_3, f_4 \in P_6$, *and* $L = span\{f_1, f_2, f_3, f_4\}$ *but dim(L) = 2.*
(11) *A is* 5×11. *Find* $dim(row(A))$, *Find* $dim(Column(A))$, *and find Rank(A).*
(12) *A is* 5×7, *b is* 5×1 *and the system* $AX = b$ *is inconsistent (write your understanding in terms of the columns of A).*
(13) *The given matrix is in echelon form but it is not in reduced echelon.*
(14) *dim(Row(A)) = dim(column(A))= 5, where A is a* 7×11 *matrix.*

QUESTION 2. *WRITE DOWN T OR F*

(1) *If A is* 4×3, *then the homogeneous system* $AX = 0$ *has infinitely many solutions.*
(2) *If A is a* 3×5 *matrix such that* $Nullity(A) = 2$, *then for every column matrix* 3×1 b *the system* $AX = b$ *has infinitely many solutions.*
(3) *If A is a* 4×7 *such that the system* $AX = b$ *is consistent for EVERY b (note that b must be* 4×1*), then Nullity(A) = 3.*
(4) *If B is an echelon form of A, then B is unique.*
(5) *If B is a reduced echelon form of A, then B is unique*
(6) *If A is a nonzero matrix* 3×6, *then* $Nullity(A) \leq 5$.
(7) *If A, B are* $n \times n$ *and they have the same reduced echelon matrix, then* $A = B$.
(8) *if* $AB = 0$, *then either* $A = 0$ *or* $B = 0$.
(9) *It is possible to have 4 independent elements in* R^3.
(10) *It is possible that the span of 6 element in* R^5 *is equal to* R^5.
(11) $(-2, \infty)$ *is a subspace of R*
(12) $dim(span\{(2, 0, 1), (-2, 0, 1), (0, 0, 2)\}) = 2$.
(13) *If A is* 7×7 *and* $AX = 0$ *has a nontrivial solution, then the columns of A are dependent*
(14) *If A is* 10×7 *and* $AX = 0$ *has only the trivial solution, then the rank(A) = 7*
(15) *if A is a* 3×5 *matrix and* $AX = \begin{bmatrix} 2 \\ 3 \\ -4 \end{bmatrix}$ *has no solution, then* $dim(row(A)) \leq 2$.
(16) *If* X, Y *are independent, then* $X, Y, X + Y$ *are independent*
(17) *Every set of 4 elements of* R^4 *form a basis for* R^4.
(18) R^3 *has a basis of the form* $\{X, X + Y, Y\}$ *where* X, Y *are some elements in* R^3.

QUESTION 3. *a) Show that* $D = \{f(x) \in P_4 \mid f'(0) = 0, f(1) = f(-1) = 0\}$ *is subspace of* P_4 *. Find a basis for* D.

b) Show that $L = \{A \in M_{3\times 2}(R) \mid a_{21} + a_{22} + a_{31} = 0\}$ *is a subspace of* $M_{3\times 2}(R)$*. Rewrite L as a span.*

c) Let $M = span\{1 + x, -1 + 2x, x^2, 3x + x^2\}$*. Find a basis for M.*

QUESTION 4. *Find a basis for* R^4 *that contains the following two independent elements* $(1, 0, -1, 3), (-1, 0, 1, 8)$.

QUESTION 5. *Consider the following system:*

$x_1 + x_3 + bx_4 = 6$

$-x_1 + x_2 + 4x_3 - x_4 = 8$

$2x_1 + 2x_3 + 8x_4 = c$

a) For what values of b, c *does the system have infinitely many solutions?*

b) For what values of b, c *is the system inconsistent?*

QUESTION 6. *Let* $A = \begin{bmatrix} 2 & -1 & 0 & 1 \\ -1 & 0 & 1 & 0 \\ 2 & 3 & 0 & 1 \end{bmatrix}$ *and* $B = \begin{bmatrix} 1 & 2 \\ -1 & 3 \\ 0 & 1 \\ -2 & 4 \end{bmatrix}$ *Find* AB.

QUESTION 7. *Show that* $S = \{(x_1, x_2, x_1 + x_2, 2x_1) \mid x_1, x_2 \in R\}$ *is a subspace of* R^4*. Find a basis for S. Rewrite S as a span of some elements in S. Find the dimension of S.*

QUESTION 8. *Let* $S = \{(x, y) \mid x \in R \text{ and } y \geq 0\}$*. Draw the set S in the XY-plane. Is S a subspace of* R^2*? Explain.*

QUESTION 9. *are* $(1, 2, 3), (-2, 0, 1), (0, 4, 7)$ *independent? explain*

QUESTION 10. *Let* $K = \{f \in P_3 \mid xf \in P_3\}$*. Show that K is a subspace of* P_3*. Rewrite K as a SPAN.*

QUESTION 11. *Find a basis for the following subspace* $S = \{(x_3 + x_2, x_2, x_3) \mid x_2, x_3 \in R\}$*. Rewrite S as a SPAN set. Find the dimension of S.*

QUESTION 12. *The following are not subspaces (vector spaces): Give me at least one reason.*

1. $S = \{(x_1, x_1 + 1, x_3) \mid x_1, x_3 \in R\}$
2. $D = \{f(x) \in P_5 \mid f(-1) = 0 \;\; OR \;\; f(0) = 0\}$.

QUESTION 13. *Is* $Span\{(1, -1, 2), (-1, 1, 0), (-1, -1, -2), (-1, 1, 2)\} = R^3$*? EXPLAIN*

QUESTION 14. *Let* $A = \begin{bmatrix} 1 & -2 & 1 & 1 & 2 \\ -1 & 3 & 0 & 2 & -2 \\ 0 & 1 & 1 & 3 & 4 \\ 1 & 2 & 5 & 13 & 5 \end{bmatrix}$ *. Find a basis for the column space of A, Find a basis for the row space of A. Find a basis for N(A).*

QUESTION 15. *(This is indeed so NICE !!!) Given A is* 3×4 *and* $N(A) = \{ \begin{bmatrix} x_3 + 2x_2 \\ x_2 \\ x_3 \\ -5x_3 + 6x_2 \end{bmatrix} \mid x_3, x_2 \in R\}$*. Find a matrix B* 4×4 *such that Rank(B) = 2 and* $AB = 0$*. Find a matrix C* 4×6 *such that Rank(C) = 1 and* $AC = 0$*. Is it possible to find a matrix D of rank 3 such that* $AD = 0$*? Explain*

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