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Quiz 1

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20/20

QUESTION 1. (a) (4 points) Solve for x, $6x = 3$ in the planet Z_9.

$a=6,\ b=3,\ n=9$

$\rightarrow \gcd(a,n) = \gcd(6,9) = 3$

Is $3 \mid 3$? YES ∴ there are 3 solutions

$\rightarrow$ gap $= \frac{n}{\gcd(a,n)} = \frac{9}{3} = 3$

∴ Solution set is $\{2, 5, 8\}$

(b) (2 points) using (a), find all possible solutions for x over the planet Z, where $6x \pmod 9 = 3$

$\Rightarrow$ Solution set is $\{2 + 3m \mid m \in \mathbb{Z}\}$.

QUESTION 2. (4 points) Find the $gcd(105, 154)$ and the $LCM(105, 154)$

$\Rightarrow \gcd(105, 154)$

$154 = 1 \times 105 + 49$ | $105 = 2 \times 49 + 7$ | $49 = 7 \times 7 + 0$

∴ $\gcd(105, 154) = 7$

$\Rightarrow LCM[105, 154] = \frac{105 \times 154}{\gcd(105 \times 154)} = 2310$

QUESTION 3. (4 points) Find all solutions for x over the planet Z, where $5x \pmod 7 = 6$.

solve :- $5x = 6$ in $\mathbb{Z}_7$

$a=5,\ b=6,\ n=7$

$\rightarrow \gcd(a,n) = \gcd(5,7) = 1$

Is $1 \mid 6$? YES

$\rightarrow$ gap $= \frac{n}{\gcd(a,n)} = \frac{7}{1} = 7$

∴ Solution set is $\{4 + 7m \mid m \in \mathbb{Z}\}$.

QUESTION 4. (6 points) Find the smallest positive integer x such that $x \pmod 9 = 6$ and $x \pmod 7 = 2$. (Show the work)

↳ $m_1 = 9,\ m_2 = 7$

as the gcd(every 2 mi's) = 1, CRT is applicable

$\rightarrow m = m_1 \times m_2 = 9 \times 7 = 63 \Rightarrow 0 \le$ smallest +ve integer < 63

$\rightarrow n_1 = \frac{m}{m_1} = \frac{63}{9} = 7 \Rightarrow 7y_1 = 1$ in $\mathbb{Z}_9 \Rightarrow y_1 = 4$

$n_2 = \frac{m}{m_2} = \frac{63}{7} = 9 \Rightarrow 9y_2 = 1$ in $\mathbb{Z}_7 \Rightarrow 2y_2 = 1$ in $\mathbb{Z}_7 \Rightarrow y_2 = 4$

∴ $x = n_1 x_1 y_1 + n_2 x_2 y_2 \pmod m$

$= [(7 \times 6 \times 4) + (9 \times 2 \times 4)] \pmod{63}$

$= 240 \pmod{63}$

$\Rightarrow 3\frac{17}{21} = 3\frac{51}{63} \Rightarrow x = 51$

∴ $x = 51$ is the smallest positive integer.