MTH 213, SECOND EXAM, SPRING 006

AYMAN BADAWI

100 Excellent

Name Nancy Salem, Id. Num. 11431., Score $\frac{}{100}$

QUESTION 1. (**6 points**) *a)Let* $n = 6^3.11^5.10^3$. *Find* $\phi(n)$.

$n = (2\times3)^3 \cdot 11^5 \cdot (2\times5)^3$

$= 2^3 \cdot 3^3 \cdot 11^5 \cdot 2^3 \cdot 5^3$

$= 2^6 \cdot 3^3 \cdot 11^5 \cdot 5^3$

$\phi(n) = (2-1)2^5 \cdot (3-1)\,3^2 \cdot (11-1)\,11^4 \cdot (5-1)\,5^2$

$= 2^5 \cdot 2 \cdot 3^2 \cdot 10 \cdot 11^4 \cdot 4 \cdot 5^2$

$\boxed{\phi(n) = 5^3 \cdot 2^9 \cdot 3^2 \cdot 11^4}$

b) Let $k = 6\phi(3000) + 3$. Find $11^k mod(3000)$(**4 points**)

$a^{\phi(n)} = 1 \bmod n$

$a^{6(\phi(n))+3} =$

$a^{6\phi(n)} \cdot a^3 = 1 \cdot a^3 \bmod n$

$\boxed{\gcd(11, 3000) = 1}$

$11^3 \bmod (3000) = 1331$

QUESTION 2. *Let* $b = (10010)_2$.

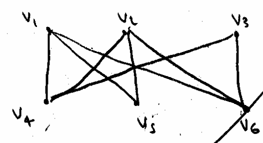
a) Find the value of b in base 10.(**4 points***)*

$2^4 + 2^1 = 16 + 2 = \boxed{18}$

b) find $7^b(mod202)$*(* **6 points***)* set answer = 1

2401	0	$7^1 \bmod 202 \to 7$	answer = answer =	$\boxed{1}$
32041	1	$7^2 \bmod 202 \to 49$	answer = (answer * 49) mod 202 =	$\boxed{49}$
15625	0	$49^2 \bmod 202 \to 179$	answer = answer =	$\boxed{49}$
3479	0	$179^2 \bmod 202 \to 125$	answer = answer = 49	
	1	$125^2 \bmod 202 \to 71$	answer = (answer * 71) mod 202 =	$\boxed{45}$

QUESTION 3. *(* **5 points***)*
Can we have a connected simple bipartite graph with exactly 8 edges and 6 vertices? If yes, then give me such graph. If no, then explain.



QUESTION 4. *a)(* **5 points***) Is it possible to have a a connected simple graph G with 7 edges such that $\overline{G}$ also have 7 edges? if yes, then GIVE me G. If no, then explain.*

$*$ $G \cup \overline{G}$ → 14 edges → $2|E| = \sum$ degrees
complete graph

$28 = n(n-1)$

$n^2 - n - 28 = 0$

Then No, if we solve this equation to get number of vertices then we get n as a non integer which is impossible.

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

b) (**5points***) How many edges does $\overline{C_{12}}$ have? DO NOT GIVE ME the graph of $\overline{C_{12}}$.*

$\boxed{C_{12}}$ → cycle with 12 edges

Complete graph (K_n) has $\frac{n(n-1)}{2} = \frac{12 \times 11}{2} = 66$ edges

Since $C_{12} \cup \overline{C_{12}} = K_{12}$

thus $\overline{C_{12}} = 66 - 12 = \boxed{54 \text{ edges}}$

c) (**5 points***) Is C_{16} a bipartite graph? if yes, then explain.*

Yes C_{16} is a bipartite graph since 16 is an even number, thus we can partition vertices into two sets

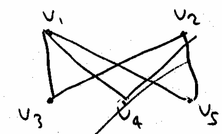
V_1 { Contains all odd vertices }

V_2 { contains all even vertices }

None of the even vertices are adjacent
" " " odd vertices " "
the only way you have an edge is between an even vertex and an odd vertex.

QUESTION 5. *a)(**6 points**) Find the adjacency matrix A for $K_{2,3}$.*

$$A = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \end{bmatrix} \end{array}$$



*b) (**2 points**) For the matrix A above, what does the number in the 2nd row and the 3rd column of A^7 mean? (there is no need to find such number)*

Number of all paths of length $\boxed{\text{seven}}$ between v_2 & v_3

*c)(**3 points**) How many edges does Q_8 have?*

$\sum \text{degrees} = 2|E|$

$$\frac{2^m \cdot m}{2} = |E| = 2^7 \cdot 8 = 2^{10} = \boxed{1024}$$

*d) (**3 points**) How many Edges does $K_{4,9}$ have?*

$K_{4,9}$ has $4 * 9$ edges $\rightarrow$ 36 edges

$2 * 7$

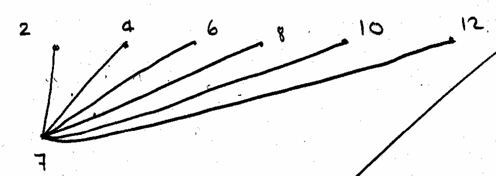
$(2-1)2^0 * (7-1)7^0$

$\boxed{6}$

QUESTION 6. *(* **10 points** *) Let $S = \{1 < a < 14 \mid gcd(a, 14) \neq 1\}$ be the set of vertices of a graph. Two vertices a, b in S are connected by an edge iff $ab(\underbrace{mod 14}) = 0$. Draw such graph. what is the degree of each Vertex? Is the graph complete? Is the graph a bipartite graph?Explain. Draw $\overline{S}$*

14

$S = \{2, 4, 6, 7, 8, 10, 12\}$



degree of each vertex is $\boxed{1}$

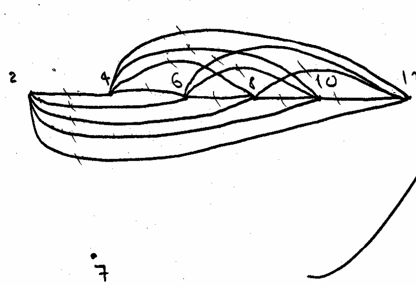
The graph is $\boxed{\text{not}}$ complete.

The graph is bipartite $V_1 = \{2, 4, 6, 8, 10, 12\}$ non adjacent

$V_2 = \{7\}$ nonadjacent

edges connect $V_1 \to V_2$ thus bipartite.

$\overline{S}$ $\Rightarrow$



QUESTION 7. *(* **10 points***) Describe all integers that have the following properties: If each number is divided by 2, then the remainder is 0. If each number is divided by 5, then the remainder is 3. If each number is divided by 11, then the remainder is 4.*

$$x \equiv \overset{b_1}{0} \bmod \overset{r_1}{2}$$
$$x \equiv \overset{b_2}{3} \bmod \overset{r_2}{5}$$
$$x \equiv \overset{b_3}{4} \bmod \overset{r_3}{11}$$

$m = 2 * 5 * 11 = 110$

$m_1 = 55$

$m_2 = 22$

$m_3 = 10.$

$55y = 1 \bmod 2 \Rightarrow \boxed{y = 3}$

$22z = 1 \bmod 5 \Rightarrow \boxed{z = 3}$

$10w = 1 \bmod 11 \Rightarrow \boxed{w = 10}$

$$S = (b_1 m_1 y + b_2 m_2 z + b_3 m_3 w) \bmod m$$

$$= \boxed{48}$$

$\frac{48}{2} \to$ remainder 0

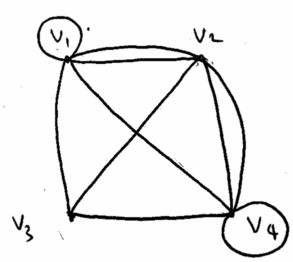
$\frac{48}{5} \to$ remainder 3

$\frac{48}{11} \to$ remainder 4

$\to$ all solutions are in the form $\boxed{48 \pm 110k}$

QUESTION 8. *(* **6 points***) Draw a graph with the following adjacency*

matrix $A = \begin{array}{c|cccc} & v_1 & v_2 & v_3 & v_4 \\ \hline v_1 & 1 & 2 & 1 & 1 \\ v_2 & 2 & 0 & 1 & 2 \\ v_3 & 1 & 1 & 0 & 1 \\ v_4 & 1 & 2 & 1 & 1 \end{array}$



QUESTION 9. *(* **10 points***) Use Math. Induction to prove* $\sum_{i=1}^{n} \frac{1}{i^2+5i+6} = \frac{n}{3n+9}$.

*. Prove it for n=1.

$$\sum_{i=1}^{1} \frac{1}{i^2+5i+6} = \frac{1}{1+5+6} = \frac{1}{12}$$

$$\frac{n}{3n+9} \rightarrow \frac{1}{3(1)+9} = \frac{1}{12} \quad ✓.$$

* assume true for every k such that $1 \leq k \leq n$ for some n.

$$\sum_{i=1}^{k} \frac{1}{i^2+5i+6} = \frac{k}{3k+9}$$

* prove it for n+1.

$$\sum_{i=1}^{n+1} \frac{1}{i^2+5i+6} \stackrel{?}{=} \frac{(n+1)}{3(n+1)+9} = \frac{n+1}{3n+12}$$

→ please see the back of the page

QUESTION 10. *(* **10 points***) Let Given* $a_0 = -6$, $a_1 = 12$, *and* $a_n = 12a_{n-1} - 36a_{n-2}$. *Find a mathematical equation for* a_n.

$P(x) \Rightarrow$

$$x^n = 12x^{n-1} - 36x^{n-2}$$

$$x^n - 12x^{n-1} + 36x^{n-2} = 0 \quad \div x^{n-2}$$

$$x^2 - 12x + 36 = 0$$

$$(x-6)(x-6) = 0 \rightarrow$$

$$(x-6)^2 \leftarrow \quad x_1 = x_2 = 6 \quad \text{root repeated twice}$$

Department of Mathematics & Statistics, American University Of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates

E-mail address: abadawi@aus.edu, www.ayman-badawi.com

$\therefore \quad a_n = (b+mn)6^n$

$a_0 = -6 = (b)$

$a_1 = (-6+m)6$

$12 = -36+6m$

$48 = 6m$

$\boxed{m=8}$

$\Rightarrow \quad \boxed{a_n = (-6+8n)6^n}$

$a_2 = \cancel{360}$

$a_2 = 12(12) - 36(-6)$

$= 360$ ✓✓

$$\sum_{i=1}^{n+1} \frac{1}{i^2+5i+6} = \sum_{i=1}^{n} \frac{1}{i^2+5i+6} + \frac{1}{(n+1)^2+5(n+1)+6}$$

now use previous step

we know

$$\sum_{i=1}^{k} \frac{1}{i^2+5i+6} = \frac{k}{3k+9}$$

let $p = k+1$

$p^2+5p+6 = (p+2)(p+3)$
$= (k+3)(k+4)$

$$\therefore \sum_{i=1}^{k} \frac{1}{i^2+5i+6} + \frac{1}{(k+1)^2+5(k+1)+6} = \frac{k}{3k+9} + \frac{1}{(k+1)^2+5(k+1)+6}$$

common denominator

$$= \frac{(\frac{1}{3}(k+4)k+1)}{((k+3)(k+4))} * \frac{3}{3}$$

$$= \frac{k^2+4k+3}{3((k+3)(k+4))}$$

$$= \frac{(k+1)\cancel{(k+3)}}{3(\cancel{(k+3)}(k+4))}$$

$$= \boxed{\frac{k+1}{3k+12}}$$

$(k+1)(k+3)$

$$\therefore \sum_{i=1}^{n} \frac{1}{i^2+5i+6} = \frac{n}{3n+9}.$$