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Quiz I

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$$Score = \frac{\quad 20 \quad}{20}$$

QUESTION 1. (13 points)

The equation of an ellipse is $\frac{(x-1)^2}{100} + \frac{(y-3)^2}{64} = 1$.

(i) Roughly, sketch the ellipse. (on the right hand side)

$$\frac{(x-x_0)^2}{(\frac{k}{2})^2} + \frac{(y-y_0)^2}{b^2}$$

(ii) Find the ellipse constant; k.

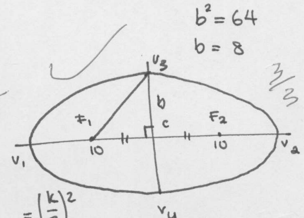
$\left(\frac{k}{2}\right)^2 = 100, \Leftrightarrow \frac{k}{2} = 10 \Leftrightarrow k = 20$ 2/2

$b^2 = 64$

$b = 8$

(iii) Find the center, C

$C = (x_0, y_0)$, $x_0 = 1$, $y_0 = 3$ ~~therefore~~ $c = (1,3)$ 1/1



3/3

(iv) Find the Foci, F_1, F_2

$c = (1,3)$. Since F_1, c, F_2 are on the H-line, y value doesn't change 3/3

$F_1 = (1-6, 3) = (-5,3)$ $F_2 = (1+6, 3) = (7,3)$ 4/4

$= \left(\frac{k}{2}\right)^2$

$|v_3F_1|^2 = |cF_1|^2 + |v_3c|^2$

(v) Find all 4 vertices, v_1, v_2, v_3, v_4.

13/13

Since v_1 and v_2 are on the H-line, the y value doesn't change.

$v_1 = (1-10, 3) = (-9,3)$; $v_2 = (1+10, 3) = (11,3)$

v_3 and v_4 are on v-line, so x value stays

$v_3 = (1, 3+8) = (1, 11)$; $v_4 = (1, 3-8) = (1, -5)$

$100 = |cF_1|^2 + 64$

$|cF_1|^2 = 100 - 64 = 36$

$|cF_1| = \sqrt{36} = 6$

QUESTION 2. (7 points) Given $(-2, 4)$ is the center of an ellipse that has $v_1 = (1, 4)$, $v_3 = (-2, 9)$ as vertices. Find the Foci and the equation of the ellipse. (Hint: it will help, if you sketch)

3pts 4pts

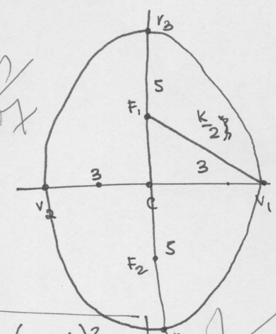
$c = (-2, 4)$; $v_1 = (1, 4)$; $v_3 = (-2, 9)$

$|v_1F_1|^2 = |v_1c|^2 + |F_1c|^2$

$k = |v_3v_4| = 10$

6.5/7

7/7



$\left(\frac{k}{2}\right)^2 = 3^2 + |F_1c|^2$

$|F_1c|^2 = 5^2 - 3^2 = 25 - 9 = 14$ 16

$|F_1c| = \sqrt{14}$ 4

$F_1 = (-2, 4+\sqrt{14})$; $F_2 = (-2, 4-\sqrt{14})$ $(-2, 8)$ $= (-2, 0)$

4/4 3/3

$$\boxed{\frac{(x+2)^2}{9} + \frac{(y-4)^2}{25} = 1}$$