

Quiz 4

Ayman Badawi

Score = 2020
20

QUESTION 1. (6 points)

Find the angle between the vectors $V_1 = \langle 2, 3 \rangle$ and $V_2 = \langle -3, 5 \rangle$.angle = θ

$$\theta = \cos^{-1} \left[\frac{V_1 \cdot V_2}{|V_1| |V_2|} \right]$$

$$V_1 = \langle 2, 3 \rangle$$

$$V_2 = \langle -3, 5 \rangle$$

$$V_1 \cdot V_2 = 2(-3) + 3(5) = 9$$

$$|V_1| = \sqrt{(2)^2 + (3)^2} = \sqrt{13}$$

$$|V_2| = \sqrt{(-3)^2 + (5)^2} = \sqrt{34}$$

$$\theta = \cos^{-1} \left[\frac{9}{\sqrt{13}\sqrt{34}} \right]$$

$$\theta = 64.65^\circ$$

6
6QUESTION 2. (6 points) Let $L_1: x = 3t + 1, y = 6t - 4, z = 3t + 9$ and $L_2: x = w - 5, y = 2w + 7, z = w - 3$ be two distinct lines. Is L_1 parallel to L_2 ? show the work.

$$L_1 \begin{cases} x = 3t + 1 \\ y = 6t - 4 \\ z = 3t + 9 \end{cases} t \in \mathbb{R}$$

 $L_1 \parallel L_2?$

$$D_1 = \langle 3, 6, 3 \rangle$$

$$D_2 = \langle 1, 2, 1 \rangle$$

$$\langle 3, 6, 3 \rangle = \langle 3 \cdot 1, 3 \cdot 2, 3 \cdot 1 \rangle$$

$$\langle 3, 6, 3 \rangle = \langle 3 \cdot 1, 3 \cdot 2, 3 \cdot 1 \rangle$$

$$\langle 3, 6, 3 \rangle = \langle 3 \cdot 1, 3 \cdot 2, 3 \cdot 1 \rangle$$

$$3 = 1c \rightarrow c = 3$$

$$6 = 2c \rightarrow \frac{6}{2} = c \rightarrow c = 3$$

$$3 = 1c \rightarrow c = 3$$

YES! $L_1 \parallel L_2$ 6
6QUESTION 3. (8 points) Let $L_1: x = 3t + 1, y = 6t - 4, z = 3t - 1$ and $L_2: x = 3w - 2, y = 2w + 2, z = 2w - 1$ be two distinct vectors. Does L_1 intersect L_2 ?

$$L_1 \begin{cases} x = 3t + 1 \\ y = 6t - 4 \\ z = 3t - 1 \end{cases} t \in \mathbb{R}$$

$$L_2 \begin{cases} x = 3w - 2 \\ y = 2w + 2 \\ z = 2w - 1 \end{cases} w \in \mathbb{R}$$

Does L_1 intersect L_2 ?

$$x_{L_1} = x_{L_2}$$

$$y_{L_1} = y_{L_2}$$

$$3t + 1 = 3w - 2$$

$$3t - 3w = -2 - 1$$

$$3t - 3w = -3 \rightarrow \textcircled{1}$$

$$6t - 4 = 2w + 2$$

$$6t - 2w = 2 + 4$$

$$6t - 2w = 6 \rightarrow \textcircled{2}$$

$$t = 2$$

$$w = 3$$

$$z \text{ in } L_1 = z \text{ in } L_2$$

$$z = 3t - 1$$

$$t = 2$$

$$z_{L_1} = 3(2) - 1$$

$$z_{L_1} = 5$$

$$z = 2w - 1$$

$$w = 3$$

$$z_{L_2} = 2(3) - 1$$

$$z_{L_2} = 5$$

YES! L_1 intersects L_2 since

$$z \text{ in } L_1 = z \text{ in } L_2$$

$$L_1 \text{ point of intersection} = (7, 8, 5)$$

$$L_2 \text{ point of intersection} = (7, 8, 5)$$

8
8

Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.

E-mail: abadawi@aus.edu, www.ayman-badawi.com