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FIRST EXAM FOR MTH 213

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QUESTION 1. **(WRITE T OR F, 38 points)**

(1) $\{\{3\},5\} \subseteq \{\{5\},\{3\},5,3\}$ T

(2) **The power set of the set** $\{2, \{2, \{7, 8\}\}, \{5\}\}$ **has exactly 8 elements.** T

$2^n = 2^3 =$

(3) $\exists! x \in Q$ **such that** $\forall y \in Z^+$, $x + y \geq xy$. F

(4) **If** A **has 4 elements and** B **has 6 elements, then** $P(A \times B)$ **has** 2^{10} **elements.** F

3 neg -> 3+7 < -300

3+7 < -300 -> 3 neg

(5) $\forall x \in R$, $\exists y! \in Z$ **such that** $Q(x, y)$ **means: I can find a unique integer y such that** $Q(x, y)$ **holds for all** $x \in R$. F

x = 4 (6) $x + 3 = 7$ **is a tautology statement.** F

(7) 3 **is a negative number if and only if** $3 + 7 < -300$ T

d | mn d | n

(8) **if** d **is a positive divisor of a positive integer** n, **then** $d \leq \sqrt{n}$. F

-45 = q 22 + r (9) **the quotient (q) of the division -45 by 22 is 3.** F

= -88 - 11

-88 mod 33 11 mod 33

(10) $-88 \equiv 11 mod(33)$ True

(11) $\neg(p \to q)$ **is logically equivalent to** $p \bigvee \neg q$. F

$p \wedge \neg q$

$p \to q$

$\neg p \vee q$

(12) **If** x **is an even integer, then** $x + 1$ **is odd integer is logically equivalent to** x **is an odd number or** $x + 1$ **is an odd integer.** T

P x = even, then x+1 is odd ≡ x is odd or x+1 is

$\neg p \vee q$

$p \to q$

$|x^2 + 3x \ln x| \leq c x^2$ (13) $(x^2 + 3xln(x))$ **is an** $O(x^2)$. T

$\frac{ab}{gcd(a,b)}$ (14) $LCM[32, 12] = 192$ F

(15) **If** a **and** b **are irrational numbers, then** $a + b$ **is an irrational number.** T

(16) **If** a **and** b **are irrational numbers, then** ab **is an irrational number.** F

-2

$-46 \bmod 7$ $\quad 3 \bmod 7 = 3$

$-46 = 7(-7) + ()$

-49

(17) $(2+3=5) \oplus (0+1=1)$ **is a true statement.** F

(18) -23 **is a solution to** $2x \equiv 3mod(7)$ True

(19) if n **is an integer, then** $\lfloor n+0.5 \rfloor < \lceil n+0.5 \rceil$ T

(14 points) **QUESTION 2.** (~~14~~ **points) Use Euclidian Algorithm to find the** $gcd(420,147)$, **then write the** $gcd(420,147)$ **as a linear combination of 420 and 147**

$420 = 147(2) + (126)$

$147 = 126(1) + \boxed{(21)}$

$126 = 21(6) + (0)$ (stop)

$$\boxed{gcd(420,147) = 21}$$

- $21 = 147 - 126$

$126 = 420 - 147(2)$

$21 = 147 - \cancel{126}(420 - 147(2))$

$$\boxed{21 = (-1)\,420 + (3)\,147}$$

$c_1 = -1$ $\quad c_2 = 3$

f^{-1} iff f 1-1

QUESTION 3. (6 **points**) *Let* $f : [-3, \infty] \longrightarrow [0, \infty]$ *such that* $f(x) = x^2 - 1$. *If* $B = \{8, 15, 24\}$, *then find* $f^{-1}(B)$.

(a) $8 = x^2 - 1$

$x^2 = 9 \Rightarrow |x| = 3$

$\boxed{x = \pm 3}$

(b) $15 = x^2 - 1$

$x^2 = 16$

$|x| = 4$

$\boxed{x = \pm 4}$ but -4 not in $\mathbb{D}$ so $x = 4$

(c) $24 = x^2 - 1$

$25 = x^2$

$|x| = 5$

$x = \pm 5$, $-5 \notin \mathbb{D}$

$\Rightarrow \boxed{x = 5}$

so $\boxed{f^{-1}(B) = \{3, -3, \cancel{4, 5}\}}$

$p \to Q$

$\neg Q \to \neg p$

$\neg p \to$ False

QUESTION 4. (7 **points**) *Let* a *be an irrational number and* b *be a rational number. Prove that* $a + b$ *is an irrational number (NOT MORE THAN 4 lines).*

P: a : irrational (&), $b = \left(\frac{m}{n}\right)$ rational, $n \neq 0$; $m, n \in \mathbb{Z}$

Q: $a + b$ irrational

1) a irrational, b rational $\Rightarrow b = \frac{m}{n}$

2) $\frac{na}{n} + \frac{m}{n} = \frac{na + m}{n}$ $\therefore$ $a+b$ is irrational

QUESTION 5. (5 **points**) *Convert number* $(123)_{10}$ *to its equivalent number in base 7.*

$123_{10} \to (\ )_7$

$123 = 7(17) + (4)$

$17 = 7(2) + (3)$

$2 = 7(0) + (2)$

$\boxed{123_{10} = \cancel{234_7}}$

$6x = 8(\ ) + (4)$ $\quad$ $4 = 8(0)$ $\boxed{4}$

18 $\quad$ 36 $\quad$ 24 $\quad$ 12 $\quad$ $6x \bmod 8$

$6x$ $\quad$ 30

42

QUESTION 6. (**8 points**) *Find all solutions for* $6x \equiv 4 mod(8)$

$0 \le x < 8$

exist $\Rightarrow$ $\gcd(6,8) \mid 4$

$2 \mid 4$ ✓

$x = 2$

$12 \equiv 4 \bmod 8$

$x = 6$

$36 \equiv 4 \bmod 8$

$\therefore$ sln $= \boxed{2 \pm k8,\ k \in \mathbb{Z}}$ $\wedge$ $\boxed{6 \pm k8,\ k \in \mathbb{Z}}$

QUESTION 7. (**6 points**) *Is* $\neg(p \to q) \vee q$ *a tautology statement? EXPLAIN..*

$p \wedge \neg q$

p	q	$p \to q$	$\neg(p \to q)$	$\neg(p \to q) \vee q$
T	T	T	F	T
T	F	F	T	T
F	T	T	F	T
F	F	T	F	F

$\therefore$ the statement is not tautology since it is not always true.

101 110
111

011
100
011 11 8

QUESTION 8. *(***7 points***) Convert* $(34661)_8$ *to its equivalent number in base 16 (hexadecimal).*

$(34661)_8$

$(011\ 100\ 110\ 110\ 001)_2$

$0011\ 1001\ 1011\ 0001$

$\boxed{(39B1)_{16}}$

QUESTION 9. *(***7 points***) Describe an algorithm that gives 220 consecutive ODD NUMBERS NON of them is a prime number.*

$k = 440$

$k = \frac{440}{2}$

$d = (440+1)! + \boxed{3} = c$

$\vdots$

$d_{k-1} = (440+1)! + 439$

this gives 220 consecutive odd numbers non prime

each time increment c by 2 (not 1).

Excellent

GOOD LUCK

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