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HW II

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QUESTION 1. Let R be a commutative ring with $1 \neq 0$ and M be a nonzero vector space over R (i.e., M is an R-module). Consider the trivial ring extension A of R, called the idealization of R with M, $A = R(+)M$, Where $(r, m) + (r_1, m_1) = (r + r_1, m + m_1)$ and $(r, m)(r_1, m_1) = (rr_1, r_1m + rm_1)$. We know (class notes) that A is a ring with identity $(1, 0)$. Prove

(i) $Nil(A) = \{(n, m) \mid n \in Nil(R)$, and $m \in M\}$

(ii) $Spec(A) = \{P(+)M \mid$ P is a prime ideal of $R\}$

(iii) $Max(A) = \{L(+)M \mid$ L is a maximal ideal of $R\}$

(iv) $U(A) = \{(u, m) \mid u$ is a unit in R and $m \in M\}$ [Hint: Note that a unit + nilpotent is a unit]

QUESTION 2. Let $A \subset B$ be an integral ring extension such that B is an integral domain. Assume A is a field. Prove that B is a field. (In class, we proved that if B is a field, then A is a field)

QUESTION 3. Prove that $Q[\sqrt{2} + \sqrt{3}]$ is a field.

QUESTION 4. Find the integral closure of $R = Z[\frac{1}{2}]$ inside $R[\sqrt[5]{\frac{1}{2}}]$

QUESTION 5. Give a Theatricality PROOF of the statement: Exists at least one element in $Q[\sqrt{0.5}]$ that is not integral over $Z[0.5]$.

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