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Exam II: MTH 111, Spring 2025

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Points = $\frac{42}{\cancel{60}\ 42}$ Excellent!!

QUESTION 1. (i) Find $Lim_{x \to 1} \frac{x^2-1}{x-1}$

$$\lim_{x\to 1} \frac{x^2-1}{x-1} = \lim_{x\to 1} \frac{(x+1)(x-1)}{x-1} = \lim_{x\to 1} x+1 = \lim_{x\to 1} x+1 = 2$$

(ii) Find $Lim_{x \to 2} \frac{\sqrt{x+2}}{x}$

$$\lim_{x\to 2} \frac{\sqrt{x+2}}{x} = \lim_{x\to 2} \frac{\sqrt{x+2}}{x} = \lim_{x\to 2} \frac{\sqrt{x+2}}{x} = 1$$

QUESTION 2. Find the equation of the tangent line to the curve $f(x) = 2x^3 + 7x + 3$ at $x = 1$.

$f(x) = 2x^3 + 7x + 3$, $x = 1$

$f'(x) = 6x^2 + 7$

$f(1) = 2 \times 1^3 + 7 \times 1 + 3$
$= 2 + 7 + 3 = 12$

$f'(1) = 6 \times 1^2 + 7$
$= 6 + 7 = 13$

$y = f'(1)(x-1) + f(1)$

$= 13x - 13 + 12 = \boxed{13x - 1}$

QUESTION 3. Find $f'(x)$ but do not simplify

$v = (2x^4 + x + 1)^2$; $v' = 2(2x^4 + x + 1) \times (8x^3 + 1)$

$u = (3x^3 + 7x + 1)^3$; $u' = 3(3x^3 + 7x + 1)^2 \times (9x^2 + 7)$

(i) $f(x) = (3x^3 + 7x + 1)^3(2x^4 + x + 1)^2$

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$f'(x) =$

(ii) $f(x) = \sqrt{3x+2} + 7x + 1$ $= (3x+2)^{1/2} + 7x + 1$

$f'(x) = \frac{1}{2}(3x+2)^{-1/2} \times 3 + 7$

(iii) Let $k(x) = f(2x^3 + 7x + 1)$ such that $f'(1) = 2025$. Find $k'(0)$.

$k(x) = f(g(x))$

$g(x) = 2x^3 + 7x + 1$

$k'(x) = f'(g(x)) \times g'(x)$

$k'(0) = f'(1) \times g'(0)$ $= k'(0) = 2025 \times 7$

$k'(0) = f'(1) \times g'(0)$

$= 2025 \times 7$

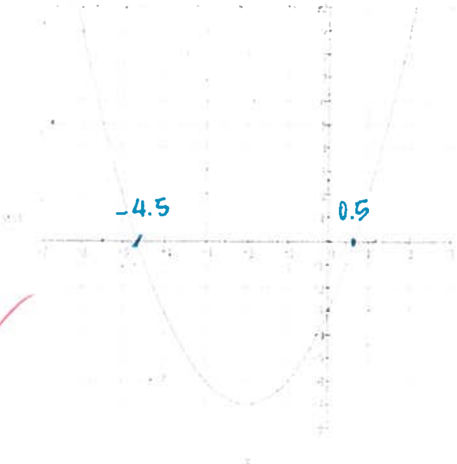
$= 14175$

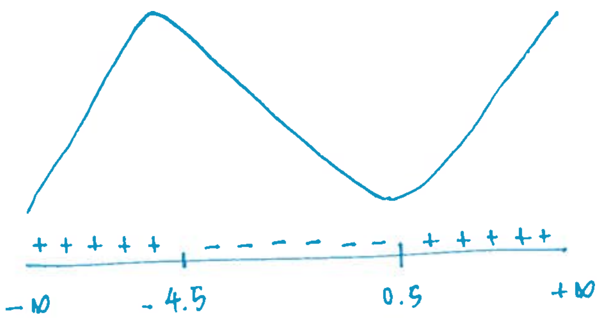
$g(0) = 2 \times 0^3 + 7 \times 0 + 1 = 1$; $g'(x) = 6x^2 + 7$

$g'(0) = 7$

$-\infty < x < \infty$

QUESTION 5. (12 points) The graph of $f'(x)$ is given below. Note that $2010 \leq x \leq 2014$, you may draw the sign of $f'(x)$ on the right hand side of the graph.





(i) For what values of x does $f(x)$ increase?

$f(x)$ increases for $x \in (-\infty; -4.5) \cup (0.5; +\infty)$

(ii) For what values of x does $f(x)$ decrease?

$f(x)$ decreases for $x \in (-4.5; 0.5)$

(iii) For what values of x does $f(x)$ have local min values?

$f(x)$ has local min values for $x = 0.5$

(iv) For what values of x does $f(x)$ have local max values?

$f(x)$ has local max values for $x = 4.5$

(v) Roughly, sketch the graph of $f(x)$. (on the right hand side)

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$3x^2 + 12x$

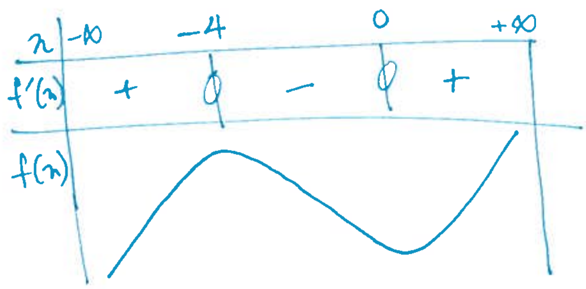
$3x(x+4)$

QUESTION 6. Let $f(x) = x^3 + 6x^2 + 4$. $\rightarrow$ $f'(x) = 3x^2 + 12x$

$x = 0$ & $x = -4$

(i) For what values of x does $f(x)$ increase?

$f(x)$ increases for $x \in (-\infty; -4) \cup (0; +\infty)$



(ii) For what values of x does $f(x)$ decrease?

$f(x)$ decreases for $x \in (-4; 0)$

(iii) For what values of x does $f(x)$ have local min values?

$f(x)$ has local min values at $x = 0$

(iv) For what values of x does $f(x)$ have local max values?

$f(x)$ has local max values at $x = -4$

(v) Roughly, sketch the graph of $f(x)$. (on the right hand side)

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