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Exam II

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SCORE = $\frac{39}{\cancel{56}\ 41}$ 41/41

QUESTION 1. (i) (**4 points**) Define "=" on $A = \{1, 2, 9, 10, 17, 18, 11, 25, 3\}$ such that for every $a, b \in A$, a "$=$" b iff $a\ mod(8) = b\ mod(8)$. Then "$=$" is an equivalence relation on A. Find all equivalence classes. If we view "=" as a subset of $A \times A$. What is the size of "="? DON'T find the elements of $=$.

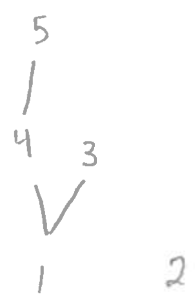
$\bar{1} = \{1, 9, 17, 25\}$

$\bar{2} = \{2, 10, 18\}$

$\bar{3} = \{3, 11\}$

$|\text{"="}| = 4^2 + 3^2 + 2^2 = 29$

(ii) (**4 points**) $\leq = \{(1,1), (2,2), (3,3), (4,4), (5,5), (1,3), (1,4), (4,5), (1,5)\}$ is a partial order relation on $A = \{1, 2, 3, 4, 5\}$. Draw the Hesse diagram of $\leq$.



(iii) **(4 points)** Given " = " = $\{(1,1),(2,2),(3,3),(4,4),(5,5),(1,3),(3,1),(2,6),(6,2),(3,5),(5,3),(1,5),(5,1)\}$ is an equivalence relation on $A = \{1,2,3,4,5,6\}$. Find all distinct equivalence classes of "=".

$\bar{1} = \{1,3,5\}$

$\bar{2} = \{2,6\}$

$\bar{4} = \{4\}$

QUESTION 2. (i) **(4 points)** Let $a_n = 6a_{n-1} - 8a_{n-2}$ such that $a_1 = 5$ and $a_2 = 89$. Find a general formula for a_n. Dont find c_1 & c_2

$-4 + -2 = -6$

$-4 \times -2 = 8$

$a_n - 6a_{n-1} + 8a_{n-2} = 0$

$\alpha^2 - 6\alpha + 8 = 0$

$a_n = c_1 4^n + c_2 2^n$

$(\alpha - 4)(\alpha - 2) = 0$

$\alpha_1 = 4 \quad \alpha_2 = 2$

(ii) **(6 points)** Let $a_n = 6a_{n-1} \cancel{-12a_{n-2}} + 40(\cancel{2^n})$ $-8a_{n-2}$ (5^n). Find a general formula for a_n. DO NOT FIND C_1, C_2.

$a_h \rightarrow \quad a_n - 6a_{n-1} + 8a_{n-2} = 0$

$a_h = c_1 4^n + c_2 2^n$

$a_p(n) = a(5^n)$

$$\frac{a(5^n) - 6a(5^{n-1}) + 8a(5^{n-2}) = 40(5^n)}{5^{n-2}}$$

$25a - 30a + 8a = 1000$

$3a = 1000$

$a = \frac{1000}{3}$

$a_n = c_1 4^n + c_2 2^n + \frac{1000}{3}(5^n)$

0 1 2 3 4 5 6 7

QUESTION 3. (i) **(3 points)** The digits 0, 1, 2, ..., 7 will be used to construct car plates, where each plate has 5 digits. How many EVEN number plates (i.e., last digit is even) can be constructed if repetition is not allowed?

7 6 5 4 4

$7 \times 6 \times 5 \times 4 \times 4$

0 1 2 3 4 5 6 7 8

(ii) **(3 points)** The digits 0, 1, 2, ..., 8 will be used to construct car plates, where each plate has 5 digits. If the first, third and forth digits must be even, the 5th digit is odd, and no repetition. How many car plates can be constructed ?

5 5 4 3 4

$5 \times 5 \times 4 \times 3 \times 4$

0 1 2 3 4 5 6 7 8 9

(iii) **(3 points)** The digits 0, 1, 2, ..., 9 will be used to construct car plates, where each plate has 5 digits. If the first, third and forth digit must be even, the 5th digit is odd, and repetition is allowed. How many car plates can be constructed?

5 10 5 5 5

$5 \times 10 \times 5 \times 5 \times 5$

(iv) **(3 points)** Let m be the number of balls that will be distributed over 101 schools. What is the minimum value of m so that a school will have at least 80 balls.

$|C| = 101$

$\left\lceil \frac{m}{101} \right\rceil = 80$

$m = (101 \times 79) + 1$

$= 7980$

(v) **(3 points)** 479 positive ODD integers are available. Then there are at least m odd integers out of the given 479 odd integers say $a_1, \ldots, a_m$ such that $a_1 \pmod 8) = a_2 \pmod 8) \cdots = a_m \pmod 8)$. What is the best value of m?

$|D| = 479$

$|C| = 4$

$C \to \begin{pmatrix} 1 \\ 3 \\ 5 \\ 7 \end{pmatrix}$

$\left\lceil \frac{479}{4} \right\rceil = 120$

QUESTION 4. (6 points)

Imagine: The following Algorithm segment. Find the exact number of additions, multiplications, and subtractions that will be performed when the algorithm is executed. Then find the complexity of the Algorithm segment.

For $k := 1$ *to* $n^2 + 3$

$1 + 1 + 1 + 1 + 1 + 1 = 6$

$L = k * m + s^2 + 2 * s - 2$

For $i := 1$ *to* $(k + 2)$

$3 + 1 + 1 + 1 + 2 + 1 + 1 = 10$

$W = s^4 + 3 * k + m^3 + i - 8$

next i

next k

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Outer loop

# of terms $= n^2 + 3 - 1 + 1 = n^2 + 3$

# of arithmatic operations $= 6(n^2 + 3)$

Inner loop

iterations $= k + 2 - 1 + 1 = k + 2$

first term $= 10\big((1) + 2\big) = 10(3) = 30$

last term $= 10\big((n^2+3) + 2\big) = 10n^2 + 50$

total $= 6(n^2 + 3) + (n^2 + 3)\left(\dfrac{10n^2 + 80}{2}\right)$

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