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# Quiz 3

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**QUESTION 1.** Use the 4th method and prove that $\sqrt{21}$ is irrational.

Deny. Hence, $\sqrt{21}$ is rational

proof:- $\sqrt{21} = \frac{a}{b}$ where $a, b$ are integers and $b \neq 0$

$21 = \frac{a^2}{b^2}$, where $a$ is odd and $b$ is odd

Hence, $a = 2k+1$ and $b = 2m+1$, where $k, m \in \mathbb{Z}$

$21 = \frac{(2k+1)^2}{(2m+1)^2} \Rightarrow 21 = \frac{4k^2+4k+1}{4m^2+4m+1}$

$\frac{-1+4k^2+4k+1 = 21(4m^2)+21(4m)+21-1}{4}$

$\underbrace{k^2+k}_{\text{even}} = \underbrace{21m^2+21m+5}_{21(m^2+m)+5}$

odd.even
=even+odd
=odd

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we got even = odd
but even is not equal odd, so there is a contradiction, proving that $\sqrt{21}$ is irrational

**QUESTION 2.** Prove that $15 \mid (2^{8n} - 1)$ for every $n \geq 1$.

(1) let $n=1$

$\Rightarrow \frac{2^{8(1)}-1}{15} = 17$

(2) we assume that $15 \mid 2^{8n}-1$ for some integer $n \geq 1$

(3) proof: $15 \mid 2^{8(n+1)}-1$

$\Rightarrow 2^{8n+8}-1$

$\Rightarrow 2^{8n} \cdot 2^8 - 2^8 + 2^8 - 1$

$\Rightarrow \underbrace{2^8(2^{8n}-1)}_{*} + \underbrace{2^8-1}_{**}$

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$*$ is $2^8(2^{8n}-1)$ is a factor of 15 proven by (2)

$**$ is $2^8-1$ is a factor of 15 proven by (1)

Hence, by math induction $15 \mid 2^{8n}-1$ for every $n \geq 1$

**QUESTION 3.** Use the 4th method and prove that $\sqrt{10}$ is irrational.

Deny. Hence, $\sqrt{10}$ is rational

proof: $\sqrt{10} = \frac{a}{b}$ where $a, b$ are integers and $b \neq 0$

$10 = \frac{a^2}{b^2}$, where $a$ is even and $b$ is odd

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Hence, $a = 2m$ and $b = 2n+1$, where $m, n \in \mathbb{Z}$

$10 = \frac{(2m)^2}{(2n+1)^2} \Rightarrow 10 = \frac{4m^2}{4n^2+4n+1} \Rightarrow \frac{4m^2 = 10(4n^2)+10(4n)+10}{4}$

$\Rightarrow \underbrace{m^2}_{\text{integer}} = \underbrace{10n^2+10n+\frac{10}{4}}_{\text{not integer}}$, we got integer = not integer which is a contradiction, hence $\sqrt{10}$ is irrational