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# Quiz One, MTH 221, Spring 2022

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**QUESTION 1.** Let $T : R^3 \to R^4$ such that $T(a_1, a_2, a_3) = (a_1 + 2a_2 - a_3, -2a_1 - 4a_2 + 2a_3, 0, -a_1 - 2a_2 + a_3)$

(i) By staring, convince me that $T$ is a linear transformation. 1/2

T is a linear transformation because it is a linear combination of $a_1, a_2, a_3$. ✓

(ii) Write the range of $T$ as span

$T(a_1, a_2, a_3) = \{a_1(1,-2,0,-1) + a_2(2,-4,0,-2) + a_3(-1,2,0,1)\}$ 1/2 ✓

Range of T = span $\{(1,-2,0,-1), (2,-4,0,-2), (-1,2,0,1)\}$

Range is a subset of co-domain ($R^4$).

(iii) What is the $dim(Range(T))$?

(back side)

dim(Range(T)) = 1 ✓ 2

(iv) Find a basis of Range(T) and then Write Range(T) as span of a basis of Range(T). 1

1/1 basis of Range(T) = $\{(1,-2,0,-1)\}$

Range(T) = span $\{(1,-2,0,-1)\}$ ✓

(v) Is $(1,1,3) \in Z(T)$? (note $Z(T) = Ker(T) = Null(T)$) explain

(back side)

1/1

**QUESTION 2.** Given $T : R^4 \to R^2$ is a linear transformation such that $T(1,1,0,0) = (2,4)$ and $T(0,0,4,0) = (-8,-12)$. By staring or by SIMPLE calculations, find

✓ (a) $T(2,2,4,0) = 2(T(1,1,0,0)) + T(0,0,4,0) = (4,8) + (-8,-12) = \underline{(-4,-4}$

✓ (b) $T(0,0,1,0) = (-2,-3)$

**QUESTION 3.** (a) Convince me that $T : R^3 \to R^4$ such that $T(a_1, a_2, a_3) = (2a_1, 4 + a_3, -2a_1)$ is not a linear transformation.

$T(a_1,a_2,a_3) = \{a_1(2,0,-2) + a_2(0,0,0) + a_3(0,1,0) + 1.(0,4,0)\}$

.5 ✓ $T(0,0,0) = (0,4,0) \notin (0,0,0)$ ∴ not a linear transformation.

(b) Convince me that $D = \{(a_1 - 2a_2, 3a_1 + a_3, 7 - a_2) \mid a_1, a_2, a_3 \in R\}$ is not a subspace of $R^3$.

.5 ✓ $D = \{a_1(1,3,0) + a_2(-2,0,-1) + a_3(0,1,0) + 1.(0,0,7)\}$

$a_1 = a_2 = a_3 = 0$ it does not $= (0,0,0)$ ∴ not a subspace of $R^3$.

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(iii) $\begin{bmatrix} 1 & -2 & 0 & -1 \\ 2 & -4 & 0 & -2 \\ -1 & 2 & 0 & 1 \end{bmatrix}$ $\begin{array}{l} -2R_1 + R_2 \to R_2 \\ R_1 + R_3 \to R_3 \end{array}$ $\begin{bmatrix} 1 & -2 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $\to$ independent

dependent

(v) $Z(T) = T(a_1, a_2, a_3) = (a_1 + 2a_2 - a_3,\ -2a_1 - 4a_2 + 2a_3,\ 0,\ -a_1, -2a_2, + a_3)$

$= (0, 0, 0, 0)$

$a_1 + 2a_2 - a_3 = 0$

$-a_1 - 2a_2 + a_3 = 0$

$a_1 = -2a_2 + a_3$

$T(a_1, a_2, a_3) = \{-2a_2 + a_3, 0, a_2, a_3 \mid a_2, a_3 \in R\}$

$Z(T) = \text{span}\{(-2, 1, 0), (1, 0, 1)\}$

$Z(T)$ is a subspace of $R^3$.

$(1, 1, 3) = C_1(-2, 1, 0) + C_2(1, 0, 1)$

$\underline{1 = C_1}$ $\quad 3 = C_2$

$1 = -2 + 3$ $\quad \therefore$ Yes, $\underline{\underline{(1, 1, 3) \in Z(T)}}$

$\underline{\underline{1 = 1}}$

ok but long

easier: check $T(1,2,3)$

$T(1,2,3) = (0,0,0,0)$.

Hence Yes.

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# Quiz Two, MTH 221 , Spring 2022

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**QUESTION 1.** Let $T : R^4 \to R^4$ such that

$$T(a_1, a_2, a_3, a_4) = (a_1 - 2a_2 + a_3 - a_4, -2a_1 + 4a_2 - a_3 + 4a_4, -a_1 + 2a_2 + 3a_4, 0)$$

It is clear that $T$ is a linear transformation.

(i) Find the standard matrix presentation of $T$, say $M$.

$$M = \begin{array}{c} \begin{array}{cccc} a_1 & a_2 & a_3 & a_4 \end{array} \\ \begin{bmatrix} 1 & -2 & 1 & -1 \\ -2 & 4 & -1 & 4 \\ -1 & 2 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \qquad T(a_1, a_2, a_3, a_4) = M \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix}$$

(ii) Find Rank(M) and write Col(M) as a span of a basis.

$\boxed{\text{Rank}(M) = 2}$

↳ # of independent row

$$\begin{array}{l} 2R_1 + R_2 \to R_2 \\ R_1 + R_3 \to R_3 \end{array} \begin{bmatrix} 1 & -2 & 1 & -1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{-R_2 + R_3 \to R_3} \begin{bmatrix} 1 & -2 & 1 & -1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

dim (Range) = rank (M) = 2

$$\text{col}(M) = \{ (1, -2, -1, 0), (1, -1, 0, 0) \}$$

is in Column 1, 3, 4

(iii) What is the $dim(Range(T))$? Write the range of $T$ as a span of a basis.

Range (T) = span{ independent columns }

dim ( Range (T) ) = rank (M) = 2

Range = span $\{ (1, -2, -1, 0), (1, -1, 0, 0) \}$

(v) Find $T(5,1,-1,1)$

$$\begin{bmatrix} 1 & -2 & 1 & -1 \\ -2 & 4 & -1 & 4 \\ -1 & 2 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 5 \\ 1 \\ -1 \\ 1 \end{bmatrix} = 5\begin{bmatrix} 1 \\ -2 \\ -1 \\ 0 \end{bmatrix} + 1\begin{bmatrix} -2 \\ 4 \\ 2 \\ 0 \end{bmatrix} - 1\begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix} + 1\begin{bmatrix} -1 \\ 4 \\ 3 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 5 \\ -10 \\ -5 \\ 0 \end{bmatrix} + \begin{bmatrix} -2 \\ 4 \\ 2 \\ 0 \end{bmatrix} + \begin{bmatrix} -1 \\ +1 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -1 \\ 4 \\ 3 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}$$

$$\boxed{T(5,1,-1,1) = (1,-1,0,0)}$$

vi) Find dim(Z(T)) and write $Z(T)$ as a span of a basis.[be careful, it could be trivial question ! :) ]

$$\left[\begin{array}{cccc|c} 1 & -2 & 1 & -1 & 0 \\ 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$$-R_2+R_1 \to R_1 \quad \left[\begin{array}{cccc|c} 1 & -2 & 0 & -3 & 0 \\ 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$a_1 - 2a_2 - 3a_4 = 0 \qquad a_3 + 2a_4 = 0$

$a_1 = 2a_2 + 3a_4 \qquad a_3 = -2a_4$

$a_1$ & $a_3$ → leading variables

$a_2$ & $a_4$ → free variables.

dim (domain) − dim (range) = dim(zt)

$\dim(Z(t)) = 2$

$Z(t) = \{ 2a_2 + 3a_4, a_2, -2a_4, a_4 \mid a_2, a_4 \in \mathbb{R} \}$

$Z(t) = \text{span}\{ (2,1,0,0), (3,0,-2,1) \}$

**QUESTION 2.** Given $T$ is a linear transformation $T : R^3 \to R^2$ such that $T(1,0,0) = (3,-2), T(0,1,0) = (6,-4), T(0,0,5) = (-15,10)$. Find the standard matrix presentation of $T$. Then write $Z(T)$ as a span of a basis.

$T(1,0,0) = (3,-2)$

$T(0,1,0) = (6,-4)$

$T(0,0,5) = (-15,10)$

$e_1 = (1,0,0) = T(1,0,0) = (3,-2)$

$e_2 = (0,1,0) = T(0,1,0) = (6,-4)$

$e_3 = (0,0,1) = \frac{1}{5}T(0,0,5) = (-3,2)$

$$M = \begin{bmatrix} 3 & 6 & -3 \\ -2 & -4 & 2 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 3 & 6 & -3 & 0 \\ -2 & -4 & 2 & 0 \end{array}\right] \xrightarrow{R_1/3} \left[\begin{array}{ccc|c} 1 & 2 & -1 & 0 \\ -2 & -4 & 2 & 0 \end{array}\right]$$

$$2R_1+R_2 \to R_2 \quad \left[\begin{array}{ccc|c} 1 & 2 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$x_1 + 2x_2 - x_3 = 0$

$x_1 = \underbrace{x_3 - 2x_2}_{\text{free}}$

leading

$Z(t) = \{ (x_3 - 2x_2, x_2, x_3) \mid x_3, x_2 \in \mathbb{R} \}$

$Z(t) = \text{Span}\{ (-2,1,0), (1,0,1) \}$

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# Quiz Three, MTH 221 , Spring 2022

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**QUESTION 1.** Find the solution set of the following system

$$x_2 + x_3 + 2x_4 + 2x_5 = -4$$
$$x_1 - 2x_2 - 2x_3 - 3x_4 - 4x_5 = 2$$
$$-x_1 - x_4 + x_5 = 4$$

$$\begin{array}{cccccc} x_1 & x_2 & x_3 & x_4 & x_5 & C \end{array}$$
$$\left[\begin{array}{ccccc|c} 0 & 1 & 1 & 2 & 2 & -4 \\ 1 & -2 & -2 & -3 & -4 & 2 \\ -1 & 0 & 0 & -1 & 1 & 4 \end{array}\right]$$

$2R_1 + R_2 \to R_2$

$$\left[\begin{array}{ccccc|c} 0 & 1 & 1 & 2 & 2 & -4 \\ 1 & 0 & 0 & 1 & 0 & -6 \\ -1 & 0 & 0 & -1 & 1 & 4 \end{array}\right]$$

$R_2 + R_3 \longrightarrow R_3$

$$\left[\begin{array}{ccccc|c} 0 & 1 & 1 & 2 & 2 & -4 \\ 1 & 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 0 & 1 & -2 \end{array}\right]$$

$-2R_3 + R_1 \longrightarrow R_1$

$$\begin{array}{cccccc} x_1 & x_2 & x_3 & x_4 & x_5 & C \end{array}$$
$$\left[\begin{array}{ccccc|c} 0 & 1 & 1 & 2 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 0 & 1 & -2 \end{array}\right]$$

5/5

READ!

$x_2 + x_3 + 2x_4 = 0$

$x_1 + x_4 = -6$

$x_5 = -2$

2/2

in terms of leading

$x_2 = -x_3 - 2x_4$

$x_1 = -x_4 - 6$

$x_5 = -2$

$x_3$ & $x_4$ are free variables.

3/3

solution set $= \{(-x_4 - 6, -x_3 - 2x_4, x_3, x_4, -2) \mid x_3, x_4 \in R\}$

**QUESTION 2.** In view of Question I, find the solution set of the following homogeneous system

$$x_2 + x_3 + 2x_4 + 2x_5 = 0$$
$$x_1 - 2x_2 - 2x_3 - 3x_4 - 4x_5 = 0$$
$$-x_1 - x_4 + x_5 = 0$$

Write the solution set as a span of some points in $R^5$.

$$\begin{array}{cccccc} x_1 & x_2 & x_3 & x_4 & x_5 & C \end{array}$$
$$\left[\begin{array}{ccccc|c} 0 & 1 & 1 & 2 & 2 & 0 \\ 1 & -2 & -2 & -3 & -4 & 0 \\ -1 & 0 & 0 & -1 & 1 & 0 \end{array}\right]$$

$\Downarrow$

$$\begin{array}{cccccc} x_1 & x_2 & x_3 & x_4 & x_5 & C \end{array}$$
$$\left[\begin{array}{ccccc|c} 0 & 1 & 1 & 2 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{array}\right]$$

$x_2 + x_3 + 2x_4 = 0$

$x_1 + x_4 = 0$

$x_5 = 0$

$x_2 = -x_3 - 2x_4$

$x_1 = -x_4$

$x_5 = 0$

5/5

Solution set $= \{(-x_4, -x_3 - 2x_4, x_3, x_4, 0) \mid x_3, x_4 \in R\}$

$= \{x_3(0, -1, 1, 0, 0), \ x_4(-1, -2, 0, 1, 0)\}$

$= \text{span}\{(0,-1,1,0,0), (-1,-2,0,1,0)\}$

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# Quiz Four, MTH 221 , Spring 2022

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$\frac{15}{15}$

**QUESTION 1.** Let $A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & -1 & -3 \\ -2 & -4 & -5 \end{bmatrix}$

(i) Find $A^{-1}$.

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ -1 & -1 & -3 & 0 & 1 & 0 \\ -2 & -4 & -5 & 0 & 0 & 1 \end{array}\right] \to \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 2 & 0 & 1 \end{array}\right]$$

$R_1 + R_2 \to R_2$

$2R_1 + R_3 \to R_3$

$-2R_2 + R_1 \to R_1$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 3 & -1 & -2 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 2 & 0 & 1 \end{array}\right] \to \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -7 & -2 & -3 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 2 & 0 & 1 \end{array}\right]$$

$-3R_3 + R_1 \to R_1$

$$A^{-1} = \begin{bmatrix} -7 & -2 & -3 \\ 1 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$$

✓ 5/5

(ii) Find the solution set of the following system of L. E.

$$\underbrace{A \times A^{-1}}_{I_3} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ -4 \end{bmatrix} \times A$$

$$I_3 \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ -1 & -1 & -3 \\ -2 & -4 & -5 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ -4 \end{bmatrix} = 2\begin{bmatrix} 1 \\ -1 \\ -2 \end{bmatrix} + 1\begin{bmatrix} 2 \\ -1 \\ -4 \end{bmatrix} + -4\begin{bmatrix} 3 \\ -3 \\ -5 \end{bmatrix}$$

$$= \begin{bmatrix} -8 \\ 9 \\ 12 \end{bmatrix}$$

3/3

solution set = $\{(-8, 9, 12)\}$

**QUESTION 2.** Find the matrix $A$, $4 \times 2$, such that

$$\underbrace{B^{-1} \times \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}}_{I_n} A^T = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & 2 & 1 & 1 \end{bmatrix} \times B^{-1}$$

assume $B = \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}$ $\quad B^{-1} = \frac{1}{-1}\begin{bmatrix} 5 & -2 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} -5 & 2 \\ 3 & -1 \end{bmatrix}$

$|B| = (1 \times 5) - (2 \times 3) = -1$

$I_n A^T = \begin{bmatrix} -5 & 2 \\ 3 & -1 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & 2 & 1 & 1 \end{bmatrix} =$

first column $= 1\begin{bmatrix} -5 \\ 3 \end{bmatrix} + -1\begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} -7 \\ 4 \end{bmatrix}$ seond $= 1\begin{bmatrix} -5 \\ 3 \end{bmatrix} + 2\begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

third $= 1\begin{bmatrix} -5 \\ 3 \end{bmatrix} + 1\begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} -3 \\ 2 \end{bmatrix}$ fourth $= \begin{bmatrix} -3 \\ 2 \end{bmatrix}$

$A^T = \begin{bmatrix} -7 & -1 & -3 & -3 \\ 4 & 1 & 2 & 2 \end{bmatrix}$ $\quad A = \begin{bmatrix} -7 & 4 \\ -1 & 1 \\ -3 & 2 \\ -3 & 2 \end{bmatrix}$

$\frac{4}{4}$

*take the transpose of both sides

**QUESTION 3.** Let $A$ be a $3 \times 3$ matrix such that $C_A(\alpha) = (\alpha - 2)^2(\alpha + 1)$ and let $B = A^2 + 2A^{-1} + 2I_3$.

(i) Find $|B|$.

eigen values : $\alpha = 2$ (twice) $\quad \alpha = -1$

for $\alpha = 2$ (twice)

$B = (2)^2 + 2\left(\frac{1}{2}\right) + 2$

$= 7$ (twice)

for $\alpha = -1$

$B = (-1)^2 + 2\left(\frac{1}{-1}\right) + 2$

$= 1$

$|B| = (7)(7)(1) = 49$

$\frac{1.5}{1.5}$

$\frac{3}{3}$

(ii) Find Trace$(B)$

Trace$(B) = 7 + 7 + 1 = 15$

$\frac{1.5}{1.5}$

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# Quiz Five, MTH 221 , Spring 2022

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**QUESTION 1.** Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

(i) Find all eigenvalues of $A$.

$C_A(\alpha) = |\alpha I_3 - A| = \left| \begin{bmatrix} \alpha-1 & -1 & -1 \\ 0 & \alpha-2 & 0 \\ 0 & 0 & \alpha-2 \end{bmatrix} \right| = (\alpha-1)(\alpha-2)^2$

2

the eigen values:

$\alpha = 1$ repeated once

$\alpha = 2$ repeated twice

2/2

(ii) For each eigenvalue $\alpha$ of $R$, find $E_\alpha$.

$$E_1 = \begin{bmatrix} 0 & -1 & -1 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \xrightarrow{-R_1 \to R_1} \begin{bmatrix} 0 & 1 & 1 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \xrightarrow{R_1 + R_2 \to R_2} \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & -1 \end{bmatrix} \xrightarrow{R_2 + R_3 \to R_3} \begin{array}{c} \begin{array}{ccc} x_1 & x_2 & x_3 \end{array} \\ \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \end{array}$$

READ!

$x_2 = -x_3 = 0$

$x_2 + x_3 = 0$

$x_3 = 0$

$0 = 0$

$E_1 = (x_1, 0, 0 \mid x_1 \in R)$

$E_1 = (x_1 (1,0,0) \mid x_1 \in R)$

$E_1 = \text{span}\{(1,0,0)\}$

4/4

$$E_2 = \begin{bmatrix} 1 & -1 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{\text{read}}$$

$x_1 - x_2 - x_3 = 0$

$x_1 = x_2 + x_3$

$E_2 = \{(x_2 + x_3, x_2, x_3) \mid x_2, x_3 \in R\}$

$E_2 = \{x_2(1,1,0) + x_3(1,0,1) \mid x_2, x_3 \in R\}$

$E_2 = \text{span}\{(1,1,0), (1,0,1)\}$

5/5

(iii) If $A$ is diagnolizable, then find a diagonal matrix $D$ and an invertible matrix $Q$ such that $Q^{-1}AQ = D$.

yes, A is diagnolizable.

$D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ 2/2

$Q = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ 2/2

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# Quiz six, MTH 221 , Spring 2022

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15/15

**QUESTION 1.** let $T : P_2 \to P_2$ such that $T(ax+b) = (2a-b)x + a + b$

(i) Convince me that $T^{-1}$ exists and find $T^{-1}$.

$L : \mathbb{R}^2 \to \mathbb{R}^2$ $\quad L(a,b) =$

$(2a-b, a+b)$

$M = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix} \to |M| = (2\times1)-(-1\times1) = 3 \neq 0$ so $M^{-1}$ exists $\to T^{-1}$ exists

$M^{-1} = \frac{1}{3}\begin{bmatrix} 1 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1/3 & 1/3 \\ -1/3 & 2/3 \end{bmatrix}$ convert

$\boxed{T^{-1} = (\frac{1}{3}a + \frac{1}{3}b)x + (-\frac{1}{3}a + \frac{2}{3}b)}$ $\to P_2 \to P_2$

$T^{-1}(ax+b) =$

90/90

(ii) Find $T^{-1}(3x+6)$

$\left.\begin{array}{l} a=3 \\ b=6 \end{array}\right\} (\frac{1}{3}(3) + \frac{1}{3}(6))x + (-\frac{1}{3}(3) + \frac{2}{3}(6)) = \boxed{3x+3}$

**QUESTION 2.** Convince me that $D = \{ \begin{bmatrix} a+2b & 2a+4b \\ -a-2b & 3a+6b \end{bmatrix} \mid a, b \in R\}$ is a subspace of $R^{2\times2}$. Then find dim(D)

$\mathbb{R}^{2\times2} \to \mathbb{R}^4$

$D = \{(a+2b, 2a+4b, -a-2b, 3a+6b) \mid a, b \in \mathbb{R}\}$

7/7

$\begin{bmatrix} 1 & 2 \\ 2 & 4 \\ -1 & -2 \\ 3 & 6 \end{bmatrix} \xrightarrow{\substack{R_2-2R_1 \to R_2 \\ R_3+R_1 \to R_3 \\ R_4-3R_1 \to R_4}} \begin{bmatrix} 1 & 2 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \to a = -2b$

$D = \{(a+2b) \mid a = b \in \mathbb{R}\}$

$= \{(a+2a) \mid a \in \mathbb{R}\}$

$= \{3a \mid a \in \mathbb{R}\}$

$\boxed{D = \text{span}\{(3)\} \to D = \text{span}\{\begin{bmatrix} 3 & 0 \\ 0 & 0 \end{bmatrix}\}}$

$\boxed{\dim(D) = 1}$

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# Quiz six, MTH 221 , Spring 2022

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**QUESTION 1.** let $T : P_2 \to P_2$ such that $T(ax+b) = (2a-b)x + a + b$ $= 2ax - bx + a + b$

(i) Convince me that $T^{-1}$ exists and find $T^{-1}$.

2/2

$T(ax+b) = (2a-b)x + a + b$

$L(\cancel{ax+b}) = (2a-b) + (a+b)$ $\longrightarrow L : R^2 \to R^2$

$(a,b)$

$$M = \begin{array}{c c c} & a & b \\ x & & \\ \end{array} \begin{array}{c} \\ \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix} \begin{array}{l} x \\ \text{no } x \end{array} \end{array}$$

$$M^{-1} = \frac{1}{|M|}\begin{bmatrix} 1 & 1 \\ -1 & 2 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 \\ -1 & 2 \end{bmatrix} = \begin{array}{c} \begin{array}{cc} a & b \end{array} \\ \begin{bmatrix} 1/3 & 1/3 \\ -1/3 & 2/3 \end{bmatrix} \end{array}$$

$\longleftarrow$ because M is invertible, L is invertible.

$\therefore$ T is invertible.

$L^{-1} : R^2 \to R^2 \longrightarrow L^{-1}(a,b) =$

$L^{-1} = (1/3a + 1/3b) + (-1/3a + 2/3b)$

$T^{-1} = (1/3a + 1/3b)x + (-1/3a + 2/3b)$

Hence $T^{-1} : P_2 \to P_2$

2/2

$T^{-1}(ax+b) = \left(\frac{1}{3}a + \frac{1}{3}b\right)x + \left(\frac{-1}{3}a + \frac{2}{3}b\right)$

(ii) Find $T^{-1}(3x+6)$

$T^{-1} = ((1/3)(3) + (1/3)(6))x + ((-1/3)(3) + (2/3)(6)) = 3x + 3$

2/2

**QUESTION 2.** Convince me that $D = \{\begin{bmatrix} a+2b & 2a+4b \\ -a-2b & 3a+6b \end{bmatrix} \mid a, b \in R\}$ is a subspace of $R^{2\times 2}$. Then find dim(D)

(a)

$$D = \left\{ a\begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix} + b\begin{bmatrix} 2 & 4 \\ -2 & 6 \end{bmatrix} \right\}$$

$$D = \text{span}\left\{ \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}, \begin{bmatrix} 2 & 4 \\ -2 & 6 \end{bmatrix} \right\}$$

because D can be written as a span of finite points, it is a subspace.

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(b) $R^{2\times 2} \approx R^4$

$$\begin{bmatrix} 1 & 2 & -1 & 3 \\ 2 & 4 & -2 & 6 \end{bmatrix} \xrightarrow{-2R_1 + R_2 \to R_2} \begin{bmatrix} 1 & 2 & -1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\boxed{\therefore \dim(D) = 1}$

3/3