Name Sara Zamzam, ID 88202

Exam 2, MTH 213 , Fall 2022

Ayman Badawi

Score = 50/50]

QUESTION 1. (8 points) The following numbers will be used to create car license plates: 1, 2, 3, 4, 5, 6, 7, 8. Each plate number must have seven digits.

a) In the event where repetition is not allowed, how many **odd** plate numbers can be created?

$2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 4 = 20{,}160$ ✓

b) How many possible license plate numbers may be made if repetition is not allowed, the first digit must be even, and the last digit must be odd?

$4 \times 2 \times 3 \times 4 \times 5 \times 6 \times 4 = 11{,}520$ ✓

c) There are 4 men and 5 women in the class, we need to form a committee of 6 persons. How many different ways can we set up this committee so that there is at least one man on it?

$9C6 = 84$ ✓ Good

d) We need to form 4-letters words from the word "read" such the letters a, e are adjacent in each word. How many words can be formed? (note that each letter must appear exactly once in each word).

a,e { at the beginning, 4 words
in the middle, 4 words
at the end, 4 Words

$4+4+4 = 12$ words can be formed ✓

QUESTION 2. (6 points) Let $a_n = -3a_{n-1} + 4a_{n-2} + 10(2^n)$. Find a general formula for a_n[no need to find c_1, c_2]

$a_h(n): \; a_n + 3a_{n-1} - 4a_{n-2} = 0$

$(a^n + 3a^{n-1} - 4a^{n-2} = 0\ )/a^{n-2}$

$a^2 + 3a - 4 = 0$

$(a-1)(a+4) = 0$

$a_1 = 1 \quad a_2 = -4$

$a_h(n) = c_1(1^n) + c_2((-4)^n)$

$a_p(n) = c(2^n) = \frac{20}{3}(2^n)$

$a_p(n) + 3a_p(n-1) - 4a_p(n-2) = 10(2^n)$

$(c(2^n) + 3c(2^{n-1}) - 4c(2^{n-2}) = 10(2^n))/2^{n-2}$

$4c + 6c - 4c = 40$

$6c = 40$

$c = 20/3$

$a_n = a_h(n) + a_p(n)$

$= c_1 + c_2((-4)^n) + \frac{20}{3}(2^n)$ ✓

QUESTION 3. (4 points) Let $a_n = -3a_{n-1} + 4a_{n-2} + 1$. Find a general formula for a_n[no need to find c_1, c_2, see Question 2]

$a_h(n) = c_1(1^n) + c_2((-4)^n)$

$a_p(n) = cn$ (because 1 is one of the roots and the particular is a polynomial)

$a_p(n) + 3a_p(n-1) - 4a_p(n-2) = 1$

$cn + 3c(n-1) - 4c(n-2) = 1$

$\cancel{cn} + \cancel{3cn} - 3c - \cancel{4cn} + 8c = 1$

$5c = 1$

$c = 1/5$

$a_n = a_h(n) + a_p(n)$

$a_n = c_1 + c_2((-4)^n) + \frac{1}{5}n$

QUESTION 4. (10 points)

(a) **(2 points)** Let $A = \{1, 2, 3, 13, 15, 22, 23\}$. Define "=" on A such that $a" = "b$ if $b - a \in \{0, 1, 2\}$. Convince me that "=" is not an equivalence relation on A.

Let $a = 1$, $b = 2$

Test for symmetry $2 - 1 = 1 \in \{0, 1, 2\}$

$1 - 2 = -1 \notin \{0, 1, 2\}$

Thus, "=" is not an equivalence relation since symmetry axiom fails.

(b)**(2 points)** Assume there is a relation, call it F, on the set $B = \{2, 4, 7\}$ such that F is both equivalence relation on B and a partial order on B. If we view F as a subset of $B \times B$. Write down all elements of F.

$\{(2,2), (4,4), (7,7)\}$

(c) Let $A = \{1, 3, 5, 6, 7, 8, 10, 12\}$ define "=" on A such that $a" = "b$ if (a mod 3) - (b mod 3) $\in \{-2, 0, 2\}$. Then "=" is an equivalence relation on A.

$3, 6, 12 \to 0$
$1, 7, 10 \to 1$
$5, 8 \to 2$

(i) **(4 points)** Find all distinct equivalence classes.

$\bar{1} = \{1, 7, 10\}$ 3 elements

$\bar{3} = \{3, 5, 6, 8, 12\}$ 5 elements

(ii) **(2 points)** If we view "=" as a subset of $A \times A$. How many elements does "=" have? (do not write down the elements)

$(3 \times 3) + (5 \times 5) = 9 + 25 = 34$ elements

QUESTION 5. (6 points) (a) **(3 points)** There are n balls in a big basket. We need to distribute these balls over 10 elementary schools. Find the minimum value of n so that at least one school will have 23 balls.

$$\left\lceil \frac{n}{10} \right\rceil = 23 \quad \Bigg| \quad n = [(23-1)(10)] + 1$$
$$= (22)(10) + 1$$
$$= 221 \text{ balls}$$

(b) **(3 points)** Assume $n = 163$. Then at least one school will have m balls. What is the best value of m?

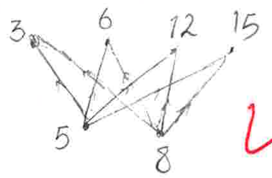
$$\left\lceil \frac{163}{10} \right\rceil = \lceil 16.3 \rceil = 17 \text{ balls}$$

QUESTION 6. (6 points) Let $A = \{3, 5, 6, 8, 12, 15\}$ define " $\leq$ " on A such that a" $\leq$ "b if $b = a$ **OR** (a mod 3) - (b mod 3) $\in \{2\}$. Then " $\leq$ " is a partial order relation on A.

3 6 12 15 → 0
5,8 → 2

(i) **(4 points)** Draw Hasse diagram of $\leq$.

3 "≤" 3
5 "≤" 3, 5, 6, 12, 15
6 "≤" 6
8 "≤" 3, 6, 8, 12, 15
12 "≤" 12
15 "≤" 15



(ii) **(2 points)** Find

(a) gcd(8, 5) = DNE

(b) LCM(8, 3) = 3

(c) gcd (5, 15) = 5

(d) gcd(3, 15) = DNE

QUESTION 7. (4 points) Use the truth table and convince me that $\overline{a \cdot b}$ is logical equivalent to $\overline{a} + \overline{b}$.

a	b	$\overline{a}$	$\overline{b}$	$a.b$	$\overline{a.b}$	$\overline{a}+\overline{b}$
0	0	1	1	0	1	1
0	1	1	0	0	1	1
1	0	0	1	0	1	1
1	1	0	0	1	0	0

QUESTION 8. (6 points) Consider the following code

$$For\ k = 1\ to\ n\ do$$

$$y = 3 * i + i^2 + 7$$

$1+1+1+1=4$

$$For\ i = 1\ to\ k\ do$$

$$x = i^4 + 2 * i$$

$3+1+1=5$

$$Next\ i$$

$$Next\ k$$

a) Find the exact number of arithmetic operations that will be executed by the code.

Outer loop $n-1+1=n$

Arithmetic $=4n$

Inner loop $k-1+1=k$

$5k$

1st $\to k=1 \sim 5(1)=5$

last $\to k=n \sim 5(n)=5n$

$\frac{5+5n}{2}(n)$

Total $\frac{(5+5n)(n)}{2}+4n$

b) Find the complexity of the code, i.e. O(Code)

$O(\text{Code}) = O(n^2)$