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Exam II

Ayman Badawi

All rings are commutative with $1 \neq 0$

QUESTION 1. Let $n \geq 2$. Prove that nZ is not an injective Z-module.

QUESTION 2. Let $F = Z_{10}$ and $R = F[[x]]$, $a = 3 + x^3 + 2x^4 \in R$. If $a^{-1} \in R$, then find a^{-1}.

QUESTION 3. Give an example of a ring R with exactly one maximal ideal M such that Nil(R) = M, but $M^k \neq \{0\}$ for every integer $k \geq 1$.

QUESTION 4. (a) Give me an example a non-finitely generated Z-module M such that $ann(M) = \{0\}$.

(b) Give me an example a non-finitely generated Z-module M such that $ann(M) = 23Z$.

QUESTION 5. Let P be a projective R-module, M be an R module, and $f : M \to P$ be a surjective R-homomorphism. Prove that M and $P \oplus Ker(f)$ are isomorphic as R-modules.

QUESTION 6. Give me an example of a Noetherian ring that is not Artinian.

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