Name——————————————————————, ID ——————

HWI

Ayman Badawi

ALL RINGS ARE COMMUTATIVE with $1 \neq 0$

QUESTION 1. Let I be a proper ideal of R. Prove:

(i) I is a prime ideal of R if and only if whenever $I_1I_2 \subseteq I$, for some proper ideals I_1, I_2 of R, then $I_1 \subseteq I$ or $I_2 \subseteq I$.

(ii) I is a prime ideal of R if and only if $R - I$ is a multiplicative closet subset of R.

(iii) Give me an example of a prime ideal I and a proper ideal $J \nsubseteq I$ of a ring R such that $I \cap J$ is not a prime ideal

QUESTION 2. Let I be a proper ideal of R. Prove:

(i) I is a primary ideal of R if and only if whenever $I_1I_2 \subseteq I$, for some proper ideals I_1, I_2 of R, then $I_1 \subseteq I$ or $I_2 \subseteq \sqrt{I}$.

(ii) Give me an example of a proper ideal I of a ring R such that $\sqrt{I}$ is a prime ideal of R, but I is not a primary ideal of R.

(iii) Let M be a maximal ideal of R. Prove that M^n is a primary ideal of R for every positive integer ≥ 1.

(iv) A prime ideal P of R is called a divided prime ideal if $P \subset rR$ for every $r \in R - P$. Assume that P is a divided prime ideal of an integral domain R, prove that P^n is a primary ideal of R for every integer $n \geq 1$.

(v) Let $I_1, ..., I_k$ be proper ideals of R. Prove that $\sqrt{I_1 \cap \cdots \cap I_k} = \sqrt{I_1} \cap \cdots \cap \sqrt{I_k}$

QUESTION 3. .

(i) Let $R = Z[X]$ and $I = (4, 9x)$ be a proper ideal of R. Find $\sqrt{I}$. Is I a primary ideal of R? explain briefly.

(ii) Let $R = \frac{Z[X]}{(30X)}$. Find $Z(R)$, and $Nil(R)$.

QUESTION 4. (a) Let $P_1, P_2, ..., P_k$ be prime ideals of a ring R such that for every $1 \leq i, j \leq k$ with $i \neq j$, we have $P_i \nsubseteq P_j$. Prove that $R - (P_1 \cup \cdots \cup P_k)$ is a multiplicative closet subset of R.

(b) Give me an example of infinite integral domain that has exactly 4 maximal ideals.

(c) Give me an example of a nonzero prime ideal, say P, of an infinite integral domain R, such that P is not a maximal ideal of R, but P_S is a maximal ideal of R_S for some multiplicative closet subset S of R.

Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: abadawi@aus.edu, www.ayman-badawi.com