Math 205, Differential Equations
Second Exam

1. *(12 points)* Find the inverse Laplace transform of the given function.

   a. $\dfrac{5s - 6}{s^2 - 3s}$

   b. $\dfrac{2s - 3}{s^2 + 2s + 10}$

2. *(16 points) Solve the following Initial Value Problem using undetermined coefficients:*

$$y'' - 10y' + 25y = 50x + 6e^{5x}, \qquad y(0) = 0, \quad y'(0) = 1.$$

3. *(20 points) Determine a suitable form for y_p if the method of undetermined coefficients is used. Do* not *evaluate the constants.*

   a. $y''' - 8y = -2e^{2x} + x^3$.

   b. $y^{(4)} + 9y'' = 5\cos 3x + 6$.

   c. $y''' - 6y'' + 12y' - 8y = 2xe^{2x}$. *4. (15 points) Given that $y_1 = x^4$ is a solution of the homogeneous equation*

$$x^2 y'' - 7xy' + 16y = 0$$

Use reduction of order to find a second linearly independent solution.

5. (18 points) A mass weighing 256 lb stretches a spring $\frac{32}{9}$ ft. A downward force given by $f(t) = 144 \sin 3t$ is applied to the weight. Assume that initially the weight is released from equilibrium position and that damping is neglected. Find the position x at any time t. Determine when does the mass first return to its equilibrium position.   6. (20 points)

6A. Without the aid of the Wronskian, determine whether the given set of function is linearly dependent or linearly independent. Explain.

a. $f_1(x) = \ln x$, $f_2(x) = \ln x^5$ on the interval $(0, \infty)$.

b. $f_1(x) = e^{x+2}$, $f_2(x) = e^{x-3}$ on the interval $(-\infty, \infty)$.

c. $f_1(x) = x$, $f_2(x) = 3x - 5x^2$, $f_3(x) = x^2$ on the interval $(-\infty, \infty)$.

6B. Assume that y_1 and y_2 are solutions for

$$y'' + P(x)y' + Q(x)y = 0$$

and $W(y_1, y_2) = 5$, what does this imply about $P(x)$ and $Q(x)$?

6C. Fill in the blank with the letter corresponding to the best description.
Use

SHM = simple harmonic motion   *OD = overdamped*

CD = critically damped   *UD = underdamped*

TSS = transient plus steady-state

a. _____ $x'' + x = 0$

b. _____ $x'' + 2x' + x = 0$

c. _____ $x'' + 2x' + x = \cos 2t$

d. _____ $x'' - 6x' + 9x = 0$

e. _____ $2x'' + 3x' + 5x = 0$

f. _____