

HW IV

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All rings are commutative with $1 \neq 0$

QUESTION 1. Let $n \geq 2$. Prove that nZ is not an injective Z-module.

QUESTION 2. Let M, N be R-modules and $f : M \to N$ be an R-homomorphism. Prove that $Ker(f)$ is an R-submodule of M.

QUESTION 3. Let P be a projective R-module, M be an R module, and $f : M \to P$ be a surjective R-homomorphism. Prove that M and $P \oplus Ker(f)$ are isomorphic as R-modules.

QUESTION 4. We know that Z_7 is not a projective Z-module. Prove that Z_7 is a projective Z_{21}-module.

QUESTION 5. Let $F = Z_8$ and $R = F[[x]]$, $a = 5 + x^2 + 4x^3 \in R$. If $a^{-1} \in R$, then find a^{-1}.

QUESTION 6. (i) Give me an example of a non-Noetherian ring. [Hint: Take $R = Z[X_1, X_2, ..., X_n, ...]$.

(ii) Give an example of a ring R with exactly one maximal ideal M such that Nil(R) = M, but $M^k \neq \{0\}$ for every integer $k \geq 1$.

QUESTION 7. Let M be a Z-module such that $ann(m) = 13Z$ for some $m \in M$. Prove that M has a submodule N such that N is isomoprphic to a field F as F-modules.

QUESTION 8. Give me an example of a Noetherian ring that is not Artinian.

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