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HW III

Ayman Badawi

All rings are commutative with $1 \neq 0$

QUESTION 1. Let R be a quasilocal ring (i.e., R has exactly one maximal ideal) that is not a field. Let M be a nonzero f.g. R-module. Prove that $IM \neq M$ for every proper ideal I of R.

QUESTION 2. (a) Let R be a Noetherian quasilocal ring (i.e., R has exactly one maximal ideal, say, N) that is not a field. Prove that $\cap_{i=1}^{\infty} N^i = \{0\}$.

(b) Let P be a nonzero prime ideal of $A = Z[X, Y, Z]$ and $R = A_P$. Let $M = P_P$. Prove that $\cap_{i=1}^{\infty} M^i = \{0\}$.

QUESTION 3. It is clear that Z_{12} and Z_{15} are Z-module. Let $f : Z_{12} \to Z_{15}$ be a Z-homomorphism such that $f(a) \neq 0$ for some $a \in Z_{12}$. Find the range(f) and the Ker(f).

QUESTION 4. Let A, B, C, D be R-modules and $f : C \to D$ be an R-homomorphism. Let $K : HOM_R(B, C) \to HOM_R(B, D)$ such that $K(g) = f\ o\ g = f(g(b))$ for every $g \in Hom(B, C)$ and every $b \in B$. Prove that K is an R-homomorphism.

QUESTION 5. Let $M = Z_4 \oplus Z_6$. Then M is a Z-module. How many Z-homomorphism are there from Z into M?

QUESTION 6. Let R be a reduced finite ring (i.e., $Nil(R) = \{0\}$). Let $x \in R$. Prove that $x = eu$, for some idempotent e of R (recall e is idempotent if $e^2 = e$) and some unit $u \in U(R)$.

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