Problem One 20 Points

Differentiate each of the following functions (**do not simplify**):

(a) $f(x) = 4x + e^{3x^2+5x-1} + 3^{2x}$

(b) $y = \left(\ln(x^4 + 2x + 1)\right)^3$

(c) $y = (e^{3x} + 1)^3 \ln(x+1)$

(d) $f(t) = \sqrt[3]{\ln(1-t^2)}$

Problem Two 7 Points

Use implicit differentiation to find an equation of the tangent line defined by

$xe^{y} - y = x^2 - 2$ at the point (0, 2).

$x^2 + 2xy - y^2 + x = 2$ at the point (1,2)

Problem Three 7 Points

The price p (in dollars) and demand x for a product are related by:

$x^2 + 2xp + 25p^2 = 82{,}500$ $\qquad x = x(t)$ and $p = p(t)$

(a). Find $\dfrac{dp}{dt}$

(b) If the demand is decreasing at a rate of 10 units per month when the demand is 100 units, what is the rate of change of the price?

Problem Four **16 Points**

Given $f(x) = \dfrac{2+x}{1-x}$, $f'(x) = \dfrac{3}{(1-x)^2}$ and $f''(x) = \dfrac{6}{(1-x)^3}$, find the following:

(a) Domain of $f(x)$.

(b) x and $y-$intercepts.

(c) Vertical and horizontal asymptotes if any.

(d) Increasing and decreasing intervals.

(e) Local extrema (local max and local min).

(f) Concave up and concave down intervals.

(g) Inflection points.

(h) Sketch the graph of the function.

Problem Five **15 Points**

Integrate the following:

1. $$\int \left(x^2 - \frac{1}{x} + 3e^x + 1 \right) dx$$

2. $$\int x^2 (x^3 + 1)^3 \; dx$$

3. $$\int (4x - 2) e^{x^2 - x} \; dx$$

Problem Six **10 Points**

(a) Find the absolute maximum and minimum of $f(x) = \frac{x^3}{3} - \frac{x^2}{2} - 2x$, on [0,3].

(b) If the marginal profit for producing x radios per day is given by:

$$P'(x) = 3x^2 + x + 2 \qquad\qquad P(0) = 0$$

(i) Find the total profit function.

(j) Find the profit on 100 radios of production per day.