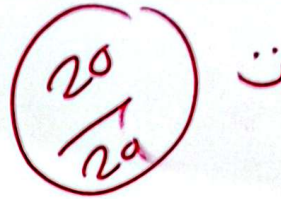


Quiz VII

Ayman Badawi



QUESTION 1. (10 points) Let $A = \{1, 5, 9, 2, 6, 3, 15, 4, 12, 16, 20\}$. Define " \equiv " on A such that for all $a, b \in A$, $a \equiv b$ if $(a \bmod 4) = (b \bmod 4)$. Then " \equiv " is an equivalence relation.

(i) Find all distinct equivalence classes of " \equiv ".

$$\bar{1} = \{1, 5, 9\}$$

$$\bar{3} = \{3, 15\}$$

$$\bar{2} = \{2, 6\}$$

$$\bar{4} = \{4, 12, 16, 20\}$$

$$4 \bmod 4 = 12 \bmod 4 = 16 \bmod 4 = 20 \bmod 4$$

$$1 \bmod 4 = 5 \bmod 4 = 9 \bmod 4$$

$$2 \bmod 4 = 6 \bmod 4$$

$$3 \bmod 4 = 15 \bmod 4$$

(ii) if we view " \equiv " as a subset of $A \times A$, how many elements does " \equiv " have? Do NOT write the elements of " \equiv ".

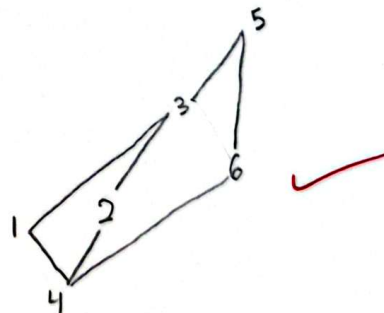
$$|\equiv| = 3^2 + 2^2 + 4^2 + 2^2$$

$$= 33$$

QUESTION 2. (10 points) The following is a partial order relation on the set $A = \{1, 2, 3, 4, 5, 6\}$

$$"\leq" = \{(1, 1), (2, 2), \dots, (6, 6), (4, 1), (4, 2), (4, 3), (4, 5), (4, 6), (2, 3), (3, 5), (2, 5), (1, 3), (1, 5), (6, 5)\}$$

(i) Draw Hasse diagram of " \leq ".



(ii) Find the minimum element of (A, \leq) and the maximum element of (A, \leq) .

Minimum element = 4

Maximum element = 5