Suggested Review Questions
Final Exam MTH 102

1. Find the following limits

a. $\lim_{x\to 4} \dfrac{x^2-3}{x}$

b. $\lim_{x\to 2} \dfrac{2+x}{x^2-4}$

c. $\lim_{x\to 2} \dfrac{|2-x|}{2-x}$

d. $\lim_{x\to 0} \dfrac{\sqrt{4-x}-2}{x}$

2. Find the indicated derivative (do not simplify):

a. $f(x) = (4x+3e^x)^4$

b. $f(x) = \dfrac{e^x}{2x^2}$

c. $f(x) = (x^2+1)e^{-x}$

d. $h(t) = e^{(3t^4+6)}$

e. $f(x) = (x^3+2x^2)(3x-1)$

f. $G(x) = \ln(x^3-1)^{-3}$

g. $f(x) = \dfrac{x^2-3x+1}{\sqrt[4]{2x+3}}$

h. $f(x) = [\ln(x^3-1)]^{-3}$

i. $f(x) = (e^x+3)^4 \ln x$

j. $y = \log_3(4x^3+5x+7)$

k. $M(x) = 8^{(4x+\ln x)}$

3. Suppose that the total cost in dollars per week for manufacturing x refrigerators is given by the function $C(x) = 8000 + 200x - 0.2x^2$

a) Find the marginal cost function?

b) Evaluate the marginal cost at $x = 250$, 500, and 600 and interpret the results?

c) Use the marginal cost function to approximate the cost of producing 301st refrigerator?

d) Find the exact cost of producing the 301st refrigerator?

e) Find the average and marginal average cost functions

f) Evaluate the average and marginal average cost functions at $x = 100$ and interpret the results?

4. Find the absolute maximum and minimum, if either exists, for each function.

a. $f(x) = x^2 - 2x + 3$

b. $f(x) = x + \frac{4}{x}$ $\quad , (0, \infty)$

c. $f(x) = -x^2 - 6x + 9$

d. $f(x) = (x-1)(x-5)^5 + 1$ on $[3,6]$

5. Let $p = 400 - 0.4x$ and $C(x) = 160x + 2{,}000$ be the price –demand equation and the cost function respectively, for the manufacturing and selling of x digital cameras per week.

a. What price should the company charge for the cameras and how many cameras should be produced to maximize the weekly revenue? What is the maximum revenue?

b. What is the maximum weekly profit? How much should the company charge for the cameras and how many cameras should be produced to realize the maximum weekly profit?

c. If the government decides to tax the company $4 for each camera it produces, how many cameras should the company manufacture each week to maximize its profit? What is the maximum profit? What should the company charge for each camera?

6. Find the equation(s) of the tangent line(s) to the graphs of the indicated equations at the given point(s)

a. $y^2 - xy - 6 = 0$; x=1

b. $(2x - y)^4 - y^3 = 8$; (-1,-2)

c. $x^2 - y = 4e^y$; (2,0)

d) $\ln(xy) = y^2 - 1$; (1,1)

7. For a company manufacturing calculators, the cost, revenue and profit equations are given by:

$C = 90{,}000 + 30x$ $\qquad R = 300x - \dfrac{x^3}{30}$ $\qquad$ P=R-C

If the production x is increasing at a rate of 500 calculators per week when production output is 6,000 calculators, find the rate of increase (decrease) in cost, revenue and profit?

8. Find each indefinite integral:

(a) $\int (3x^2 - \frac{2}{x^2})dx$

(b) $\int \frac{e^x - 3x^2}{2} dx$

(c) $\int (\sqrt[3]{x^2} - \frac{4}{x^3})dx$

(d) $\int_0^1 x^2 e^{x^3} dx$

(e) $\int_e^{e^2} \frac{dx}{x(\ln(x))^3}$

(f) $\int x^3 \sqrt[4]{x^4 + 8}\ dx$

9. Find the particular antiderivative of each derivative that satisfies the given condition:

(a) $C'(x) = x^2 - 2x \quad ; \; C(0) = 3000$

(b) $\dfrac{dx}{dt} = 4e^t - 2 \qquad ; \; x(0) = 1$

10. Given the equation of the marginal cost function:

$$C'(x) = (2x - 4)^2$$

Find the cost function if the monthly fixed costs at 0 output are 30,000.
What is the cost of producing 4000 units.

11. For the function: $f(x, y) = e^{x^2+2y} + xy$

Find f_x , f_{xx}, f_{xy}, f_y, f_{yx} f_{yy}

12. A food company produces two types of coffee: mild and robust. It has determined that the daily demand x and y (in kilos) for its mild and robust coffee is given by

p = 42-6x+2y

q = 57+x-6y

where p and q are the price in kilo of mild and robust coffee, respectively.

(a) How many kilos of each type of coffee should be produced each day to maximize the revenue?

(b) How much should the company charge for each kilos of each type of coffee to maximize daily revenue?

13. For the functions

a. $f(x) = \dfrac{x}{x^2 - 1}$,

b. $f(x) = x(x-3)(x+3)$

Find, <u>if any</u>:

(a) the domain

(b) the x intercepts and the y intercept

(c) the vertical asymptote(s) and the horizontal asymptote(s)

(d) critical values

(e) the increasing and decreasing intervals

(f) the concave upward and downward intervals

(g) the inflection point(s)

(k) Graph $f(x)$