Name Samira Steitie, ID g00104859

MTH 213, Discrete Math, Spring 2025, 1–1

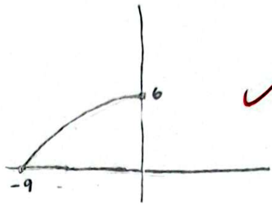
Quiz IV

Ayman Badawi

$\frac{20}{20}$

QUESTION 1. Consider the function $f(x) : [-9,0] \to [0,6]$ such that $f(x) = 2\sqrt{x+9}$.

(i) **(2 points)** Sketch $f(x)$, roughly



2

(ii) **(3 points)** Is $f(x)$ ONTO? Is $f(x)$ one-to-one? you may use the staring-method and some tests.

* yes by horizontal line test it is one to one as each input has only one output

* yes it is onto as the co-domain is equal to the range.

3

(iii) **(5 points)** If f^{-1} exists, find f^{-1}. Then find its domain, and its co-domain.

f^{-1} exists as $f(x)$ is onto and one to one.

$x = 2\sqrt{y+9}$

$\left(\frac{x}{2}\right)^2 = \left(\sqrt{y+9}\right)^2$

$\frac{x^2}{4} = y+9$

$f^{-1}(x) = y = \frac{x^2}{4} - 9$

$\frac{5}{5}$

domain = $[0,6]$

co-domain = $[-9,0]$

QUESTION 2. Let $f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 3 & 4 & 7 & 8 & 1 & 10 & 9 & 2 & 5 & 6 \end{pmatrix}$

a) **(6 points)** Find the smallest integer n such that $f^n = I(x)$

$\frac{10}{10}$

$(1\ 3\ 7\ 9\ 5)(2\ 4\ 8)(6\ 10)$

$n = \text{LCM}(5,3,2)$

$\gcd(5,3) = 1$

$\gcd(1,2) = 1$

$\text{LCM} = \frac{5 \times 3 \times 2}{\gcd = 1} = 30$

b) **(2 points)** Find the smallest integer m such that $f^m = f^{-1}$.

$m = n-1 = 30-1 = 29$

c) **(2 points)** find f^{-1} $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 5 & 8 & 1 & 2 & 9 & 10 & 3 & 4 & 7 & 6 \end{pmatrix}$