Name————————————————————, ID ————————

Exam I

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All rings are commutative with $1 \neq 0$

QUESTION 1. Let M be a Z-module. Assume (*) $0 \to M \to Z_{20} \to Z_4 \to 0$ is a short exact sequence. Prove that (*) is a split short exact sequence. (note we know that Z_{20}, Z_4 are Z-modules)

QUESTION 2. Let R be a Noetherian integral domain and I be a nonzero proper ideal of R. Assume M is a f.g R-module that is torsion free. Prove that $\cap_{n=1}^{\infty} I^n M = \{0\}$ and $\cap_{n=1}^{\infty} I^n = \{0\}$

QUESTION 3. Let M be an R-module and m be a nonzero element of M such that whenever $n \in M$, $n \neq 0$ and $ann(m) \subseteq ann(n)$, then $ann(m) = ann(n)$. Prove that $ann(m)$ is a prime ideal of R.

QUESTION 4. (a) Let M be a f.g Z-module such that $ann(M) = 11Z$. Prove that $|M| < \infty$. Can you give a general formula for $|M|$?

(b) Give me an example a non-finitely generated Z-module M such that $ann(M) = \{0\}$.

(b) Give me an example a non-finitely generated Z-module M such that $ann(M) = 2025Z$.

QUESTION 5. (a) Let R be a GCD-domain with quotient field K. Prove that R is an integrally closed domain.

(b) Let $R = Z[X]$ and K be the quotient field of R. Let $\alpha \in K - R$. prove that $R[\alpha]$ is not a finitely generated R-module.

(c) Give me an example of an integral domain R with quotient field K such that R is not integrally closed inside K.

QUESTION 6. Let M_1, M_2, M_3 be R-modules such that the short exact sequence $0 \to M_1 \to M_2 \to M_3 \to 0$ splits. Prove that M_2 and $M_1 \oplus M_3$ are isomorphic as R-modules.

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