Name Lana Talo, ID 91350

Quiz One, MTH 205 , Spring 2022

Ayman Badawi

15/15

QUESTION 1. Use Laplace and solve for $y(t)$, where $y^{(2)} - 6y' + 9y = U_2(t)e^{3(t-2)}, y(0) = 1, y'(0) = 6$

$\ell[y^{(2)} - 6y' + 9y] = \ell[U(t-2)e^{3(t-2)}]$

$s^2Y(s) - sy(0) - y'(0) - 6[sY(s) - y(0)] + 9Y(s) = \frac{e^{-2s}}{s-3}$ 1/1

$e^{3(t)}$

$\ell[U(t-2)\underbrace{e^{3(t-2)}}_{g(t)}]$

$= e^{-2s} L\{e^{3t}\}$

$= e^{-2s}\frac{1}{s-3}$

$s^2Y(s) - s - \cancel{6} - 6sY(s) + \cancel{6} + 9Y(s) = \frac{e^{-2s}}{s-3}$ 2/2

$Y(s)(s^2 - 6s + 9) = \frac{e^{-2s}}{s-3} + \frac{s}{1}$

$Y(s) = \frac{e^{-2s}}{(s-3)^3} + \frac{s}{(s-3)^2}$

$Y(t) = \frac{1}{2!}\ell^{-}\left(\frac{e^{-2s}\,2!}{(s-3)^3}\right) + \ell^{-}\left(\frac{s}{(s-3)^2}\right)$ 2/2

$\frac{1}{(s-3)^3}$ $L(t^2) = \frac{2!}{s^3}$

$L(e^{3t}t^2) = \frac{2!}{(s-3)^3}$

$f(t-a) = e^{3(t-2)}(t-2)^2$

$U(t-2)$

$\boxed{y(t) = \frac{1}{2}e^{3(t-2)}(t-2)^2\, U(t-2) + e^{3t} + 3te^{3t}}$

$\ell^{-}\left(\frac{s}{(s-3)^2}\right) = \frac{a}{(s-3)} + \frac{b}{(s-3)^2}$

$b = 3$ $a = 1$ $= \frac{1}{s-3} + \frac{3}{(s-3)^2}$

$= e^{3t} + 3\ell^{-}\left(\frac{1}{(s-3)^2}\right)$

$e^{3t} + 3te^{3t}$

QUESTION 2. Use Laplace and solve for $y(t)$, where $y' - 3y = \delta_3(t) + 4, y(0) = 0$, ~~$y'(0) = 0$~~

$\ell[y' - 3y] = \ell[\delta_3(t) + 4]$ 1/1

$sY(s) - \cancel{y(0)}^{0} - 3Y(s) = e^{-3s} + \frac{4}{s}$

$Y(s)(s-3) = e^{-3s} + \frac{4}{s}$

2/2 $Y(s) = \frac{e^{-3s}}{s-3} + \frac{4}{s(s-3)}$

$y(t) = \ell^{-}\left[\frac{e^{-3s}}{s-3} + \frac{4}{s(s-3)}\right] =$

$F(s) = \frac{1}{s-3}$

$f(t) = e^{3t}$

2/2 $= e^{3(t-3)}U(t-3) + 4\ell^{-}\left[-\frac{1}{3s} + \frac{1}{3(s-3)}\right]$

$\ell^{-}\left(\frac{1}{s(s-3)}\right)$ $y(t)\ \boxed{= e^{3(t-3)}U(t-3) - \frac{4}{3} + \frac{4}{3}e^{3t}}$

$= \frac{a}{s} + \frac{b}{s-3}$

$a = -\frac{1}{3}$ $b = \frac{1}{3}$

QUESTION 3. Use Laplace and solve for $y(t)$, where $y^{(2)} - 4y' + 13y = 0, y(0) = 1, y'(0) = 2$

$\ell[y^{(2)} - 4y' + 13y] = 0$

$s^2Y(s) - sy(0) - y'(0) - 4[sY(s) - y(0)] + 13Y(s) = 0$ 1/1

$s^2Y(s) - s - 2 - 4sY(s) + 4 + 13Y(s) = 0$

2/2 $Y(s)(s^2 - 4s + 13) = s - 2$

$Y(s) = \frac{s-2}{s^2 - 4s + 13} = \frac{s-2}{s^2 - 4s + 2^2 - 2^2 + 13}$

$L'[F(s-a)]$

$= e^{at}f(t)$

$= \frac{s-2}{(s-2)^2 + 9}$

2/2 ... ..

$y(t) = \ell^{-}\left(\frac{s-2}{(s-2)^2+9}\right) = \boxed{e^{2t}\cos 3t}$

Name Mohamad Chehab, ID @00090578

Quiz TWO, MTH 205 , Spring 2022

Ayman Badawi

15/15

QUESTION 1. Solve for $y(t)$, where $y(t) = cos(t) + 2\int_{r=0}^{r=t} cos(t-r)y(r)\, dr$

$\mathcal{L}\{\cos(t)\} + 2\mathcal{L}\left\{\int_0^t \cos(t-r)\, y(r)\, dr\right\} = \mathcal{L}\{y(t)\}$

$Y(s) = \frac{s}{s^2+1} + 2\mathcal{L}\{\cos(t) * y(t)\} = \mathcal{L}\{\cos(t)\}\cdot\mathcal{L}\{y(t)\} = \frac{s}{s^2+1}\cdot Y(s)$

$Y(s) = \frac{s}{s^2+1} + 2\left[\frac{s}{s^2+1}\cdot Y(s)\right]$

2/2

$\frac{s^2+1}{s^2+1} - \frac{2s}{s^2+1} = \frac{s^2-2s+1}{s^2+1}$

$Y(s) - \frac{2s}{s^2+1}Y(s) = \frac{s}{s^2+1}$

$Y(s)\left[1 - \frac{2s}{s^2+1}\right] = \frac{s}{s^2+1}$

2/2

$Y(s) = \frac{s}{\cancel{(s^2+1)}} \times \frac{\cancel{(s^2+1)}}{s^2-2s+1}$

$Y(s) = \frac{s}{s^2-2s+1}$

$y(t) = \mathcal{L}^{-1}\left\{Y(s) = \frac{s}{s^2-2s+1}\right\}$

$\frac{s}{s^2-2s+1} = \frac{s-1+1}{(s-1)^2}$

$y(t) = \mathcal{L}^{-1}\left\{\frac{s-1}{(s-1)^2} + \frac{1}{(s-1)^2}\right\} = \mathcal{L}^{-1}\left\{\frac{\frac{1}{s-1}}{\cancel{(s-1)^2}}\right\} + \mathcal{L}^{-1}\left\{\frac{1}{(s-1)^2}\right\}$ $\quad \frac{s-1}{(s-1)^2} + \frac{1}{(s-1)^2}$

$\boxed{y(t) = e^t + e^t \cdot t}$

$\mathcal{L}^{-1}\left\{\frac{1}{s-1}\right\} = e^t$

$\mathcal{L}^{-1}\left\{\frac{1}{(s-1)^2}\right\} = e^t t$

3/3

QUESTION 2. Solve for $x(t)$ and $y(t)$

① $x'(t) + y(t) = 2, x(0) = 1$

② $x(t) + y'(t) = 2t, y(0) = 1$

①* $\mathcal{L}\{x'(t)\} + \mathcal{L}\{y(t)\} = 2\mathcal{L}\{1\} \longrightarrow sX(s) - \cancel{x(0)}^{1} + Y(s) = \frac{2}{s}$

②* $\mathcal{L}\{x(t)\} + \mathcal{L}\{y'(t)\} = 2\mathcal{L}\{t\} \longrightarrow X(s) + sY(s) - \cancel{y(0)} = \frac{2}{s^2}$

$sX(s) + Y(s) = \frac{2}{s} + 1$

$X(s) + sY(s) = \frac{2}{s^2} + 1$

3/3

$X(s) = \dfrac{\begin{vmatrix} \frac{2}{s}+1 & 1 \\ \frac{2}{s^2}+1 & s \end{vmatrix}}{\begin{vmatrix} s & 1 \\ 1 & s \end{vmatrix}}$

~~$X(s) = \frac{2+s-(\frac{2}{s^2}+1)}{s^2-1} = \frac{2+s-\frac{2}{s^2}-1}{s^2-1}$~~

~~$X(s) = \frac{1+s-\frac{2}{s^2}}{s^2-1} = \frac{\frac{s^2+s^3-2}{s^2}}{s^2-1} = \frac{s^3+s^2-2}{s^2(s^2-1)}$~~

$\frac{1}{1+s} \quad \frac{1}{s+1}$

$Y(s) = \dfrac{\begin{vmatrix} s & \frac{2}{s}+1 \\ 1 & \frac{2}{s^2}+1 \end{vmatrix}}{\begin{vmatrix} 1 & s \\ s & 1 \end{vmatrix}} = \frac{\frac{2}{s}+1-\frac{2}{s}-s}{1-s^2} = \frac{1-s}{1-s^2}$

$Y(s) = \frac{s-1}{s^2-1} = \frac{\cancel{s-1}}{\cancel{(s-1)}(s+1)} = \frac{1}{s+1}$

$y(t) = \mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\} = e^{-t}$

$y'(t) = -e^{-t}$

$x(t) - e^{-t} = 2t$

$x(t) = 2t + e^{-t}$

3/3

$\boxed{\begin{aligned} y(t) &= e^{-t} \\ x(t) &= 2t + e^{-t} \end{aligned}}$

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ame Sara Tareh Adel, ID 91827

Excellent ☺

15/15

Quiz THREE, MTH 205 , Spring 2022

Ayman Badawi

QUESTION 1. Given $t^3y'' - 5t^2y' - 7ty = 0,\ t > 0.$ Solve for $y(t)$

Solve for $y = t^n$ $\quad y' = nt^{n-1}$ $\quad y'' = (n^2-n)t^{n-2}$

$t^3(n^2-n)t^{n-2} - 5t^2 n t^{n-1} - 7t\,t^n = 0$

$(n^2-n)t^{n+1} - 5nt^{n+1} - 7t^{n+1} = 0$

$t^{n+1}[n^2-n-5n-7] = 0 \Rightarrow t^{n+1}(n^2-6n-7) = 0$

$t \neq 0 \to t > 0 \quad , \quad n^2-6n-7 = 0$

$(n-7)(n+1) = 0 \to n = 7 \quad n = -1$

$y(t) = C_1 t^7 + C_2 t^{-1}$ ✓ 5

QUESTION 2. Given $ty'' - 9y' + \frac{25y}{t} = 0,\ t > 0.$ Solve for $y(t)$ Solve for $y = t^n$

$t(n^2-n)t^{n-2} - 9nt^{n-1} + \frac{25t^n}{t} = 0$

$(n^2-n)t^{n-1} - 9nt^{n-1} + 25t^{n-1} = 0$

$t^{n-1}[n^2-n-9n+25] = 0 \Rightarrow t \neq 0 \quad t > 0$

$n^2-n-9n+25 = 0 \Rightarrow n^2-10n+25 = 0$

$(n-5)(n-5) = 0 \to n = 5 \quad n = 5$

$y(t) = C_1 t^5 + C_2 \ln(t) t^5$ ✓ 5

QUESTION 3. Given $y'' + \frac{7y'}{t} + \frac{25y}{t^2} = 0,\ t > 0.$ Solve for $y(t)$ Solve for $y = t^n$

$(n^2-n)t^{n-2} + \frac{7nt^{n-1}}{t} + \frac{25t^n}{t^2} = 0$

$(n^2-n)t^{n-2} + 7nt^{n-2} + 25t^{n-2} = 0$

$t^{n-2}[n^2-n+7n+25] = 0 \Rightarrow t^{n-2}[n^2+6n+25] = 0$

$\cancel{n^2}\ t^{n-2} \neq 0 \to t > 0, \quad n^2+6n+25 = 0$

$n = -3+4i \quad n = -3-4i \quad a = -3 \quad b = \pm 4i$

$y(t) = t^{-3}[C_1\cos(4\ln(t)) + C_2\sin(4\ln(t))]$ 5

Name Nada Nassen, ID 900089520

$\frac{15}{15}$

Quiz Four, MTH 205 , Spring 2022

Ayman Badawi

QUESTION 1. Given $t^2y'' + 2ty' = 1$, $t > 0$. Find the general solution for $y(t)$

[1] Find y_h: Let $y = t^n$, $y' = nt^{n-1}$, $y'' = (n^2-n)t^{n-2}$

$t^2y'' + 2ty' = 0$

$t^2(n^2-n)t^{n-2} + 2t(nt^{n-1}) = 0$

$t^n(n^2-n) + 2nt^n = 0$

$t^n[n^2 - n + 2n] = 0$

$n^2 + n = 0$

$n(n+1) = 0$

$n = 0 \longrightarrow y_1 = t^0 = 1$

$n = -1 \longrightarrow y_2 = t^{-1}$

$\boxed{y_h = c_1 + c_2t^{-1}}$ 3/3

[2] Find y_p =

$y_p = v_1y_1 + v_2y_2$

(1) $v_1' + t^{-1}v_2' = 0$

(2) $0 + -1t^{-2}v_2' = t^{-2}$

$-t^{-2}v_2' = t^{-2}$

$t^2 \times \frac{-v_2'}{t^2} = \cancel{t^2} \times \cancel{t^2}$

$-v_2' = 1$

$\boxed{v_2' = -1} \rightarrow v_2 = \int -1\,dt = -t$ 1.5/1.5

$v_1' + (t^{-1})(-1) = 0$

$v_1' - t^{-1} = 0$

$v_1' = t^{-1}$

$v_1 = \int t^{-1}\,dt = \ln(t)$ 1.5/1.5

$\boxed{v_2 = -t}$

$\boxed{v_1 = \ln(t)}$

(1) $y_p = -t + \ln(t)t^{-1}$

(1) $\boxed{y_g = c_1 + c_2t^{-1} - t + \ln(t)t^{-1}}$

QUESTION 2. Given $y'' - y' = te^t$, $t > 0$. Solve for $y(t)$

te^{-t}

[1] Find y_h: $y = e^{mt}$

$y'' - y' = 0$

$m^2 - m = 0$

$m(m-1) = 0$

$m = 0 \rightarrow y_1 = e^{0t} = 1$

$m = 1 \rightarrow y_2 = e^t$

$\boxed{y_h = c_1 + c_2e^t}$ 2/2

[2] Find y_p:

$y_p = v_1y_1 + v_2y_2$

(1) $v_1' + e^tv_2' = 0$

(2) $0 + v_2'e^t = \frac{te^{-t}}{1}$

$\frac{v_2'e^t}{e^t} = \frac{te^{-t}}{e^t}$

$v_2' = te^{-2t}$

$v_2 = \int te^{-2t}\,dt$

$'$	$\int$
t	e^{-2t}
1	$\frac{e^{-2t}}{-2}$
0	$\frac{e^{-2t}}{4}$

$v_2 = -\frac{1}{2}te^{-2t} - \frac{1}{4}e^{-2t}$ 1.5/1.5

$v_1' + e^t(te^{-2t}) = 0$

$v_1' = -te^{-t}$

$v_1 = \int -te^{-t}$

$'$	$\int$
$-t$	e^{-t}
-1	$-e^{-t}$
0	e^{-t}

$\boxed{v_1 = te^{-t} + e^{-t}}$ 1.5/1.5

$$y_p = \left(te^{-t} + e^{-t}\right) + \left(-\frac{1}{2}te^{-2t} - \frac{1}{4}e^{-2t}\right)e^{t}$$

$$\boxed{y_g = c_1 + c_2 e^{t} + te^{-t} + e^{-t} + \left(-\frac{1}{2}te^{-2t} - \frac{1}{4}e^{-2t}\right)e^{t}}$$

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Q_5 MTH 205

Name Reem Alshareef , ID = 900087222

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$f(t)$

$p(t)$

$Q_1.\quad y' + \left(\frac{2t}{t^2+1}\right) y = \sin(t)$

$I = e^{\int p(t)\,dt}$

$I = e^{\int \frac{2t}{t^2+1}\,dt} = e^{\ln|t^2+1|} = t^2+1$ 4/4

8/8

$y = \frac{\int I \cdot f(t)\,dt}{I}$

$y = \frac{\int (t^2+1)\sin t}{t^2+1} \longrightarrow$

D	I
t^2+1 (+)	$\sin t$
$2t$ (−)	$-\cos t$
2 (+)	$-\sin t$
0	$\cos t$

4/4

$\int (t^2+1)\sin t =$

$$\boxed{y = \frac{-\cos t\,(t^2+1) + 2t\sin t + 2\cos t + C}{t^2+1}}$$

$= \frac{\sin t}{\cos t} \times \frac{1}{\sin t}$

$Q_2 - \sin(t)\, y' + \cos(t)\, y = \tan(t) \quad \div \sin(t)$

$p(t)$ $f(t)$

$y' + \left(\frac{\cos(t)}{\sin(t)}\right) y = \frac{1}{\cos t}$

$I = e^{\int p(t)\,dt} = e^{\int \frac{\cos t}{\sin t}\,dt} = e^{\ln|\sin(t)|} = \sin(t)$ 4/4

$y = \frac{\int I \cdot f(t)}{I}$

$y = \frac{\int \sin t \cdot \frac{1}{\cos t}}{\sin t}$ $\xrightarrow{}\tan t$ $\Rightarrow$ $\boxed{y = \frac{-\ln|\cos(t)| + c}{\sin t}}$ 3/4

7/7

Name Reem Tariq, ID 87222

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MTH 205 , Spring 2022, 1–2

Quiz Six MTH 205 , Spring 2022

Ayman Badawi

I meant e^t not te^t

$a_1(t)$ $a_0(t)$

QUESTION 1. Given $y' - \left(\frac{1}{2t}\right)y = e^t y^3$, $t > 0$. Find the general solution for $y(t)$

$k(t)$

$$y' - \frac{1}{2t}y = te^t y^3$$

$n = 3$, $1 - n = -2$

$W = y^{1-n} \Rightarrow \boxed{W = y^{-2}}$

$W' - \frac{1}{2t}(-2)W = (-2)(te^t)$

$W' + \frac{1}{t}W = -2te^t$

$p(t)$ $f(t)$

$I = e^{\int p(t)dt} = e^{\int \frac{1}{t}dt} = e^{\ln t} = t$

$W = \frac{\int I \cdot f(t)\, dt}{I} = \frac{\int t \cdot -2te^t\, dt}{t}$

$W = \frac{\int -2t^2e^t\, dt}{t} = \frac{-2\int t^2e^t dt}{t} = \frac{-2[t^2e^t - 2te^t + 2e^t] + c}{t}$

$W = \frac{-2t^2e^t + 4te^t - 4e^t + c}{t}$

	$\int$
t^2 (+)	e^t
$2t$ (−)	e^t
2 (+)	e^t
0	e^t

$t^2e^t - 2te^t + 2e^t$

$(W)^{-\frac{1}{2}} = (y^{-2})^{-\frac{1}{2}}$ $\therefore y = W^{-\frac{1}{2}}$

$$\boxed{y = \left(\frac{-2t^2e^t + 4te^t - 4e^t + c}{t}\right)^{-\frac{1}{2}}}$$

OK

7/7

QUESTION 2. Find the general solution to

$$\underbrace{[3 + 2y + e^x + 2x]}_{Fx}dx + \underbrace{[2x + 3y^2 + e^y]}_{Fy}dy = 0$$

$Fxdx + Fydy = 0$

$F_{xy} = F_{yx}$ equal so, use Exact

$2 = 2$

2/2

integrate Fy w.r.to y

$\int 2x + 3y^2 + e^y\, dy$

$2xy + \frac{3y^3}{3} + e^y + k(x) = F(y, x)$

$F(x) = 2y + k'(x)$

Compare $2y + k'(x) = 3 + 2y + e^x + 2x$

$k'(x) = 3 + e^x + 2x$

$k(x) = \int 3 + e^x + 2x\, dx$

$k(x) = 3x + e^x + x^2 + c$

$\therefore \boxed{2xy + y^3 + e^y + 3x + e^x + x^2 + c = 0}$

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Name Lana, ID 91350

Excellent ☺

Quiz Seven, MTH 205 , Spring 2022

15

Ayman Badawi

QUESTION 1. Given $\frac{dy}{dx} = \frac{x+1}{e^{(2y-x)}}$

$(x+1)\,dx = e^{2y}e^{-x}\,dy$

$\int e^{x}(x+1)\,dx = \int e^{2y}\,dy$

$\frac{e^{2y}}{2} = \int xe^{x} + e^{x}\,dx$

3/3

$u = x \quad dv = e^{x}$

$du = 1 \quad v = e^{x}$

$= xe^{x} - \int e^{x}$

$= xe^{x} - e^{x}$

$x \;\; + \;\; e^{x}$

$1 \;\; - \;\; e^{x}$

$0 \;\; e^{x}$

7/7

$\frac{e^{2y}}{2} = xe^{x} - \cancel{e^{x}} + \cancel{e^{x}} + C$

4/4

$\frac{e^{2y}}{2} = xe^{x} + C_1$

$C = 2C_1$

$e^{2y} = 2xe^{x} + C$

$2y = \ln(2xe^{x} + C)$

$\boxed{y = \frac{\ln(2xe^{x}+C)}{2}}$

QUESTION 2. Imagine: A tank has a capacity of 700 gallons. Initially it contains 250 gallons of brine (water and salt) that contains 10 g of salt (i.e., A(0) = 10). A solution containing 2g of salt per gallon is pumped into the tank at rate 3 gallon/min. and the solution is pumped out at rate 2 gallons per min. Find the amount of salt at any time t. Find the concentration of salt $c(t)$ at anytime t.

$A(t) =$ grams of salt

9/9

$c(t) = \frac{\text{grams}}{\text{gallon}} = \frac{A(t)}{250 + (3-2)t} = \frac{A(t)}{250 + t}$

2/2

$A'(t) = \text{In} - \text{out} = 3(2) - 2\,C(t)$

$= 6 - \frac{2\,A(t)}{250+t}$

Answer :

$A(t) = 500 + 2t - \frac{3.1\times10^{7}}{(250+t)^{2}}$

4/4

$A'(t) + \frac{2}{250+t}A(t) = 6$

$I = e^{\int \frac{2}{250+t}dt} = e^{2\int \frac{1}{250+t}dt} = e^{2\ln 250+t} = e^{\ln(250+t)^2} = (250+t)^2$

$= (250+t)^2$

4/4

$A(t) = \frac{6\int (250+t)^2 dt}{(250+t)^2} = \frac{6\left(\frac{(250+t)^3}{3} + C_1\right)}{(250+t)^2} = \frac{2(250+t)^3 + C}{(250+t)^2}$

$A(t) = 2(250+t) + \frac{C}{(250+t)^2}$

$10 = 2(250) + \frac{C}{250^2}$

$10 = 500 + \frac{C}{250^2}$

$C = -3.1\times10^{7}$