EXAM ONE, MTH102 SUMMER 007

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QUESTION 1. **(20 points) Find the limits of the following**

(1) $lim_{x\to -2}\frac{x^3+8}{(x+1)^2-1}$

$= \lim_{x\to -2} \frac{3x^2}{2(x+1)} = \lim_{x\to -2} \frac{3\times 4}{2\times -1} = \lim_{x\to -2} \frac{12}{-2} = \underline{\underline{-6}}$

(2) $lim_{x\to 1}\frac{\sqrt{x+8}-3}{x^3-1}$

$\lim_{x\to 1} \frac{\frac{1}{2\sqrt{x+8}}}{3x^2} = \lim_{x\to 1} \frac{\frac{1}{2\times 3}}{3} = \underline{\underline{\frac{1}{18}}}$

(3) $lim_{x\to 4}\frac{x-4}{x^2-5x+4}$

$= \lim_{x\to 4} \frac{\cancel{x-4}}{\cancel{(x-4)}(x-1)} = \underline{\underline{\frac{1}{3}}}$

$P = 4 \lesssim^{-4}_{-1}$
$S = -5$

(4) $lim_{x\to 3^+}\frac{x^2-4x+3}{|6-2x|}$

$= \lim_{x\to 3^+} \frac{x^2-4x+3}{-(6-2x)}$

$= \lim_{x\to 3^+} \frac{\cancel{(x-3)}(x-1)}{-2\cancel{(3-x)}}$

$= \lim_{x\to 3^+} \frac{x-1}{2}$

$= 1$

$P = 3 \lesssim^{-3}_{-1}$
$S = -4$

QUESTION 2. (**20 points**) **Find the first derivative for each of the following. DO NOT SIMPLIFY.**

1. $f(x) = (2x^3 + x - 13)^{12} + 3x - 10$

$f'(x) = 12(2x^3 + x - 13)^{11} \cdot [6x^2 + 1] + 3.$

2. $f(x) = \frac{x^3 - 7x + 3}{2x^5 + 7x + 13}$

$$f'(x) = \frac{3x^2 - 7(2x^5 + 7x + 13) - (10x^4 + 7)(x^3 - 7x + 3)}{(2x^5 + 7x + 13)^2}.$$

3. $f(x) = (3x + 10)(7x - 9)^6$

~~$f'(x) = 3(7x-9)^6 + (3x+10)($~~

$f'(x) = 3(7x-9)^6 + 6(7x-9)^5 7 \cdot (3x+10)$

$= 3(7x-9)^6 + 42(7x-9)^5(3x+10)$

4. $f(x) = 3\sqrt{x^2 + 7} + \frac{3}{x^2} - 12x + 13$

$f(x) = 3(x^2+7)^{1/2} + 3x^{-2} - 12x + 13.$

$f'(x) = 3 \times \frac{1}{2}(x^2+7)^{-1/2} \cdot 2x - 6x^{-3} - 12.$

$$= \frac{3}{2\sqrt{x^2+7}}(2x) - \frac{6}{x^3} - 12$$

QUESTION 3. (**10 points) Find the equation of that tangent line to** $f(x) = 36\sqrt{x} + 81/x + 3$ **at** $x = 9$

$x = 9$; $f(x) = 36 \times 3 + \frac{81}{9} + 3$.

$= 108 + 9 + 3 = 120.$

$\therefore$ Point at which the tangent line is to be determined : $(9, 120)$

Slope $(m) = f'(x) = \overset{18}{\cancel{36}} \cdot \frac{1}{\cancel{2}\sqrt{x}} - \frac{81}{x^2}$

$= \frac{18}{\sqrt{x}} - \frac{81}{x^2}.$

$f'(9) = \frac{18}{3} - \frac{81}{81} = 6 - 1 = 5.$

$y = mx + b \Rightarrow 120 = 5 \times 9 + b$

$b = 120 - 45 = 75$

Equation of the tangent line

$y = \underline{5x + 75}$

QUESTION 4. (**10 points) Let** x **be the number of items from Product** A **and** $P(x)$ **be the profit function in dollars. Given** $P(x) = 4(x + \sqrt{x})^3 - 10x + 80$. **Use the marginal profit concept to approximate the profit on item number 10.**

$P(x) = 4(x + \sqrt{x})^3 - 10x + 80.$

$P'(x) = 12(x + \sqrt{x})^2 \cdot \left(1 + \frac{1}{2\sqrt{x}}\right) - 10$

$P'(9) = \left[12(9+3)^2 \cdot \left(1 + \frac{1}{6}\right)\right] - 10.$

$= \underline{\$2006 \text{ (approx.)}}$

$1 + \frac{1}{2}$

$\frac{6+1}{6}$

$\frac{7}{6}$

QUESTION 5. (**20 points**) **Let** $f(x) = 2x^4 - 8x^3 + 10$ **be defined on [-2, 6].**

a) **For what values of** x **does** $f(x)$ **increase (decrease)?**

$f'(x) = 8x^3 - 24x^2$.

$f'(x) = 0$

$8x^2(x-3) = 0$.

$x^2 = 0$; $x - 3 = 0 \Rightarrow x = 0$; $x = 3$.

(-) (-) +

-2 0 3 6.

$f'(-1) = -8-24 < 0$; $f'(1) = 8-24 < 0$ $f'(5) = 400 > 0$

$\therefore f(x)$ increases from $(3,6)$ and decreases from $(-2,3)$

b) **Find all local max. and all local min.**

Local max : ~~LAR~~ Local min.

~~$x = -2$~~ . $16 \times 2 - 8(-8) + 10 = 106$. ~~$x = 3$~~ ; $f(x) = -44$

$x = 6$ $= 874$.

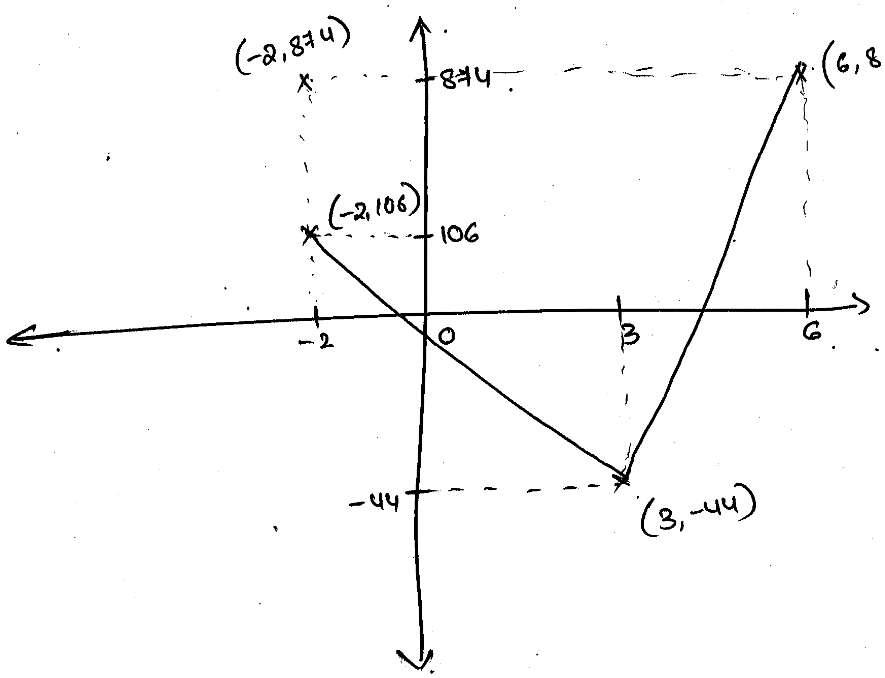
$\therefore f(x)$ has a local max of 106 at $x = -2$.
and 874 at $x = 6$.
$f(x)$ has a local min. of -44 at $x = 3$.

c) **Find the absolute max. value of** $f(x)$ **and the absolute min. value of f(x).**

Absolute max. Value of $f(x) = 874$ at $x = 6$

Absolute min. Value of $f(x) = -44$ at $x = 3$

d) **Sketch a rough graph of f(x).**



QUESTION 6. (**20 points) Let** $C(x) = 8\sqrt{x} + \frac{32}{x} - 6$ **be the total cost of** x **items.**

a) **Find the total cost of 4 items.**

$C(4) = 8 \times 2 + \frac{32}{4} - 6$

$= 16 + 8 - 6 = 18$

b) **Find the marginal cost of 4 items.**

$C(x) = 8\sqrt{x} + \frac{32}{x} - 6.$

$C'(x) = 8\frac{1}{2\sqrt{x}} - \frac{32}{x^2} = \frac{4}{\sqrt{x}} - \frac{32}{x^2}.$

$C'(4) = \frac{4}{2} - \frac{32}{16} = 2 - 2 = 0.$

c) **Use (a and b) to approximate the total cost of 5 items.**

$C(5) = C(4) + C'(4)$

$= 18 + 0 = 18.$

d) **What is the exact cost of 5 items?**

$C(5) = 8\sqrt{5} + \frac{32}{5} - 6.$

$= 17.89 + 6.4 - 6.$

$= \underline{18.29}$

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