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Final Exam

Ayman Badawi

All rings are commutative with $1 \neq 0$

QUESTION 1. Let R be an integral domain contained in a field L. If L is integral over R, then R is afield.

QUESTION 2. Give me an example of an integral domain that has exactly 3 maximal ideals.

QUESTION 3. Let R be a finite commutative ring. Prove that R is ring-isomorphic to $R_1 \times R_2 \cdots R_n$ for some $n < \infty$, such that each R_i is quasi-local with maximal ideal M_i where $M_i^{k_i} = \{0\}$ for some $k_i \geq 1$.

QUESTION 4. Let P be a projective R-module, M be an R module, and $f : M \to P$ be a surjective R-homomorphism. Prove that M and $P \oplus Ker(f)$ are isomorphic as R-modules.

QUESTION 5. Let R be an Artinian ring. Prove that R is Noetherian.

QUESTION 6. Let $R = Q[X, Y]$ and K be the quotient field of R. Prove that $R[\frac{x}{y}]$ is not finitely generated R-module.

QUESTION 7. Let M be a finitely generated R-module such that $J(R)M = M$. Prove that $M = \{0\}$.

QUESTION 8. Give me an example of a free R-module M of dimension 2 that has a submodule L of dimension 2, but $L \neq M$.

QUESTION 9. Let $A \in Z_5^{3\times 3}$ such that $C_A(\alpha) = (\alpha + 3)^3$. Find all possible rational forms and Jordan forms of A.

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