MTH 733, Commutative Algebra, Spring 2025, 1-1

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All rings are commutative with $1 \neq 0$

QUESTION 1. Let $A = \begin{bmatrix} 0 & 0 & 5 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \in (Z_7)^{3 \times 3}$. Find A^9 .

QUESTION 2. (1) Give me an example of a quasilocal ring R that is not Artinian.

(2) Give me an example of a quasilocal ring R that is Artinian.

QUESTION 3. Let R be a commutative ring and $A \in \mathbb{R}^{n \times n}$. Prove that A is invertible iff $|A| \in U(\mathbb{R})$.

QUESTION 4. Let A be the companion matrix of the polynomial $f(x) = (x - 1)^2(x - 3) \in Z_{13}[x]$. Find the Jordan form and the rational form of A.

QUESTION 5. Assume that $R = R_1 \times R_2$ such that R_1, R_2 are quasilocal and Noetherian. Then R is Artinian. prove or disprove.

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