Name————————————, ID ——————

MTH 111 Mathematics for Architects 2021, 1–190

MTH111-Course Portfolio-Fall-2021

Ayman Badawi

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Section 1: Course Syllabus

AUS | الجامعة الأمريكية في الشارقة
American University of Sharjah

COURSE SYLLABUS

A	Course Title & Number	Mathematics for Architects - **MTH 111**
B	Pre/Co-requisite(s)	Prerequisites: MTH 001 or MTH 003 or Architecture Math Placement Test or Engineering Math Placement Test or SAT II Math Level 1 test with score 600 and above
C	Number of credits	3-0-3
D	Faculty Name	**Ayman Badawi**
E	Term/ Year	Fall 2021
F	Sections	

CRN	Course	Days	Time	Location
10941	MTH 111	UTR	11-11:50	First 2 weeks ONLINE/Starting Sept 12 hybrid (I explain more during the first class), Nab 007

G Instructor Information

Instructor	Office	Telephone	Email
Ayman Badawi	NAB 262	---------	abadawi@aus.edu

Office Hours: ONLINE ONLY

UTR: 14:15-15:10 (by APPOINTMENT ONLY/just EMAIL me/then we meet on Blackboard Collaborate. If you like to meet with me on diff time, also email me and we set a time)

H Course Description from Catalog

Introduces the topics of geometry and calculus needed for architecture. Reviews trigonometry, areas and volumes of elementary geometric figures, and the analytic geometry of lines, planes and vectors in two and three dimensions. Covers differential and integral calculus, including applications on optimization problems, and areas and volumes by integration. Restricted to CAAD

I Course Learning Outcomes

Learning Outcomes Upon completion of the course, students will be able to:	**Assessment Instruments**
1. Solve problems involving comic sections (Parabola, Ellipse, and Hyperbola).	Midterm 1& Final
2. Find the derivative of a function and apply it to solve a variety of problems involving optimization and curve sketching	Midterm 2 & Final

		3. Apply the Fundamental Theorem of Calculus to find the area under a curve and compute volumes of revolution. 4. Apply the analytic geometry of conic sections to solve word problems 5. Express geometric quantities using vectors and their standard operations in 2 and 3 dimensions 6. Solve geometric problems involving lines and planes in 2 and 3 dimensions.	Final Midterm 1 Midterm 1& Final Midterm 1& 2 & Final
J	**Textbook and other Instructional Material and Resources**	Class notes are the main source. Class notes (very crucial) , Materials posted on I-Learn , and my personal webpage (for old quizzes, exams, finals, see http://ayman-badawi.com/MTH%20%20111.html)	
K	**Teaching and Learning Methodologies**	This is a traditional lecture based course. **All lectures will be recorded and available on Ilearn.** All old class notes , quizzes, exams, and finals are available on my personal webpage http://ayman-badawi.com/MTH%20%20111.html	
L	**Grading Scale, Grading Distribution, and Due Dates**	See below	

Grading Distribution

Assessment	Weight	Date
Quizzes	15%	Every Thursday Starting Sept 9
Exam one	25%	October 18 at 18/ Group A F2F and Group B on line at the same time)
Exam Two	25%	November 29 at 18/ Group B F2F and Group A on line at the same time
Final Exam	35%	TBA F2F ALL STUDENTS
Total	100%	

Grading Scale

93.00 – 100	A	4.0
89.00 – 92.99	A-	3.7
86.00 – 88.99	B+	3.3
81.00 – 85.99	B	3.0
77.00 – 80.99	B-	2.7
73 .00– 76.99	C+	2.3
66 .00– 72.99	C	2.0
60 .00– 65.99	C-	1.7
50.00 – 61.99	D	1.0
Less than 50	F	0

M	Explanation of Assessments	There will be two exams, final, and quizzes. **The lowest quiz-score will be dropped.** • ***No make-up exam will be given***. With a valid written excuse and making immediate arrangements with the instructor, a missed exam might be replaced with the grade of the final exam and/or the average grade of all tests (including final)
N	Student Academic Integrity Code Statement	All students are expected to abide by the Student Academic Integrity Code as articulated in the AUS undergraduate catalog
0	Attendance Policy	Students in this course are required to follow the AUS Attendance Policy as outlined in the AUS Undergraduate Catalog 20-21 (p.27). ***During the face to face component of the course, wearing mask is a must and not optional. Students are required to attend according to their designated group (A or B). Students will not be allowed to change their designated group or switch between F2F and online.***

SCHEDULE

CHAPTER	Week
Conic sections, ellipse, parabola, and hyperbola	One
Continue: Conic sections, ellipse, parabola, and hyperbola	• Two
Lines in 2D , Vectors in 2 D , and projection	• Three
Dot Product, Cross Product and applications	• Four
Line and planes in 3 dimensional space , and Parametric Equations	• Five
Continue: Line and planes in 3 dimensional space, and Parametric Equations	• Six
Definition of derivatives and apply derivative to polynomials, exponential function, and logarithms	• Seven
Tangent lines and normal lines, product formula, quotient formula, and chain rule	Eight
Applications of Derivatives: Maximize and Minimize	Nine

Integration (anti-derivative), techniques and properties	• Ten
Integration by substitution	Eleven
Calculating areas by definite integrals	Twelve
	Thirteen
Volume by definite integrals	Fourteen
Voume /Area and Reviews	
Final Exam	

2 Section 3: Instructor Teaching Material

2.1 HANDOUTS

2.1.1 The Course's Questions and Solutions

HEIDY TAREK
@g00093162

Q1 $x^2 - 4x - 4y^2 - 8y = 2$

$x^2 - 4x - [4(y^2 + 2y)] = 2$

$(x-2)^2 - 4 - [4[(y+1)^2 - 1]] = 2$

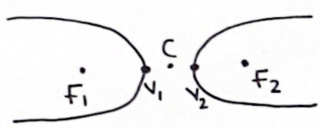
$(x-2)^2 - 4 - 4(y+1)^2 + 4 = 2$

$(x-2)^2 - 4(y+1)^2 = 2$

$\frac{(x-2)^2}{2} - \frac{4(y+1)^2}{2} = 1$

$\frac{(x-2)^2}{2} - \frac{(y+1)^2}{1/2} = 1$

- Hyperbola



x - positive $\rightarrow$ horizontal

$C(2,-1)$

F_1, V_1, C, V_2, F_2
↳ same y

$\left(\frac{K}{2}\right)^2 = 2$

$\frac{K}{2} = \sqrt{2}$

$K = 2\sqrt{2}$

$CV_1 = \frac{K}{2} = \sqrt{2}$

$CF_1 = \sqrt{a^2 + b^2}$
$= \sqrt{2 + 1/2}$
$= \sqrt{2.5}$

V_1 (2- [redacted] $\sqrt{2}$, -1)
V_2 (2+ [redacted] $\sqrt{2}$, -1)

[redacted]

$F_1(2 - \sqrt{2.5}, -1)$

$F_2(2 + \sqrt{2.5}, -1)$

Q2

HEIDY TAREK
@900093162

$4x^2+y^2+16x=20$

$4x^2+16x+y^2=20$

$4[x^2+4]+y^2=20$

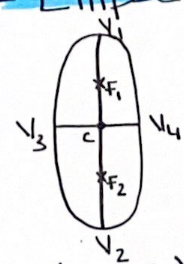
$4[(x+2)^2-4]+y^2=20$

$4(x+2)^2-16+y^2=20$

$4(x+2)^2+y^2=36$

$\frac{(x+2)^2}{9}+\frac{y^2}{36}=1$

- Ellipse



$b^2>a^2 \rightarrow$ vertical

$C(-2,0)$

$\left(\frac{k}{2}\right)^2=36$

$\frac{k}{2}=6$

$k=12$

$V_1, F_1, C, F_2, V_2 \rightarrow$ same x

[redacted]

$V_1(-2,6)$

$V_2(-2,-6)$

$CF_1=\sqrt{36-9}$
$=\sqrt{27}$

$F_1(-2,\sqrt{27})$

$F_2(-2,-\sqrt{27})$

HEIDY TAREK
@g00093162

Q3 $2y^2-6x^2+8y=30$

$2y^2+8y-6x^2=30$

$2[y^2+4y]-6x^2=30$

$2[(y+2)^2-4]-6x^2=30$

$2(y+2)^2-8-6x^2=30$

$2(y+2)^2-6x^2=38$

$\frac{(y+2)^2}{19}-\frac{6x^2}{38}=1$

$\frac{(y+2)^2}{19}-\frac{x^2}{19/3}=1$

- Hyperbola

y-positive → vertical

$C(0,-2)$

$(\frac{K}{2})^2=19$

$K=2\sqrt{19}$

$V_1(0,-2+\sqrt{19})$

$V_2(0,-2-\sqrt{19})$

$CF_1=\sqrt{a^2+b^2}$

$=\sqrt{19+19/3}=\frac{2\sqrt{57}}{3}$

$F_1(0,-2+\frac{2\sqrt{57}}{3})$ $F_2(0,-2-\frac{2\sqrt{57}}{3})$

Krstin Raed
g00078656

Quiz II MTH 111, Spring 2019

Ayman Badawi

15/15

QUESTION 1. Consider the parabola $y = 3x^2 - 6x + 2$

3 (i) Write the equation above in the standard form.

$y = 3x^2 - 6x + 2$

$y = (3(x^2 - 2x)) + 2$

$3((x-1)^2 - 1^2) + 2$

$y = 3(x-1)^2 - 3 + 2$

$y = 3(x-1)^2 - 1$

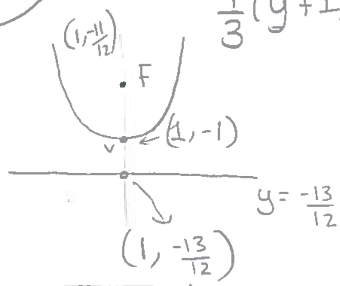
$\frac{(y+1)}{3} = \frac{3(x-1)^2}{3}$

$\frac{1}{3}(y+1) = (x-1)^2$

2 (ii) Sketch the graph (roughly)

y = so up or down

$4d = \frac{1}{3}$ $d = \frac{1}{12}$ so up



1 (iii) Find the vertex.

$\boxed{(1, -1)}$

1 (iv) Find the FOCUS.

$F = (1, -1 + \frac{1}{12})$ $\boxed{(1, \frac{-11}{12})}$

1 (v) Find the equation of the directrix line

$\boxed{y = \frac{-13}{12}}$

QUESTION 2. Consider the parabola $-12(x+4) = (y-2)^2$.

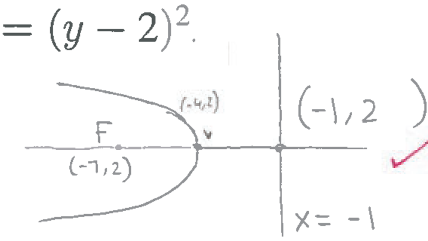
4 (i) Sketch (rough graph).

$4d = -12$

$\boxed{d = -3}$

(x so its right or left)

negative so left



1 (ii) Find the focus

$(-4-(3), 2)$ $\boxed{(-7, 2)}$

1 (iii) Find the vertex

$\boxed{(-4, 2)}$

1 (iv) Find the equation of the directrix line.

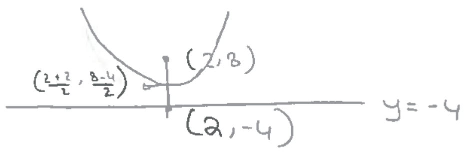
$\boxed{x = -1}$

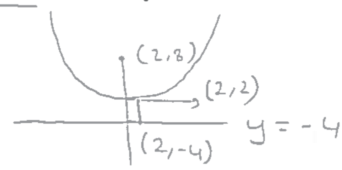
Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: abadawi@aus.edu, www.ayman-badawi.com

QUESTION 3. Given $y = -4$ is the directrix of a parabola that has the point $F = (2, 8)$ as its focus point.

a) **(2 points)** Roughly, sketch such parabola.





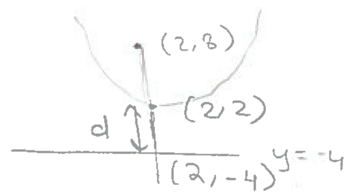
d = 6 & its up

4

b)**(4 points)** Find the equation of the parabola

$$4d(y-2) = (x-2)^2$$
$$4(6)(y-2) = (x-2)^2$$
$$24(y-2) = (x-2)^2$$

$\boxed{d=6}$



d = (-4-2
-6

c) **(2 points)** Find the vertex of the parabola, say V.

$$V = (2,2)$$

QUESTION 4. Given $y = 4x^2 + 24x - 3$ is an equation of a parabola.

a)**(3 points)** Write the equation in the standard form.

$$y = 4x^2 + 24x - 3$$
$$y = 4(x^2 + 6x) - 3$$
$$y = 4((x+3)^2 - 9) - 3$$
$$y = 4(x+3)^2 - 36 - 3$$
$$y = 4(x+3)^2 - 39$$
$$\frac{1}{4}(y+39) = \frac{4(x+3)^2}{4}$$
$$\frac{1}{4}(y+39) = (x+3)^2$$

$$4d = \frac{1}{4}$$
$$d = \frac{1}{4\times 4}$$
$$\boxed{d = \frac{1}{16}}$$

so +

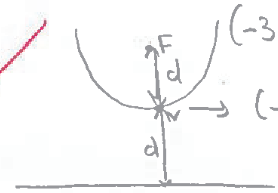
b) **(2 points)** Find the equation of the directrix line.

$$y = -\frac{625}{16}$$

c)**(2 points)** Find the focus, say F

$$F = (-3, -39 + \tfrac{1}{16}) = (-3, -\tfrac{623}{16})$$

d)**(2 points)** Roughly, sketch the graph of such parabola.



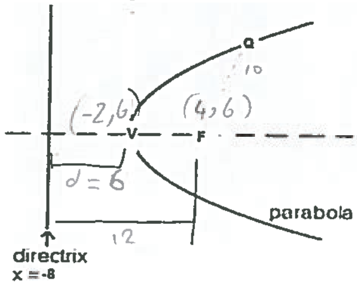
$(-3, -39 + \frac{1}{16})$

$(-3, -39)$

$(-3, -39 - \frac{1}{16})$

$(-3, -\frac{625}{16})$

QUESTION 3. (4 points) Stare at the following graph.



Given $F = (4,6)$, the directrix line, L is $x = -8$, and $|QF| = 10$.

(i) Find $|QL| = |QF| = 10$

(ii) Find $v = (-2, 6)$

(iii) Find the equation of the parabola

$$24(x+2) = (y-6)^2$$

Quiz I: MTH 111, Spring 2018

Ayman Badawi

$\frac{(y-y_0)^2}{10} + (x-x_0)^2$

15/15

QUESTION 2. Consider the parabola $y = 3x^2 + 18x + 5$

(i) Sketch, roughly.

Standard form

$y = 3[x^2 + 6x] + 5$

$y = 3[(x+3)^2 - 9] + 5$

$y = 3(x+3)^2 - 27 + 5$

$y = 3(x+3)^2 - 22$

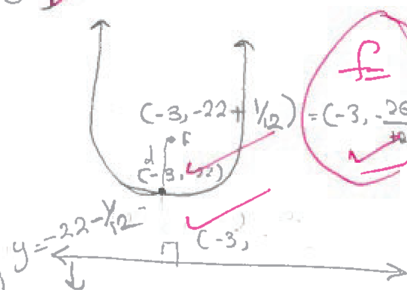
$y + 22 = 3(x+3)^2$

$\frac{1}{3}(y+22) = (x+3)^2$

vertex $= (-3, -22)$

$4d = \frac{1}{3}$ $d = \frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$

5



f

(ii) Find the focus.

write answers here!

(iii) Find the directrix line.

$y = -\frac{265}{12}$

QUESTION 3. Consider the parabola $-12(x+2) = (y-4)^2$

vertex $= (-2, 4)$

(i) Sketch, roughly.

$4d = -12$

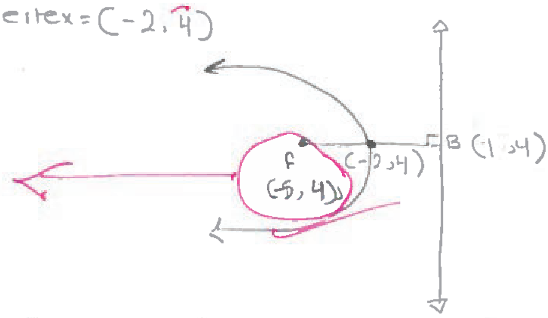
$\boxed{d = -3}$

(ii) Find the focus.

$(-5, 4)$

5

(iii) Find the directrix line. $x = 1$



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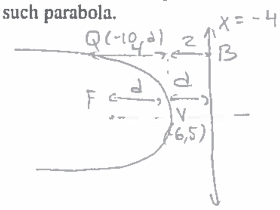
(1, 4)

(-5, 4)

QUESTION 3. Given $x = -4$ is the directrix of of a parabola that has the point $(-6, 5)$ as its vertex point.

a) **(2 points)** Roughly, sketch such parabola.



$|d| = 2$

b)**(4 points)** Find the equation of the parabola

$4d(x - x_0) = (y - y_0)^2$

$-4(2)(x + 6) = (y - 5)^2$

$\boxed{-8(x+6) = (y-5)^2}$

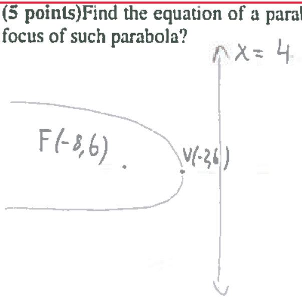
c) **(2 points)** Find the focus of the parabola, say F.

$\boxed{F(-8, 5)}$

d) **(2 points)** Given $Q = (-10, b)$ is a point on the curve of the parabola. Find $|QF|$ (HINT: You should know how to do this QUICKLY!, you do not need the value of b)

$|QL| = |QB| = |QF| = \boxed{6}$

(vii) **(5 points)** Find the equation of a parabola that has x = 4 as its directrix line and (-2, 6) as its vertex. What is the focus of such parabola?



$d = |-2-4| = 6$

$-4d\,(x - x_0) = (y - y_0)^2$

$\boxed{-24(x+2) = (y-6)^2}$

$\boxed{F(-8,6)}$

Inaam Shamma

Quiz I: Math. for the Architects, MTH 111, Spring 2017

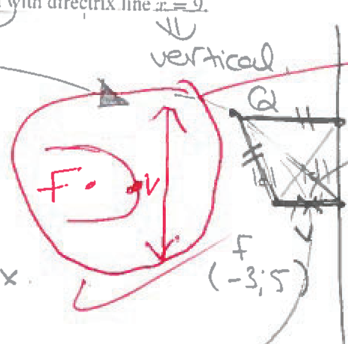
QUESTION 2. Given $(-3, 5)$ is the focus of a parabola with directrix line $x = 9$.

$x = 9$

vertical

graph sketched on the back

(i) Sketch (rough sketch)



$V(3;5)$

$B(9;5)$.

(ii) Find the equation of the Parabola.

eq: $4d(x - x_1) = (y - y_1)^2$.

vertix.

3

midpt of $|FB|$ is the vertix.

$x_V = \frac{x_F + x_B}{2} = \frac{-3+9}{2} = 3$.

$|FV| = |VB| = |d| = |\Delta x| = |-3-3| = |-6| = 6$.

since on the left side $d < 0$.

$\boxed{d = -6}$

~~(iii) If Q is a point on the curve of the parabola. What is the distance between Q and the directrix?~~

$|QF| = |QL|$ QL we draw $\perp$ to L. intersect at point E $E(9;?)$

2

$4(-6)(x-3) = (y-5)^2$

$\boxed{-24(x-3) = (y-5)^2}$.

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E-mail: abadawi@aus.edu, www.ayman-badawi.com

iii) find the distance between vertix and directrix.

$|VB| = \sqrt{\Delta x^2} = |\Delta x| = |9-3| = 6$.

Haya Alshamsi

Exam I: MTH 111, Fall 2017

Ayman Badawi

Points = $\frac{70}{70}$ Excellent

QUESTION 1. (6 points) Given $y = 11$ is the directrix of of a parabola that has the point $(6, 5)$ as its vertex point.

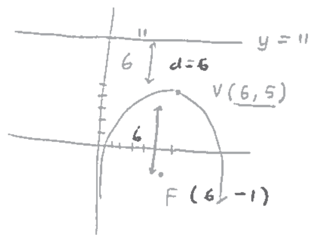
a) Find the equation of the parabola

$\boxed{d = 6}$

$$-4d(y - y_1) = (x - x_1)^2$$

$$-4(6)(y - 5) = (x - 6)^2$$

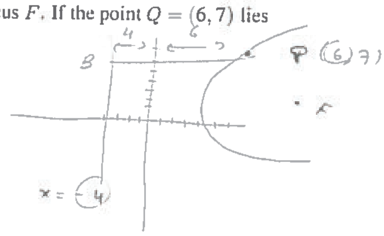
$$-24(y - 5) = (x - 6)^2 \checkmark$$



b) Find the focus of the parabola.

F $(6, -1)$ ✓

QUESTION 2. (3 points) Given that $x = -4$ is the directrix of a parabola that has focus F. If the point $Q = (6, 7)$ lies on the curve of the parabola, find $|QF|$ (i.e., find the distance between F and Q).



$|QL| = |QF|$

$|QB| = |QF|$

$|QF| = 10$ units ✓

Exam I MTH 111, Fall 2016

Ayman Badawi

$\frac{93}{95}$ Excellent

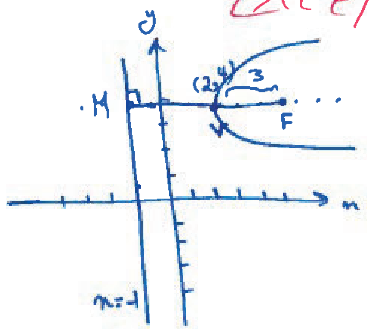
QUESTION 1. Given $12(x-2) = (y-4)^2$.

(i) Roughly, Sketch the graph of the given parabola.

$V = (2,4)$

$4d = 12 \to d = 3$

$M = (-1,4)$



(ii) What is the directrix line?

directrix: $\boxed{x = -1}$

(iii) What is the focus?

$\boxed{F = (5,4)}$

QUESTION 2. Given $y = x^2 - 6x + 4$

(i) Roughly, Sketch the graph of the given parabola.

$y - 4 = (x-3)^2 - 9 \to (y+5) = (x-3)^2$

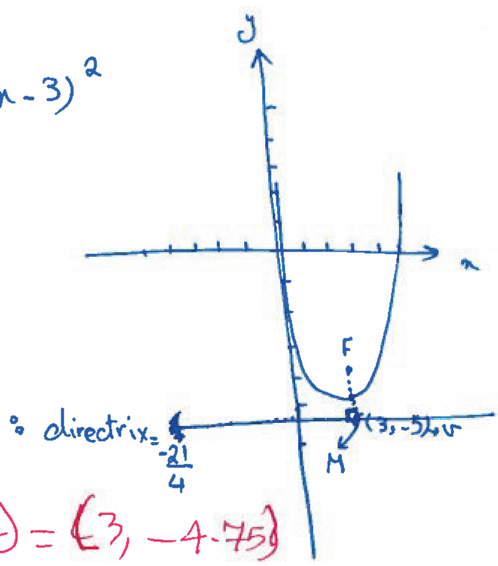
$\boxed{V = (3,-5)}$

$\boxed{4d = 1 \to d = \frac{1}{4}}$

(ii) What is the directrix line?

$M = (3, -5-\frac{1}{4})$

directrix: $y = -5 - \frac{1}{4} = \boxed{-5.25}$



(iii) What is the focus?

$\to (3, -5 + \frac{1}{4}) = (3, -4.75)$

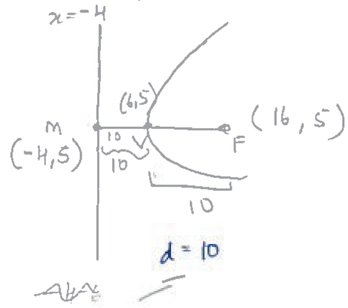
QUESTION 8. (6 points) Given $x = -4$ is the directrix of of a parabola that has the point $(6, 5)$ as its vertex point.

a) Find the equation of the parabola

$4(10)(x-6) = (y-5)^2$

$= 40(x-6) = (y-5)^2$

6/6



b) Find the focus of the parabola.

$F(16, 5)$

QUESTION 9. (6 points) Consider the parabola $x = -0.25(y + 3)^2 + 4$ [hint: first write it in the standard form].

$x = -0.25(y+3)^2 + 4$

$(x-4) = -0.25(y+3)^2$

$-4(x-4) = (y+3)^2$

$4d = -4$

$d = -1$

a) Find the focus.

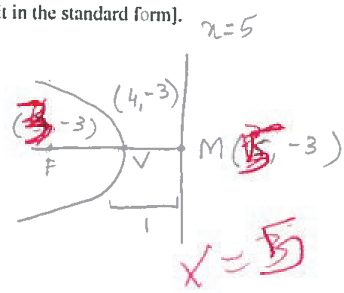
~~F(5, -3)~~ $F = (3, -3)$

b) Find the equation of the directrix

$x = \cancel{3}\ 5$

6/6

c) Draw the parabola



more ...

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: abadawi@aus.edu, www.ayman-badawi.com

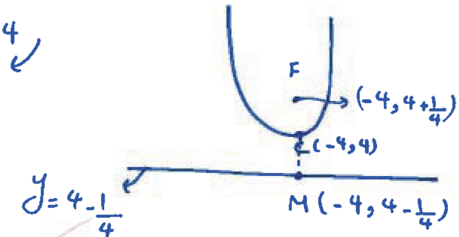
QUESTION 7. (8 points). Given $y = x^2 + 8x + 20$

(i) Roughly, Sketch the graph of the given parabola.

$y = (x+4)^2 - 16 + 20 \Rightarrow y = (x+4)^2 + 4$

$(y-4) = (x+4)^2$

$4d(y-y_0) = (x-x_0)^2$



(ii) What is the directrix line?

$4d = 1 \Rightarrow \boxed{d = \frac{1}{4}}$ ⇝ directrix line: $\boxed{y = 4 - \frac{1}{4} = \frac{15}{4}}$

$\boxed{C = (-4, 4)}$

(iii) What is the focus?

$\boxed{F = (-4, 4 + \frac{1}{4})}$

Krstin Raed

Quiz I MTH 111, Spring 2019

Ayman Badawi

15/15 :)

$CF^2 = \left(\frac{k}{2}\right)^2 - b^2$

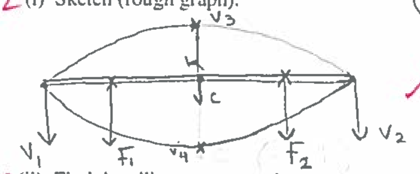
$CF^2 = 25 - 9$
$= 16$
$\boxed{CF = 4}$

QUESTION 1. Consider the ellipse $\frac{(x+2)^2}{25} + \frac{(y-1)^2}{9} = 1$

$25 = \left(\frac{k}{2}\right)^2$, $9 = b^2$

2 (i) Sketch (rough graph).

$c = (-2, 1)$



2 (ii) Find the ellipse-constant, k

$\left(\frac{k}{2}\right)^2 = 25 \qquad \frac{k}{2} = \sqrt{25} \rightarrow k = 5 \times 2 \rightarrow \boxed{k = 10}$

2 (iii) Find all 4 vertices

$b^2 = 9$
$\boxed{b = 3}$

$V_1 \ (-2-5, 1) \quad (-7, 1)$

$V_2 \ (-2+5, 1) \quad (3, 1)$

$V_3 \ (-2, 4)$

$V_4 \ (-2, -2)$

2 (iv) Find the Foci

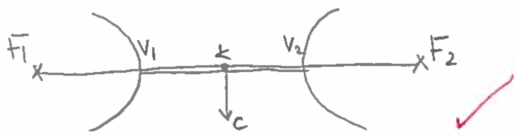
$F_2 \ (-2+4, 1)$
$(2, 1)$

$F_1 \ (-2-4, 1)$
$(-6, 1)$

QUESTION 2. Consider the hyperbola $\frac{(x-3)^2}{9} - \frac{(y+2)^2}{16} = 1.$

2 (i) Sketch (rough graph).

$c = (3, -2)$



1 (ii) Find the heperbola-constant, k

$\left(\frac{k}{2}\right)^2 = 9 \qquad \frac{k}{2} = \sqrt{9} \qquad \boxed{k = 3 \times 2 = 6}$

2 (iii) Find all vertices

$V_1 \ (3-3, -2)$
$(0, -2)$

$V_2 \ (3+3, -2)$
$(6, -2)$

2 (iv) Find the Foci

$CF^2 = \left(\frac{k}{2}\right)^2 + b^2$

$CF^2 = 9 + 16$

$CF^2 = 25$
$\boxed{CF = 5}$

$F_1 \ (3-5, -2)$
$(-2, -2)$

$F_2 \ (3+5, -2)$
$(8, -2)$

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Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
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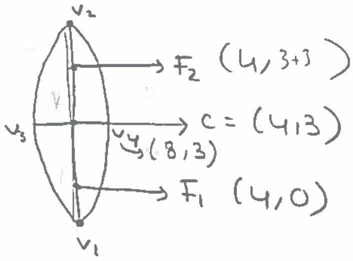
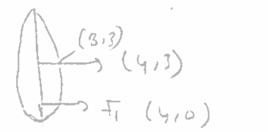
QUESTION 5. An ellipse is centered at $(4,3)$, $F_1 = (4,0)$ is one of the foci, and $(8,3)$ is one of the vertices.

(i) **(2 points)** Roughly, sketch such ellipse.

x does not change





$CF^2 = \left(\frac{k}{2}\right)^2 - b^2$

$3^2 = \left(\frac{k}{2}\right)^2 - 4^2$

$25 = \left(\frac{k}{2}\right)^2$

$\boxed{CF = 3}$

$\boxed{b = 4}$

(ii) **(3 points)** Find the ellipse-constant K.

$$CF^2 = \left(\frac{k}{2}\right)^2 - b^2$$

$$3^2 = \left(\frac{k}{2}\right)^2 - 4^2$$

$$\boxed{k = 10}$$

(iii) **(2 points)** Find the second foci of the ellipse.

$$F_2 = (4, 3+3)$$
$$(4, 6)$$

(iv) **(3 points)** Find the remaining three vertices of the ellipse

$$V_1 = \left(4, 3 - \frac{10}{2}\right) \quad \boxed{(4, -2)}$$

$$V_2 \left(4, 3 + \frac{10}{2}\right) \quad (4, 8)$$

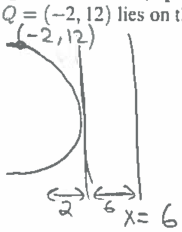
$$V_3 (0, 3)$$

(v) **(3 points)** Find the equation of the ellipse.

$$\frac{(y-3)^2}{\left(\frac{10}{2}\right)^2} + \frac{(x-4)^2}{4^2} = 1$$

$$\frac{(y-3)^2}{25} + \frac{(x-4)^2}{16} = 1$$

QUESTION 11. (4 points) Given that $x = 6$ is the directrix line of a parabola that has F as its focus point. If the point $Q = (-2, 12)$ lies on the parabola. Find $|QF|$ (i.e., the distance between Q and F).



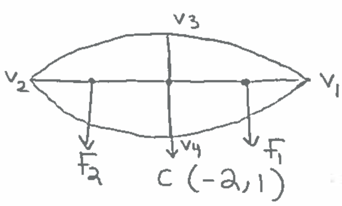
$|QF| = |QL| = 8$

QUESTION 12. (6 points) Consider the ellipse $\frac{(y-1)^2}{9} + \frac{(x+2)^2}{25} = 1.$

b^2 → big # so its $\left(\frac{k}{2}\right)^2$

so the shape is

(i) Sketch (roughly)



(ii) Find the foci of the ellipse

$CF^2 = \left(\frac{k}{2}\right)^2 - b^2$

$= 25 - 9$

$= 16$

$CF^2 = 16$

so $\boxed{CF = 4}$

so $F_1\ (-2+4\ ,\ 1)$

$(2, 1)$

$F_2\ (-2-4, 1)$

$(-6, 1)$

(iii) Find all four vertices of the ellipse.

$\left(\frac{k}{2}\right)^2 = 25$ $\boxed{\pm\frac{k}{2}}$

$\boxed{\frac{k}{2} = 5}$ $\boxed{\pm b}$

$b^2 = 9$ $\boxed{b = 3}$

$V_1 = (-2+5\ ,\ 1)$

$(3, 1)$

$V_2 = (-2-5, 1)$

$(-7, 1)$

$V_3 = (-2, 1+3)$ $(-2, 4)$

$V_4 = (-2, 1-3)$ $(-2, -2)$

QUESTION 13. (4 points) Given $Q = (1, 6, 4)$ is not on the line $L : x = t + 1, y = 2t + 4, z = -5t + 3\ (t \in R)$. Find $|QL|$.

$D = \langle 1, 2, -5 \rangle$

$I = \langle 1, 4, 3 \rangle$

$IQ = \langle 0, 2, 1 \rangle$
$Q - I$

$|QL| = \frac{|D \times IQ|}{|D|} = \frac{\sqrt{12^2 + 1^2 + 2^2}}{\sqrt{1^2 + 2^2 + 5^2}}$

$= \frac{\sqrt{149}}{\sqrt{30}}$

$IQ \times D = \begin{vmatrix} i & j & k \\ 0 & 2 & 1 \\ 1 & 2 & -5 \end{vmatrix}$

$= -12i + 1j - 2k$

Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: abadawi@aus.edu, www.ayman-badawi.com

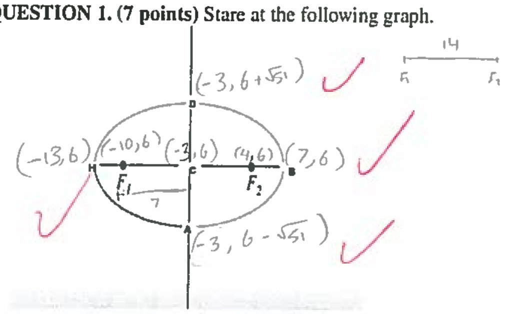
Name Haya Sujaa, ID g00082558

Final Exam, MTH 111, Spring 2019

Ayman Badawi

$$Score = \frac{75}{78}$$

QUESTION 1. (7 points) Stare at the following graph.



(i) Given $F1 = (-10, 6)$, $F2 = (4, 6)$ and the ellipse-constant is 20.

(ii) Find the center $c =$

$|F_1 C| = 7 \quad \therefore C = (-3, 6)$

(iii) Find the vertices $A = (-3, -1.14)$, $D = (-3, 13.14)$, $H = (-13, 6)$, and $B = (7, 6)$

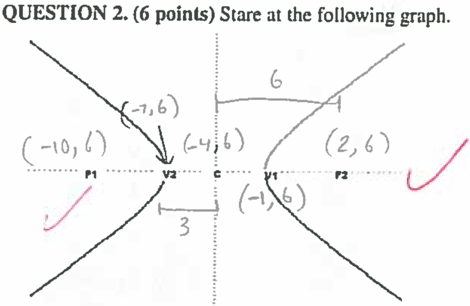
$100 - 49 = b^2$

$b = \sqrt{51}$

(iv) Find the equation of the ellipse.

$$\frac{(x+3)^2}{100} + \frac{(y-6)^2}{51} = 1$$

QUESTION 2. (6 points) Stare at the following graph.



Given $c = (-4, 6)$, $|cv2| = 3$, and $F2 = (2, 6)$.

(i) Find $v1 = (-1, 6)$ $F1 = (-10, 6)$, $v2 = (-7, 6)$, and the hyperbola-constant $k = 6$

$|CF_1| = \sqrt{(6/2)^2 + b^2} = 6$

$\sqrt{9 + b^2} = 6$

$9 + b^2 = 36$

$b^2 = 36 - 9$

$b^2 = 27$

(ii) Find the equation of the hyperbola

$$\frac{(x+4)^2}{9} - \frac{(y-6)^2}{27} = 1$$

alleem AlAli

Quiz I: MTH 111, Spring 2018

Ayman Badawi

$\frac{(y-y_0)^2}{10} + (x-x_0)^2$

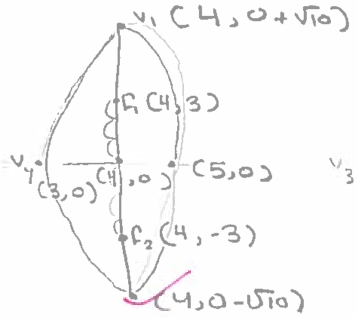
15/15

QUESTION 1. Consider the ellipse given by $\frac{y^2}{10} + (x-4)^2 = 1$

$C = (4, 0)$

(i) Sketch, roughly.

$b^2 = 1 \quad b = 1$



(ii) Find the ellipse-constant K. $\sqrt{\left(\frac{K}{2}\right)^2} = \sqrt{10}$ $\quad \boxed{K/2 = \sqrt{10}}$

$K = 2\sqrt{10}$

(iii) Find the foci. $|CF_1| = \sqrt{(K/2)^2 - b^2} = \sqrt{10-1} = 3$

5 $F_1\ (4, 3) \quad F_2 = (4, -3)$

(iv) Find all vertices. $V_1 = (4, 0+\sqrt{10}) \quad V_3 = (4+1, 0)$

$V_2 = (4, 0-\sqrt{10}) \quad V_4 = (4-1, 0)$

QUESTION 2. Consider the parabola $y = 3x^2 + 18x + 5$

(i) Sketch, roughly. Standard form

$y = 3[x^2 + 6x] + 5$

5 $y = 3[(x+3)^2 - 9] + 5$

$y = 3(x+3)^2 - 27 + 5$

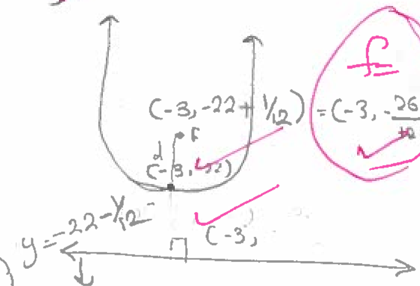
$y = 3(x+3)^2 - 22$

$y + 22 = 3(x+3)^2$

$\frac{1}{3}(y+22) = (x+3)^2$

vertex $= (-3, -22)$

$4d = \frac{1}{3} \quad d = \frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$



f

(ii) Find the focus.

Write answers here!

(iii) Find the directrix line.

$y = \frac{-265}{12}$

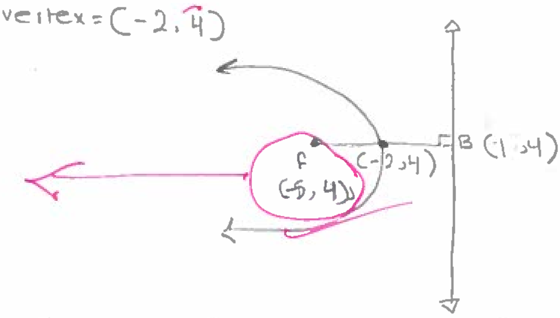
QUESTION 3. Consider the parabola $-12(x+2) = (y-4)^2$

vertex $= (-2, 4)$

(i) Sketch, roughly.

$4d = -12$

$\boxed{d = -3}$



(ii) Find the focus.

5 $(-5, 4)$

(iii) Find the directrix line. $x = 1$

Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: abadawi@aus.edu, www.ayman-badawi.com

$x = 1$

$(1, 4)$

$(-5, 4)$

QUESTION 4. Given $y = x^2 - 6x - 1$ is an equation of a parabola.

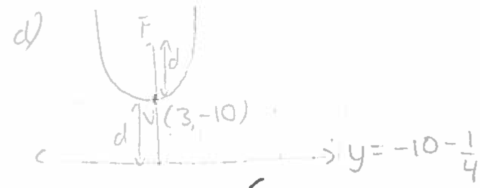
a)**(3 points)** Write the equation in the standard form.

$y = (x-3)^2 - 9 - 1$

$y = (x-3)^2 - 10$

$\boxed{(y+10) = (x-3)^2}$

$4d = 1 \Rightarrow d = 1/4$



b) **(2 points)** Find the equation of the directrix line.

$\boxed{y = -10 - \frac{1}{4} = -\frac{41}{4}}$

c)**(2 points)** Find the focus, say F

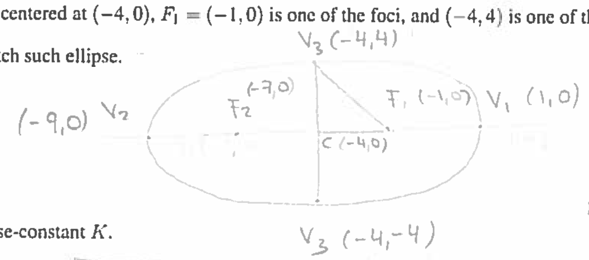
$\boxed{F(3, -10 + \frac{1}{4})} \to \boxed{F(3, -\frac{39}{4})}$

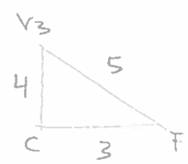
d)**(2 points)** Roughly, sketch the graph of such parabola.

(see picture)

QUESTION 5. An ellipse is centered at $(-4, 0)$, $F_1 = (-1, 0)$ is one of the foci, and $(-4, 4)$ is one of the vertices.

(i) **(2 points)** Roughly, sketch such ellipse.





(ii) **(3 points)** Find the ellipse-constant K.

$|V_3F_1| = \frac{K}{2} = 5 \Rightarrow \boxed{K = 10}$

(iii) **(2 points)** Find the second foci of the ellipse.

$\boxed{F_2 (-7, 0)}$

(iv) **(3 points)** Find the remaining three vertices of the ellipse

$\boxed{V_3(-4,-4)}$

$\boxed{V_1(1,0)}$

$\boxed{V_2(-9,0)}$

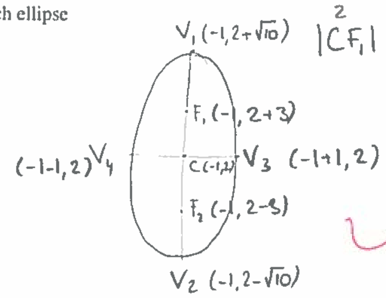
(v) **(3 points)** Find the equation of the ellipse.

$$\boxed{\frac{(x+4)^2}{25} + \frac{y^2}{16} = 1}$$

(x) Consider the ellipse $(x+1)^2 + \frac{(y-2)^2}{10} = 1$

$C(-1,2)$
$\frac{k}{2} = \sqrt{10}$
$|CF_1| = \sqrt{10-1} = 3$

a. **(2 points)** Roughly, draw such ellipse



b. **(2 points)** Find the foci

$F_1(-1,5)$
$F_2(-1,-1)$

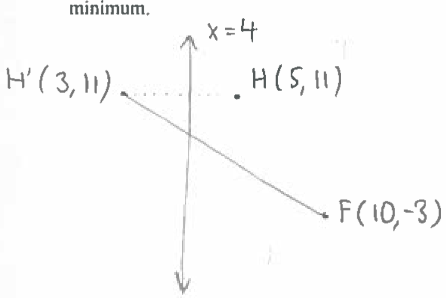
c. **(2 points)** Find the ellipse constant

$\boxed{k = 2\sqrt{10}.}$

d. **(2 points)** Find all four vertices

$V_1(-1, 2+\sqrt{10})$ $V_3(0,2)$
$V_2(-1, 2-\sqrt{10})$ $V_4(-2,2)$

(xi) **(6 points)** Let $H = (5, 11)$ and $F = (10, -3)$. Find a point Q on the vertical line $x = 4$ such that $|HQ| + |QF|$ is minimum.



$m = \frac{-3-11}{10-3} = -2$

$11 = -2(3) + b$
$b = 17$

$y = -2x + 17$
$y = -2(4) + 17 = 9$

$Q(4,9)$

Inaam Shamma

Quiz I: Math. for the Architects, MTH 111, Spring 2017

Ayman Badawi

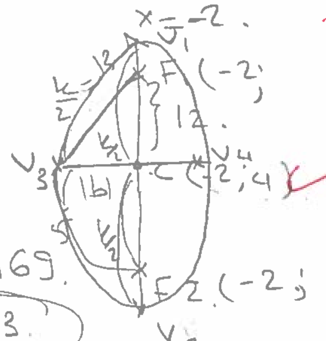
15/15

QUESTION 1. Consider the Ellipse $\frac{(x+2)^2}{25} + \frac{(y-4)^2}{169} = 1$

$\left(\frac{k}{2}\right)^2$

(i) Sketch (rough sketch)

2



$\left(\frac{k}{2}\right)^2 = 169$

$\frac{k}{2} = 13$

$\boxed{k=26}$

$hyp^2 = side^2 + side^2$

$169 = 25 + |F_1C|^2$

$|F_1C|^2 = 144$

$\boxed{|F_1C| = 12}$

$C(-2;+4)$

$b^2 = 25$

$\boxed{b=5}$

(ii) Find the Foci

2

$F_1(-2;16)$

$F_2(-2;-8)$

(iii) Find the ellipse-contant k

2

$\boxed{k=26}$

(iv) Find all 4 vertices.

2

$V_3(-7;4)$ $V_4(3;4)$ $V_1(-2;17)$ $V_2(-2;-9)$

QUESTION 2. Given $(-3, 5)$ is the focus of a parabola with directrix line $x = 9$.

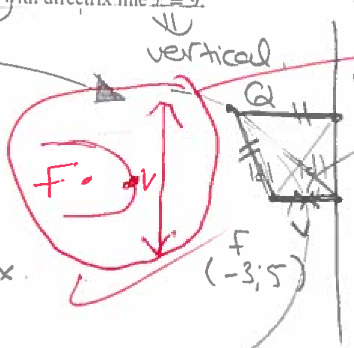
vertical

$x=9$

graph sketched on the back

(i) Sketch (rough sketch)

2



(ii) Find the equation of the Parabola.

eq: $4d(x-x_1) = (y-y_1)^2$.

vertix.

3

midpt of $|FB|$ is the vertix.

$x_v = \frac{x_F + x_B}{2} = \frac{-3+9}{2} = 3$.

$|FV| = |VB| = |d| = |\Delta x| = |-3-3| = |-6| = 6$.

since on the left side $d<0$.

$\boxed{d=-6}$

$4(-6)(x-3) = (y-5)^2$

$\boxed{-24(x-3) = (y-5)^2}$

~~(iii) If Q is a point on the curve of the parabola. What is the distance between Q and the directrix?~~

2

$|QF| = |QL|$ QL we draw $\perp$ to L.

intersect at point E

$E(9; ?)$

Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: abadawi@aus.edu, www.ayman-badawi.com

iii) find the distance between vertix and directrix.

$|VB| = \sqrt{\Delta x^2} = |\Delta x| = |9-3| = 6$.

Haya Alshamsi

Exam I: MTH 111, Fall 2017

Ayman Badawi

$$\text{Points} = \frac{70}{70}$$

Excellent

QUESTION 1. (6 points) Given $y = 11$ is the directrix of of a parabola that has the point $(6, 5)$ as its vertex point.

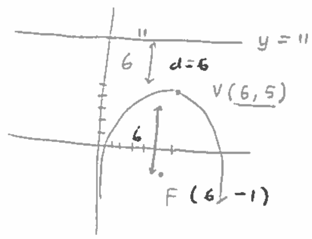
a) Find the equation of the parabola

$\boxed{d = 6}$

$$-4d(y - y_1) = (x - x_1)^2$$

$$-4(6)(y - 5) = (x - 6)^2$$

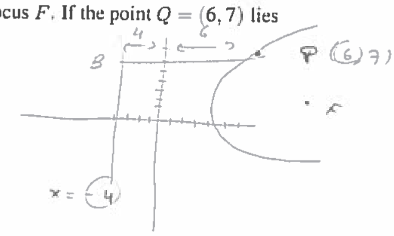
$$-24(y - 5) = (x - 6)^2$$



b) Find the focus of the parabola.

F $(6, -1)$

QUESTION 2. (3 points) Given that $x = -4$ is the directrix of a parabola that has focus F. If the point $Q = (6, 7)$ lies on the curve of the parabola, find $|QF|$ (i.e., find the distance between F and Q).



$|QL| = |QF|$

$|QB| = |QF|$

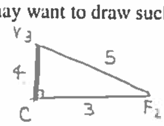
$|QF| = 10$ units

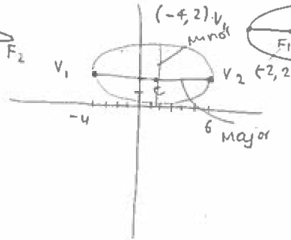
QUESTION 3. (8 points) Given $(-4, 2)$, $(6, 2)$ are the vertices of the major axis of an ellipse (recall major axis is the longer axis) and $(4, 2)$ is one of the foci.

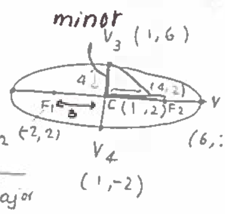
(i) Find the vertices of the minor axis (shorter axis). (you may want to draw such ellipse (roughly)).

V_3 $(1, 6)$

V_4 $(1, -2)$







(ii) Find the ellipse-constant K.

C $(1, 2)$; V_2 $(6, 2)$

$\frac{K}{2} = 5 \Rightarrow \boxed{K = 10}$

(iii) Find the second foci of the ellipse.

F_1 $(-2, 2)$

(iv) Find the equation of the ellipse.

horizontal ellipse ; $K = 10$; ~~(b = 3)~~ $b = 4$

$$\frac{(x - 1)^2}{25} + \frac{(y - 2)^2}{16} = 1$$

Final Exam: MTH 111, Fall 2017

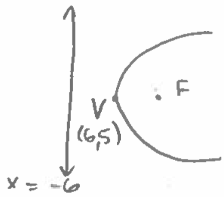
Ayman Badawi

Katia

Points = $\frac{81}{82}$

QUESTION 1. (6 points) Given $x = -6$ is the directrix of of a parabola that has the point $(6, 5)$ as its vertex point.

a) Find the equation of the parabola



$|VL| = |-6-6| = |-12| = 12$

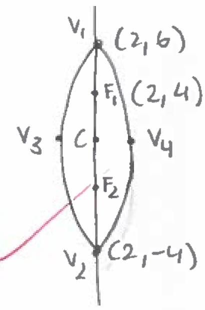
$4(12)(x-6) = (y-5)^2 \Rightarrow 48(x-6) = (y-5)^2$

b) Find the focus of the parabola.

$|VF| = 12 \rightarrow F(18,5)$

QUESTION 2. (8 points) Given $(2, -4), (2, 6)$ are the vertices of the major axis of an ellipse (recall major axis is the longer axis) and $(2, 4)$ is one of the foci.

(i) Find the vertices of the minor axis (shorter axis). (you may want to draw such ellipse (roughly)).



$|V_1V_2| = K = |6+4| = 10 \rightarrow \frac{K}{2} = 5 = |V_1C|$

$C = (2,1) \rightarrow |F_1C| = |4-1| = 3 \rightarrow b^2 = \left(\frac{K}{2}\right)^2 - |F_1C|^2$

$b^2 = 5^2 - 3^2 = 16 \rightarrow V_3(18,1), V_4(-14,1)$

(ii) Find the ellipse-constant K.

$K = 10$

(iii) Find the second foci of the ellipse.

$F_2(2,-2)$

(iv) Find the equation of the ellipse.

$$\frac{(x-2)^2}{16} + \frac{(y-1)^2}{25} = 1$$

QUESTION 3. (5 points) Given $y = 3x^2 + 12x + 9$ is an equation of a parabola. Write the equation of the parabola in the standard form and find the equation of its directrix.

-1

$y = 3x^2 + 12x + 9 \rightarrow y = 3(x^2 + 4x + 3) \rightarrow y = 3[(x+2)^2 - 4 + 3]$

$y = 3(x+2)^2 - 1(3) \rightarrow \frac{1}{3}(y+3) = (x+2)^2$

$4d = \frac{1}{3} \rightarrow d = \frac{1}{12}$

$y = -3 - \frac{1}{12}$

$V = (-2,-3) \rightarrow$ directrix $\rightarrow x = -2 - \frac{1}{12} \Rightarrow \boxed{\frac{-25}{12} = x}$

Quiz 6 MTH 111, Spring 2019

Ayman Badawi

KAMYA KANSRA
81881

15/15

QUESTION 1. Find $f'(x)$ and DO NOT SIMPLIFY

2 a) $f(x) = 10(3x^4 + 12x^3 - 10x + 5)^{11}$

$= 10 \cdot 11\,(3x^4 + 12x^3 - 10x + 5)^{10} \cdot (12x^3 + 36x^2 - 10)$

3 b) $f(x) = \sqrt[5]{x^2} + \frac{12}{x^{10}} + 7x - 3$

$= x^{2/5} + 12x^{-10} + 7x - 3$

$= \frac{2}{5}x^{-3/5} - 120x^{-11} + 7$

8/8

3 in c)Given $k(x) = f(2x^2 + x - 16)$. Find $k'(3)$ if $f'(5) = -7$.

$k'(x) = f'(2x^2 + x - 16) \cdot (4x + 1)$

$k'(3) = f'(2 \times 9 + 3 - 16) \cdot (12 + 1)$

$= f'(5) \cdot 13$

$= -7 \cdot 13 = -91$

QUESTION 2. Let $f(x) = x^3 - 6x^2 - 15x + 1$.

3 a) Find the sign of $f'(x)$.

$f'(x) = 3x^2 - 12x - 15$; For critical value: $f'(x) = 0$

$\Rightarrow 0 = 3x^2 - 12x - 15$

$= 3(x^2 - 4x - 5)$; $\Rightarrow 0 = x^2 - 4x - 5 \Rightarrow 0 = (x-5)(x+1)$

as $3 \neq 0$

$\Rightarrow x - 5 = 0$ OR $x + 1 = 0 \Rightarrow x = 5$ OR $x = -1$

sign of $f'(x)$:

At $x = -2$; $f'(-2) = 12 + 24 - 15 = 21 > 0$

At $x = 0$; $f'(0) = -15 < 0$

At $x = 6$; $f'(6) = 3 \times 36 - 72 - 15 = 21 > 0$

+ + + + , − − − − , + + +
−2 −1 0 5 6

7/7

1 b)By staring at (a) find the critical values.

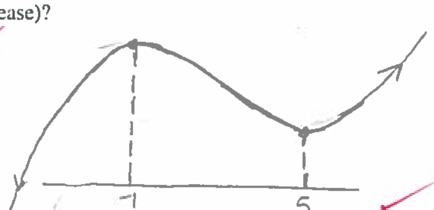
$x = 5$ or $x = -1$

2 c) By staring at (a), for what values of x does $f(x)$ increase (decrease)?

$\therefore f(x)$ increases : $(-\infty, -1) \cup (5, \infty)$

$f(x)$ decreases : $(-1, 5)$

1 d) By staring at (a), sketch $f(x)$ (roughly).



Faculty information

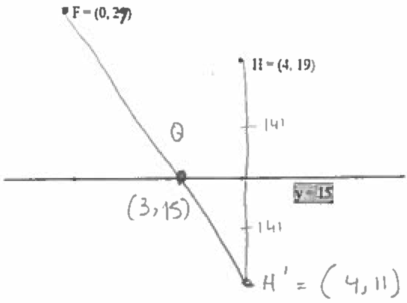
Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: abadawi@aus.edu, www.ayman-badawi.com

Quiz 7 MTH 111, Spring 2019

Ayman Badawi

15/15 :)

QUESTION 1. Given $H = (4, 19)$, $F = (0, 27)$. Find a point on the line $y = 15$, say Q, such $|FQ| + |QH|$ is minimum.



$Q = (3, 15)$

$y = mx + b$

$m = \frac{27 - 11}{0 - 4} = -4$

$y = -4x + b$

$11 = -4(4) + b$

$b = 27$

$y = -4x + 27$

$Q = (x, 15)$

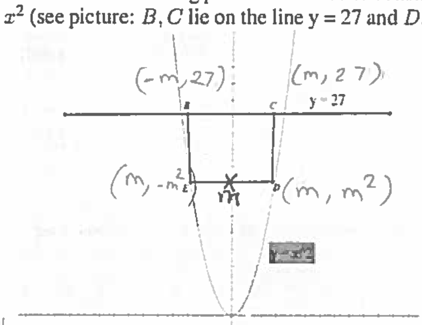
$15 = -4x + 27$

$x = \frac{15 - 27}{-4}$

$= 3$

8/8

QUESTION 2. Consider the following picture. We need to construct a rectangle B, C, D, E with maximum area between $y = 27$ and $y = x^2$ (see picture: B, C lie on the line y = 27 and D, E lie are the curve $y = x^2$. Also note that $mE = mD$).



Find $|BC|, |CD|$.

$A = 2(3) \cdot (27 - 3^2)$

$= 108$ units^2

$|BC| = 2m$

$= 6$ units

$|CD| = 27 - 3^2$

$= 18$ units

$A = |cd||ED|$

$A = 2m \cdot (27 - m^2)$

$A = 54m - 2m^3$

$A' = 54 - 6m^2$

$0 = 54 - 6m^2$

$m^2 = -\frac{54}{-6}$

$= 9$

$m = \pm 3$

check test.

$A'' = -12m$

$-12(3) = - = \max$

$\therefore 3$ is m

7/7

Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: abadawi@aus.edu, www.ayman-badawi.com

 Haya

QUESTION 4. (5 points) Let $f(x) = ln(5x-4)+4$. Find the equation of the tangent line to the curve of $f(x)$ at $x = 1$.

$f(1) = \ln(5-4)+4$
$= 4$
point $= (1, 4)$

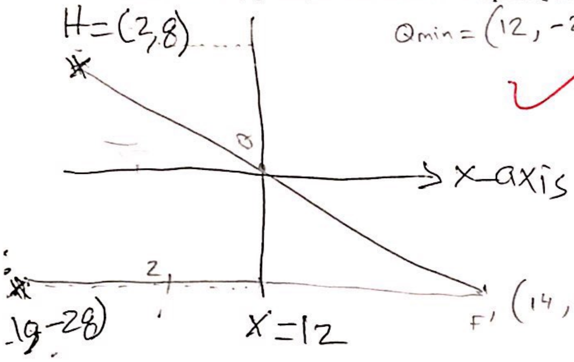
$f'(x) = \frac{5}{5x-4}$

① → $f'(1) = m = \frac{5}{1}$

$y = 5x + b$
$4 = 5(1) + b$
② → $b = 4 - 5$
$= -1$

③ → $\boxed{y = 5x - 1}$ is the equation of the tangent line.

QUESTION 5. (7 points) Given H and F. Find a point Q on the line $x = 12$ such that $|HQ| + |FQ|$ is minimum.



$Q_{min} = (12, -22)$

(H=(2,8),
F=(10,-28))

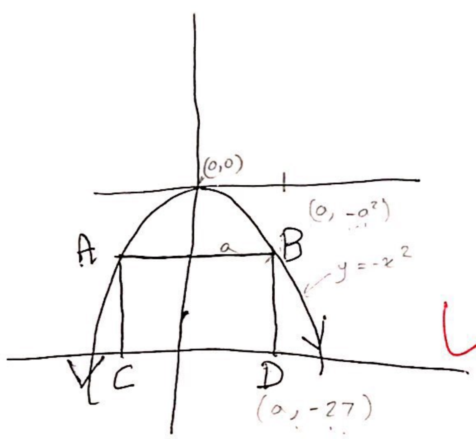
$m = \frac{-28-8}{14-2} = -3$

① → $y = -3x + b$
$8 = -3(2) + b$
$b = 8 + 6$
$= 14$
$y = -3x + 14$

|AC|

$Q = (12, y)$
$y = -3(12) + 14$
$= -22$
$Q_{min} = (12, -22)$

QUESTION 6. (7 points) Consider the following picture. Find $|AB|$ and $|\cancel{CD}|$ so that the area is MAXIMUM.



The curve is $y = -x^2$,
the line $y = -27$

$A = |BD|\,|AB|$

① → $A = [-a^2 - (-27)] \cdot 2a$
$A = (-a^2 + 27) \cdot 2a$
$A = -2a^3 + 54a$
$A' = -6a^2 + 54$
$-6a^2 + 54 = 0$
$-6a^2 = -54$
$a^2 = -54/-6$
$a = \sqrt{9}$
$a = +3$

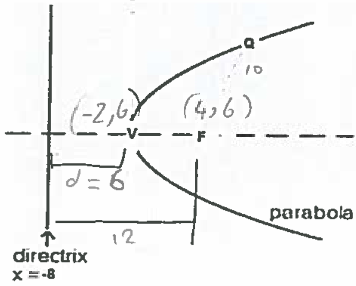
$A'' = -12a$
② → $A''(3) = - \therefore$ max.

$A = (-3^2 + 27)(6)$
$= 108$ units^2
$|AB| = 2a$
$= 6$ units
$|BD| = (-a^2 + 27)$
$= 18$ units

Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: abadawi@aus.edu, www.ayman-badawi.com

QUESTION 3. (4 points) Stare at the following graph.



Given $F = (4, 6)$, the directrix line, L is $x = -8$, and $|QF| = 10$.

(i) Find $|QL| = |QF| = 10$

(ii) Find $v = (-2, 6)$

(iii) Find the equation of the parabola

$$24(x+2) = (y-6)^2$$

QUESTION 4. (6 points). Find y' and do not simplify

(i) $y = ln[(4x+3)^{10}(-5x+30)^3]$

$y = \ln(4x+3)^{10} + \ln(-5x+30)^3$

$y = 10\ln(4x+3) + 3\ln(-5x+30)$

$y' = \dfrac{10 \cdot 4}{4x+3} + \dfrac{3 \cdot -5}{-5x+30}$

$y' = \dfrac{40}{(4x+3)} + \dfrac{-15}{(-5x+30)}$

(ii) $y = e^{(6x^3+x^2-1)} + 10x^2 - x + 23$

$y = \left[\left[e^{(6x^3+x^2-1)} \cdot (18x^2 + 2x)\right] + 20x - 1\right]$

(iii) $y = (21 + 5x - 6x^3)^7$

$y' = 7(21 + 5x - 6x^3)^6 \cdot (5 - 18x^2)$

QUESTION 5. (6 points).

(i) Find $\int x e^{(x^2+1)}\, dx$

$u = x^2 + 1$

$u' = 2x$

$\frac{1}{2}\left(e^{(x^2+1)}\right) + C$

(ii) Find $\int \frac{e^{2x}+1}{(e^{2x}+2x-5)^3}\, dx$

$\int (e^{2x}+1)(e^{2x}+2x-5)^{-3}\, dx$

$u = e^{2x} + 2x - 5$

$u' = 2e^{2x} + 2$

$\frac{1}{2}\int 2(e^{2x}+1)(e^{2x}+2x-5)^{-3}\, dx$

$\frac{1}{2} \cdot \frac{1}{-2}\left(e^{2x} + 2x - 5\right)^{-2} + C$

(iii) Find $\int (6x+3)(x^2+x-5)^{11}\, dx$

$u = x^2 + x - 5$

$u' = 2x + 1$

$3 \cdot \frac{1}{12}(x^2 + x - 5)^{12} + C$

Quiz 6 MTH 111, Spring 2019

KAMYA KANSRA
81881

Ayman Badawi

15/15

QUESTION 1. Find $f'(x)$ and DO NOT SIMPLIFY

2 a) $f(x) = 10(3x^4 + 12x^3 - 10x + 5)^{11}$

$= 10 \cdot 11\,(3x^4 + 12x^3 - 10x + 5)^{10} \cdot (12x^3 + 36x^2 - 10)$

3 b) $f(x) = \sqrt[5]{x^2} + \frac{12}{x^{10}} + 7x - 3$

$= x^{2/5} + 12x^{-10} + 7x - 3$

$= \frac{2}{5}x^{-3/5} - 120x^{-11} + 7$

8/8

3 in c)Given $k(x) = f(2x^2 + x - 16)$. Find $k'(3)$ if $f'(5) = -7$.

$k'(x) = f'(2x^2 + x - 16) \cdot (4x + 1)$

$k'(3) = f'(2 \times 9 + 3 - 16) \cdot (12 + 1)$

$= f'(5) \cdot 13$

$= -7 \cdot 13 = -91$

QUESTION 2. Let $f(x) = x^3 - 6x^2 - 15x + 1$.

3 a) Find the sign of $f'(x)$.

$f'(x) = 3x^2 - 12x - 15$; For critical value: $f'(x) = 0$

$\Rightarrow 0 = 3x^2 - 12x - 15$

$= 3(x^2 - 4x - 5)$; as $3 \neq 0$ $\Rightarrow 0 = x^2 - 4x - 5 \Rightarrow 0 = (x-5)(x+1)$

$\Rightarrow x - 5 = 0$ OR $x + 1 = 0 \Rightarrow x = 5$ OR $x = -1$

sign of $f'(x)$:

At $x = -2$; $f'(-2) = 12 + 24 - 15 = 21 > 0$

At $x = 0$; $f'(0) = -15 < 0$

At $x = 6$; $f'(6) = 3 \times 36 - 72 - 15 = 21 > 0$

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−2 −1 0 5 6

7/7

1 b)By staring at (a) find the critical values.

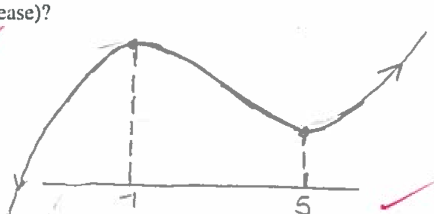
$x = 5$ or $x = -1$

2 c) By staring at (a), for what values of x does $f(x)$ increase (decrease)?

$\therefore f(x)$ increases : $(-\infty, -1) \cup (5, \infty)$

$f(x)$ decreases : $(-1, 5)$

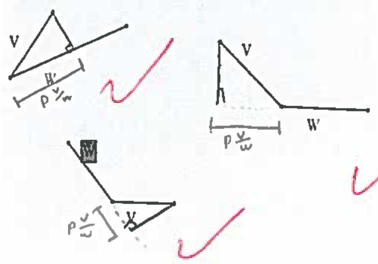
1 d) By staring at (a), sketch $f(x)$ (roughly).



Faculty information

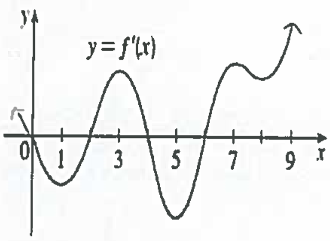
Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: abadawi@aus.edu, www.ayman-badawi.com

QUESTION 12. (4.5 points) Stare at the following picture.



Draw the projection of V over W.

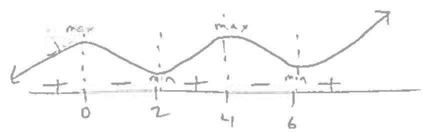
QUESTION 13. (7.5 points) Stare at the following graph of $y = f'(x)$.



critical values = 0, 2, 4, 6

$\nearrow = (-\infty, 0), (2, 4), (6, +\infty)$

$\searrow = (0, 2), (4, 6)$



(i) At what value(s) of x does $f(x)$ have local max.?

at $x = 0$ and $x = 4$

(ii) At what value(s) of x does $f(x)$ have local min.?

at $x = 2$ and $x = 6$

(iii) For what values of x does $f(x)$ increase?

$(-\infty, 0) \cup (2, 4) \cup (6, +\infty)$

(iv) For what values of x does $f(x)$ decrease?

$(0, 2) \cup (4, 6)$

(v) For what values of x will the normal lines have positive slope.

Normal line will have a + slope when the tangent line has − slope

$\therefore$ when the function x is decreasing $\therefore (0, 2) \cup (4, 6)$

QUESTION 14. (5 points) Given $L_1 : x = 2t, y = t + 1, z = 3t$ is perpendicular to $L_2 : x = 4w + 6, y = -2w, z = aw + 1$ and they intersect at a point Q. Find the value of a and find the point Q.

$L_1 : \left.\begin{array}{l} x = 2t \\ y = t + 1 \\ z = 3t \end{array}\right\} t \in \mathbb{R}$ $\quad L_2 : \left.\begin{array}{l} x = 4w + 6 \\ y = -2w \\ z = aw + 1 \end{array}\right\} w \in \mathbb{R}$

$Q = (2, 2, 3)$

$a = -2$

$x = x \quad y = y$

$2t = 4w + 6 \quad | \quad t + 1 = -2w$

$\quad\quad\quad\quad\quad\quad | \quad t + 2w = -1$

$2t - 4w = 6$

$t + 2w = -1$

$t = \dfrac{\begin{vmatrix} 6 & -4 \\ -1 & 2 \end{vmatrix}}{\begin{vmatrix} 2 & -4 \\ 1 & 2 \end{vmatrix}} \quad w = \dfrac{\begin{vmatrix} 2 & 6 \\ 1 & -1 \end{vmatrix}}{\begin{vmatrix} 2 & -4 \\ 1 & 2 \end{vmatrix}}$

$t = 1$

$w = -1$

$x = 2$

$y = 2$

$z = 3$

$z = aw + 1$

$3 = a(-1) + 1$

$3 - 1 = a(-1)$

$2 = a \cdot -1$

$a = 2/{-1} \quad a = -2$

Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: abadawi@aus.edu, www.ayman-badawi.com

Name ________________, ID ________

Krstin Raed

15/15 :)

Quiz I MTH 111, Spring 2019

Ayman Badawi

$CF^2 = \left(\frac{k}{2}\right)^2 - b^2$

$CF^2 = 25 - 9$

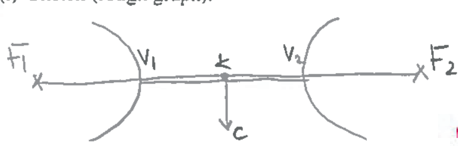
QUESTION 1. Consider the ellipse $\frac{(x+2)^2}{\ } + \frac{(y-1)^2}{\ } = 1$

$(2, 1)$

QUESTION 2. Consider the hyperbola $\frac{(x-3)^2}{9} - \frac{(y+2)^2}{16} = 1$.

2 (i) Sketch (rough graph).

$c = (3, -2)$



1 (ii) Find the heperbola-constant, k

$\left(\frac{k}{2}\right)^2 = 9$ $\qquad \frac{k}{2} = \sqrt{9}$ $\qquad \boxed{k = 3 \times 2 = 6}$

2 (iii) Find all vertices

$V_1 \quad (3-3, -2)$
$(0, -2)$

$V_2 \quad (3+3, -2)$
$(6, -2)$

2 (iv) Find the Foci

$CF^2 = \left(\frac{k}{2}\right)^2 + b^2$

$CF^2 = 9 + 16$

$CF^2 = 25$

$\boxed{CF = 5}$

$F_1 \quad (3-5, -2)$
$(-2, -2)$

$F_2 \quad (3+5, -2)$
$(8, -2)$

Faculty information

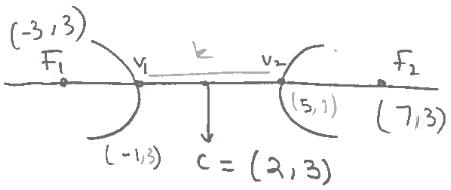
Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: abadawi@aus.edu, www.ayman-badawi.com

QUESTION 6. Consider the hyperbola $\frac{(x-2)^2}{9} - \frac{(y-3)^2}{16} = 1$.

$9 = \left(\frac{k}{2}\right)^2$, $16 = b^2$

a) **(2 points)** Draw the hyperbola, roughly.

under x so right left



b) **(2 points)** Find the hyperbola-constant K.

$\left(\frac{k}{2}\right)^2 = 9$

$\frac{k}{2} = \sqrt{9}$

$k = 3 \times 2$

$\boxed{k = 6}$

c) **(3 points)** Find the two vertices of the hyperbola.

$V_2 = (2+3, 3)$
$(5,3)$

$V_1 = (2-3, 3)$
$(-1,3)$

d) **(3 points)** Find the foci of the hyperbola.

$F_1 = (2-5, 3)$ $(-3,3)$

$F_2 = (2+5, 3)$ $(7,3)$

$CF^2 = \left(\frac{k}{2}\right)^2 + b^2$

$CF^2 = 9 + 16$

$= 25$

$\boxed{CF = 5}$

QUESTION 7. (4 points) Given two lines $L_1: x = t+1, y = 2t+4, z = -5t+3$ ($t \in R$) and $L_2: x = 2w \ldots$

Name Haya Sujaa , ID g00082558

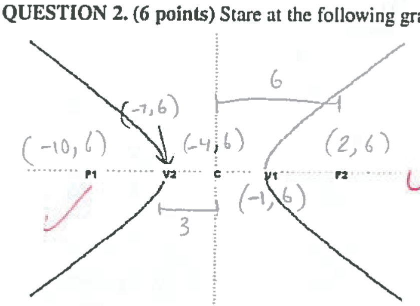
MTH 111 Math for Architects Spring 2019, 1–5

Final Exam, MTH 111, Spring 2019

Ayman Badawi

$$Score = \frac{75}{78}$$

QUESTION 2. (6 points) Stare at the following graph.



Given $c = (-4, 6)$, $|cv2| = 3$, and $F2 = (2, 6)$.

(i) Find $v1 = (-1, 6)$ $F1 = (-10, 6)$, $v2 = (-7, 6)$, and the hyperbola-constant $k = 6$

$|CF_1| = \sqrt{(k/2)^2 + b^2} = 6$

$\sqrt{9 + b^2} = 6$

$9 + b^2 = 36$

$b^2 = 36 - 9$

$b^2 = 27$

(ii) Find the equation of the hyperbola

$$\frac{(x+4)^2}{9} - \frac{(y-6)^2}{27} = 1$$

Quiz II: MTH 111, Spring 2018

Ayman Badawi

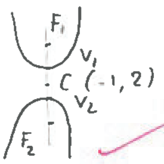
$$\frac{(y-y_0)^2}{(\frac{k}{2})^2} - \frac{(x-x_0)^2}{b^2} = 1$$

$C(-1,2)$

15/15

QUESTION 1. Consider the hyperbola given by $\frac{(y-2)^2}{9} - \frac{(x+1)^2}{16} = 1$

(i) Sketch, roughly.



$(\frac{k}{2})^2 = 9 \Rightarrow \frac{k}{2} = 3$

$\Rightarrow k = 6$

$|CF_1| = \sqrt{16+9}$

$= 5$

(ii) Find the ellipse-constant K.

$\boxed{k=6}$

(iii) Find the foci.

$F_1(-1, 2+5) \Rightarrow \boxed{F_1(-1,7)}$ $F_2(-1, 2-5) \Rightarrow \boxed{F_2(-1,-3)}$

(iv) Find all vertices.

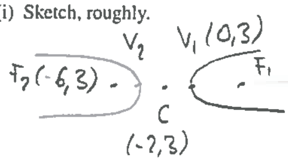
$V_1(-1, 2+3) \Rightarrow \boxed{V_1(-1,5)}$

$V_2(-1, 2-3) \Rightarrow \boxed{V_2(-1,-1)}$

QUESTION 2. Given a ~~parabola~~ hyperbola centered at $(-2, 3)$ such that one of the vertices is $(0, 3)$ and one of the foci is (-6, 3).

$$\frac{(x-x_0)^2}{(\frac{k}{2})^2} - \frac{(y-y_0)^2}{b^2} = 1$$

(i) Sketch, roughly.



$|CF_1| = \sqrt{(\frac{k}{2})^2 + b^2}$ $|CF_1| = 4$

$b^2 = |CF_1|^2 - (\frac{k}{2})^2$

$b^2 = 4^2 - 4 \Rightarrow b^2 = 12$

(ii) Find the constant K.

$\frac{k}{2} = |C_1V_2| = 2 \Rightarrow \boxed{k=4}$

(iii) Find the second focus and the second vertex.

$\boxed{V_2(-4,3)}$

$\boxed{F_1(2,3)}$

(iv) Write down the equation of the hyperbola.

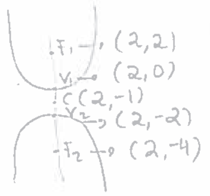
$$\frac{(x+2)^2}{4} - \frac{(y-3)^2}{12} = 1$$

Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: abadawi@aus.edu, www.ayman-badawi.com

QUESTION 6. Consider the hyperbola $(y+1)^2 - \frac{(x-2)^2}{8} = 1$.

a) **(2 points)** Draw the hyperbola, roughly



$|CF_1| = \sqrt{1+8} = 3$

b) **(2 points)** Find the hyperbola-constant K.

$\left(\frac{K}{2}\right)^2 = 1$

$\frac{K}{2} = 1 \Rightarrow \boxed{K = 2}$ ✓

c)**(3 points)** Find the two vertices of the hyperbola.

$\boxed{V_1\ (2,0)}$

$\boxed{V_2\ (2,-2)}$ ✓

d) **(3 points)** Find the foci of the hyperbola.

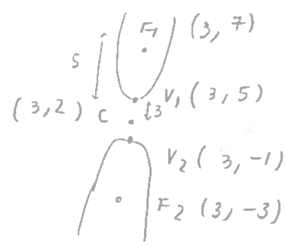
$\boxed{F_1\ (2,2)}$

$\boxed{F_2\ (2,-4)}$ ✓

QUESTION 4. (8 points)

Draw roughly the hyperbola $\frac{(y-2)^2}{9} - \frac{(x-3)^2}{16} = 1$. Then find

positive $y \Rightarrow$



a) The hyperbola-constant K.

$\left(\frac{K}{2}\right)^2 = 9 \rightarrow \frac{K}{2} = 3$

$\boxed{K = 6}$

b) The two vertices of the hyperbola.

V_1 (3, 5)

V_2 (3, -1)

c) The foci of the hyperbola. $|CF_1| = \sqrt{9 + 16} = 5$

F_1 (3, 7)

F_2 (3, -3)

$4 \times 3 = 12$ $2\sqrt{3}$

QUESTION 7. (8 points) First draw the hyperbola $\frac{y^2}{4} - \frac{(x-1)^2}{12} = 1$. Then find

a) The hyperbola-constant K.

$\left(\frac{K}{2}\right)^2 = 4$ $\frac{K}{2} = 2$ $\underline{\underline{K = 4}}$

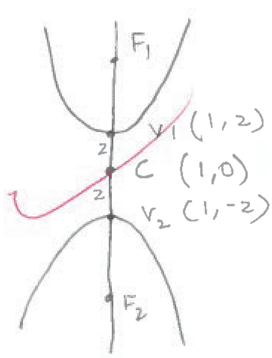
b) The two vertices of the hyperbola.

$V_1(1,2)$ //

$V_2(1,-2)$ //

8/8

$b^2 = 12$
$b = \sqrt{12}$
$= 2\sqrt{3}$



c) The foci of the hyperbola.

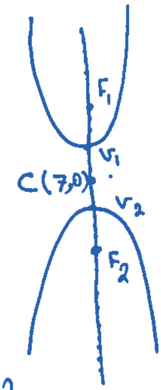
$F_1C = \sqrt{b^2 + (4/2)^2} = \sqrt{\cancel{16+144}} = \cancel{128\ 64}\ \sqrt{12+4} = \sqrt{16}$
$= 4$

$F_1(1,4)$ //

$F_2(1,-4)$ //

QUESTION 3. Given the hyperbola $\frac{y^2}{4} - \frac{(x-7)^2}{5} = 1$

(i) Roughly, Sketch the graph of the given hyperbola.



(ii) Find the two vertices, V_1 and V_2

$\left(\frac{K}{2}\right)^2 = 4 \rightarrow \frac{K}{2} = 2 \rightarrow \boxed{K = 4} \rightarrow |V_1V_2| \rightarrow |CV_1| = |CV_2| = 2$

$\boxed{V_1 = (7,2)}$ / $\boxed{V_2 = (7,-2)}$

(iii) Find the two Foci: F_1, F_2.

$|CF_1| = |CF_2| = \sqrt{\left(\frac{K}{2}\right)^2 + b^2} = \sqrt{5+4} = \sqrt{9} = \boxed{3}$

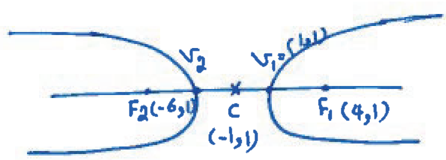
$\boxed{F_1 = (7,3)}$ / $\boxed{F_2 = (7,-3)}$

QUESTION 4. Given $F_1 = (4,1)$, $F_2 = (-6,1)$ are the foci of a hyperbola and $V_1 = (1,1)$ is one of the vertices.

(i) Find the hyperbola-constant K.

$C = \left(\frac{-6+4}{2}, 1\right) = (-1,1)$

$\frac{K}{2} = 2 \rightarrow \boxed{K = 4}$



(ii) Find the second vertex of the hyperbola.

$|CV_2| = K/2 \rightarrow \boxed{V_2 = (-3,1)}$

(iii) Find the equation of the hyperbola.

$|CF_1| = |CF_2| = 5 = \sqrt{\left(\frac{K}{2}\right)^2 + b^2} \rightarrow 5 = \sqrt{4 + b^2} \rightarrow 25 = 4 + b^2 \rightarrow \boxed{b^2 = 21}$

equation: $\boxed{\frac{(x+1)^2}{4} - \frac{(y-1)^2}{21} = 1}$

each of the conic sections are summarized below.

EQUATIONS OF CONIC SECTIONS

Conic Section	**Characteristic**	**Examp**
Parabola	Either $A = 0$ or $C = 0$, but not both.	$y =$ $x =$
Circle	$A = C \neq 0$	$x^2 +$
Ellipse	$A \neq C, AC > 0$	$\frac{x^2}{16}$
Hyperbola	$AC < 0$	x^1 -

The following chart summarizes our work with conic sections.

In order lo recognize the type of graph that a given conic section has, it is sometimes necessary to transform the equation into a more familiar form, as shown in the next examples.

Example 1

DETERMINING THE TYPE OF A CONIC SECTION FROM ITS EQUATION

Decide on the type of conic section represented by each of the following equations, and sketch each graph.

(a) $25y^2 - 4x^2 = 100$.

Divide each side by 100 to get

$$\frac{y^2}{4} - \frac{x^2}{25} = 1.$$

This is a hyperbola centered at the origin, with foci on the y-axis, and y-intercepts 2 and -2 The points $(5,2)$ $(5,-2)$, (-52) $(-5,-2)$ determine the fundamental rectangle. The diagonals of the rectangle are the asymptotes, and their equations are

$$y = \frac{\pm 2}{5}\ x.$$

The graph is shown in Figure 3.44

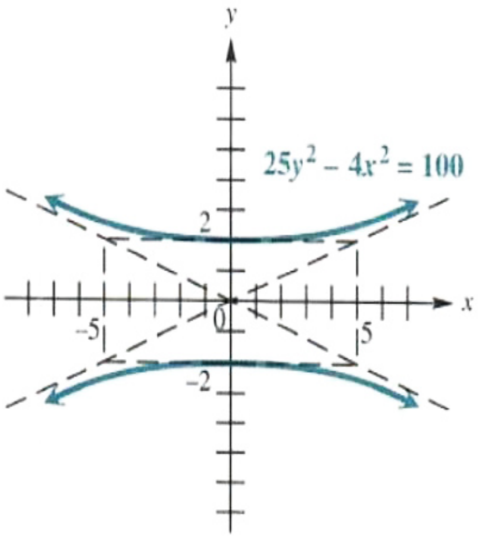


Figure 3.44

(b) $x^2 = 25 + 5\ y^2$

Rewriting the equation as

$$x^2 - 5\ y^2 = 25$$

or $$\frac{x^2}{25} - \frac{y^2}{5} = 1$$

shows that the equation represents a hyperbola centered at the origin, with asymptotes

$$y = \frac{\pm b}{a} x$$

or $$y = \frac{\pm\sqrt{5}}{5} x$$

The x-intercepts are ± 5; the graph is shown in Figure 3.45.

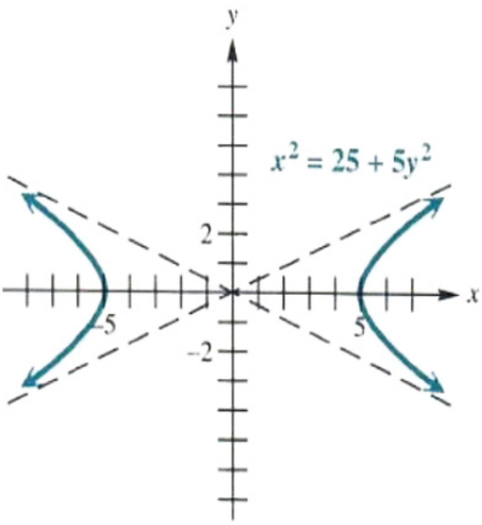


Figure 3.45

(c) $4x^2 - 16x + 9y^2 + 54y = -61$

Since the coefficients of the x^2 and y^2 terms are unequal and both positive, this equation might represent an ellipse. (It might also represent a single point or no points at all.) To find out, complete the square on x and y.

$4(x^2 - 4x) + 9(y^2 + 6y) = -61$
Factor out a 4; Factor out a 9.

$4(x^2 - 4x + 4 - 4) + 9(y^2 + 6y + 9 - 9)$
Add and subtract the same quantity.

$4(x^2 - 4x + 4) - 16 + 9(y^2 + 6y + 9) - 8$

Regroup and distribute.

$$4\,(x-2)^2+9\,(y+3)^2=36$$
Add 97 and factor.

$$\frac{(x-2)^2}{9}+\frac{(y+3)^2}{4}=1$$ Divide by 36.

This equation represents an ellipse having center at $(2,-3)$ and graph as shown in Figure 3.46.

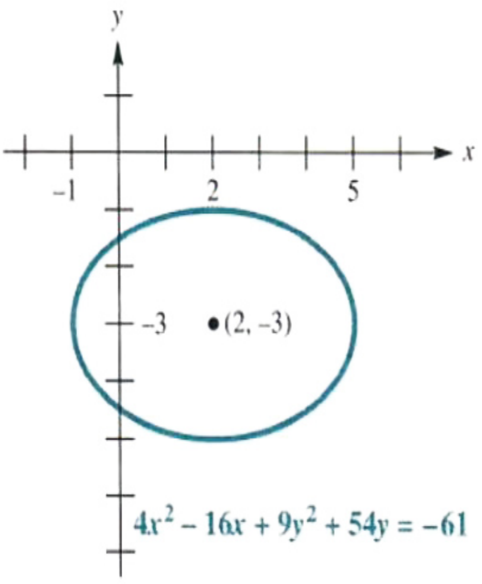


Figure 3.46

(d) $x^2-8\,x+y^2+10\,y=-41$

Complete the square on both x and y, as follows

$$\left(x^2-8\,x+16\right)+\left(y^2+10\,y+25\right)=-41$$

$$(x-4)^2+(y+5)^2=0.$$

This result shows that the equation is that of a circle of radius 0; that is, the point $(4,-5)$. Had a negative number been obtained on the right (instead of 0), the equation would have represented no points at all, and there would be no graph.

(e) $x^2 - 6x + 8y - 7 = 0$

Since only one variable is squared (x, and not y), the equation represents a parabola. Rearrange the terms to get the term with y (the variable that is not squared) alone on one side. Then complete the square on the other side of the equation.

$8y = -x^2 + 6x + 7$

$8y = -\left(x^2 - 6x\right) + 7$ Regroup and factor out -1.

$8y = -\left(x^2 - 6x + 9\right) + 7 + 9$ Add 0 in the form $-9 + 9$.

$8y = -(x-3)^2 + 16$ Factor.

$y = \frac{-1}{8}(x-3)^2 + 2$ Multiply both sides by $\frac{1}{8}$.

The parabola has vertex at $(3, 2)$, and opens downward, as shown in Figure 3.47.

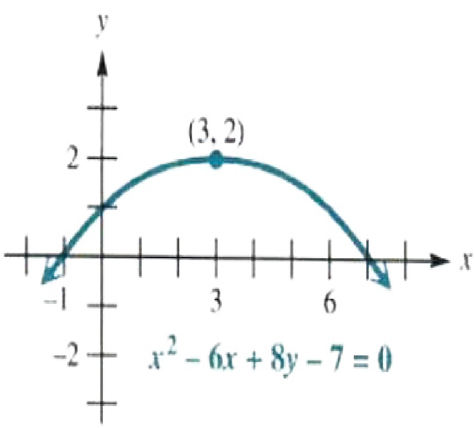


Figure 3.47

CAUTION The next example is designed to serve as a warning about a very common error.

Example 2

DETERMINING THE TYPE OF A CONIC SECTION FROM ITS EQUATION

Graph $4y^2 - 16y - 9x^2 + 18x = -43$.

Complete the square on x and on y.

$$4\left(y^2 - 4y\right) - 9\left(x^2 - 2x\right) = -43$$

$$4\left(y^2 - 4y + 4\right) - 9\left(x^2 - 2x + 1\right) = -43 -$$

$$4(y-2)^2 - 9(x-1)^2 = -36$$

Because of the -36, it is very tempting to say that this equation does not have a graph. However, the minus sign in the middle on the left shows that the graph is that of a hyperbola. Dividing through by --36 and rearranging terms gives

$$\frac{(\_\,x)^2}{4} - \frac{(y-2)^2}{9} = 1$$

a hyperbola centered at $(1, 2)$, with graph as shown in Figure 3.48.

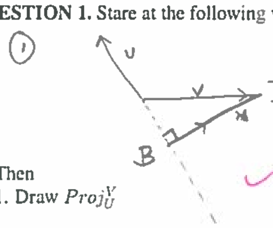
Quiz III: MTH 111, Spring 2018

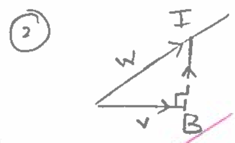
Ayman Badawi

Archa Alukka
75223

14.5 / 15

QUESTION 1. Stare at the following vectors.





Projections are IB

Then
1. Draw $Proj_U^V$

3

2. Draw $Proj_V^W$

QUESTION 2. Given $(1,2,4)$ and $(7,-4,3)$ lie on a line L.

a) Find a parametric equations of L.

$D = (7-1, -4-2, 3-4) = (6,-6,-1)$

$(1,2,4)$ and $(6,-6,-1)$

$(1+6L, 2-6L, 4-L)$

$X = 1+6L \quad Y = 2-6L \quad Z = 4-L$

b) Find a symmetric equations of L.

$6L = 2-Y$

3

$L: \dfrac{X-1}{6} = \dfrac{2-Y}{6} = \dfrac{4-Z}{1}$

c) Does the point $(1,4,8)$ lie on the line L.

- $\dfrac{X-1}{6} = \dfrac{1-1}{6} = 0$
- $\dfrac{4-8}{6} = \dfrac{-2}{6} = -\dfrac{1}{3}$
- $4-8 = -4$

1

$\therefore$ It doesn't lie on the line L because the values varies when subtituted.

QUESTION 3. Let $V =< 1,1,2 >$ and $W =< -2,2,-1 >$. Find $Proj_V^W$. Will it be in the direction of V ?

3.5

$Proj_V W = \dfrac{U.W}{|V|^2} \times V \qquad |V| = \sqrt{1^2+1^2+2^2} = \sqrt{1+1+4} = \sqrt{6}$

$|V|^2 = 6$

$U.W = 1(-2)+1(2)+2(-1) = -2+2-2 = -2$

$Proj_V W = \dfrac{-2}{6} \times (1,1,2) = \left(\dfrac{-2}{6}, \dfrac{-2}{6}, \dfrac{-4}{6}\right)$

Yes, it will be in the direction of V

No

opposite

Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: abadawi@aus.edu, www.ayman-badawi.com

Name Krstin Raed , ID 78656

MTH 111 Math for Architects Spring 2019, 1–6

Exam I: MTH 111, Spring 2019

$F = V \times W$

Ayman Badawi

Points = $\frac{87}{87}$

QUESTION 1. b) **(4 points)** Given $A = (6, 10)$, $B = (-7, 3)$, and $C = (-4, -2)$ are the vertices of a triangle. Find the area of the triangle ABC.

Area of the triangle ABC = $\frac{1}{2} |AB \times AC|$

$AB = \langle -13, -7 \rangle$ (B−A)

$AC = \langle -10, -12 \rangle$ (C−A)

$AB \times AC = \begin{vmatrix} i & j & k \\ -13 & -7 & 0 \\ -10 & -12 & 0 \end{vmatrix} = 0i - 0j + 86k = 86$

Area of $\Delta ABC = \frac{1}{2} 86 = \boxed{43 \text{ units}^2}$

c) **(3 points)** Find a vector F that is perpendicular to both vectors $V = < 2, 6, -3 >$ and $W = < 5, -4, 1 >$ such that $|F| = 111$.

$F = V \times W = \begin{vmatrix} i & j & k \\ 2 & 6 & -3 \\ 5 & -4 & 1 \end{vmatrix} = -6i - 17j - 38k$

$|F| = 111 = \frac{111}{|F|} F$

$= \boxed{\frac{111}{42} \langle -6, -17, -38 \rangle}$

QUESTION 2. a) **(4 points)** The line $L_1 : x = -2t - 3, y = -3t + 3, z = 4t - 2$ ($t \in R$) intersects the line $L_2 : x = 2w - 13, y = 4w - 15, z = 4w - 6$ ($w \in R$) in a point Q. Find Q.

$L_1 : x = -2t - 3$
$y = -3t + 3$
$z = 4t - 2$

$L_2 : x = 2w - 13$
$y = 4w - 15$
$z = 4w - 6$

use substitution method

find pt of intersection: $-2t - 3 = 2w - 13$

$\frac{-2t}{-2} = \frac{2w - 13 + 3}{-2}$

$\boxed{t = -w + 5}$

sub in second equ

$-3(-w + 5) + 3 = 4w - 15$

$3w - 15 + 3 = 4w - 15$

$4w - 3w = -15 + 15 + 3$

$\boxed{1w = 3}$

$t = -3 + 5$

$\boxed{t = 2}$

• now sub in each line to get intersection pt

$-2(2) - 3 = 2(3) - 13$

$\boxed{-7 = -7}$

$-3(2) + 3 = 4(3) - 15$

$\boxed{-3 = -3}$

$4(2) - 2 = 4(3) - 6$

$\boxed{6 = 6}$

$\boxed{\text{Intersection pt} = Q = (-7, -3, 6)}$

b) **(2 points)** Are the lines in (a) perpendicular? Explain

$D_1 = \langle -2, -3, 4 \rangle$

$D_2 = \langle 2, 4, 4 \rangle$

$D_1 \cdot D_2 = (-2 \times 2) + (-3 \times 4) + (4 \times 4)$
$= 0$

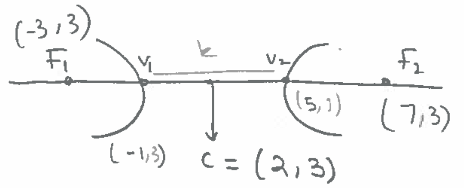
So they are perpendicular because their dot product is zero & they intersect

QUESTION 6. Consider the hyperbola $\frac{(x-2)^2}{9} - \frac{(y-3)^2}{16} = 1$.

$9 = \left(\frac{k}{2}\right)^2 \qquad 16 = b^2$

a) **(2 points)** Draw the hyperbola, roughly.

under x so right left



b) **(2 points)** Find the hyperbola-constant K.

$\left(\frac{k}{2}\right)^2 = 9$

$\frac{k}{2} = \sqrt{9}$

$k = 3 \times 2$

$\boxed{k = 6}$

c) **(3 points)** Find the two vertices of the hyperbola.

$V_2 = (2+3, 3)$
$(5,3)$

$V_1 = (2-3, 3)$
$(-1,3)$

d) **(3 points)** Find the foci of the hyperbola.

$F_1 = (2-5, 3) \quad (-3,3)$

$F_2 = (2+5, 3) \quad (7,3)$

$CF^2 = \left(\frac{k}{2}\right)^2 + b^2$

$CF^2 = 9 + 16$

$= 25$

$\boxed{CF = 5}$

QUESTION 7. (4 points) Given two lines $L_1: x = t+1, y = 2t+4, z = -5t+3$ ($t \in R$) and $L_2: x = 2w-1, y = 4w+1, z = -10w+13$ ($w \in R$). Is L_1 parallel to L_2? Explain (show the work)

• 2 lines are // if they have cst & they do not intersect

L_1: $x = t+1$
$y = 2t+4$
$z = -5t+3$

L_2: $x = 2w-1$
$y = 4w+1$
$z = -10w+13$

$D_1 \langle 1, 2, -5 \rangle$ $\qquad$ $D_2 \langle 2, 4, -10 \rangle$

$1 = c2$ $\qquad c = \frac{1}{2}$
$2 = c4$ $\qquad c = \frac{1}{2}$
$-5 = c(-10)$ $\qquad c = \frac{1}{2}$

they have a cst

take $t = 0$

$1 = 2w - 1$
$4 = 4w + 1$
$3 = -10w + 13$

$2w = 2$
$w = 1$
$4w = 4-1$
$w = \frac{3}{4}$
$10w = 13-3$
$10w = 10$
$w = 1$

$2w = 2$
$\boxed{w = 1}$
$4 - 1 = 4w$
$3 = 4w$
$\boxed{w = \frac{3}{4}}$
$3 - 13 = -10w$
$-10 = -10w$

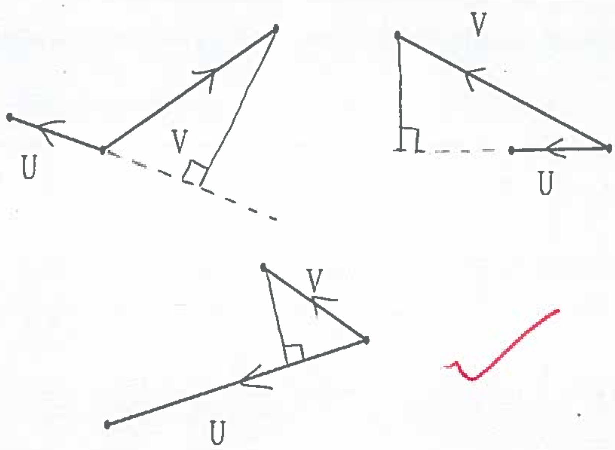
they do not intersect

$L_1 \mathbin{/\!/} L_2$

QUESTION 8. (6 points)

proj_U^V

Stare at the below. Then find Projection of V over U



QUESTION 9. (4 points) Find the equation of the plane that contains the points $Q_1 = (4,4,0)$. $Q_2 = (0,2,6)$ and $Q_3 = (4,0,8)$.

$N = \overrightarrow{Q_1Q_2} \times \overrightarrow{Q_1Q_3}$

$\langle -4, -2, 6 \rangle \times \langle 0, -4, 8 \rangle$

$$\begin{vmatrix} i & j & k \\ -4 & -2 & 6 \\ 0 & -4 & 8 \end{vmatrix} = 8i + 32j + 16k$$

$\langle 8, 32, 16 \rangle$

choose a pt

$Q_1 = (4,4,0)$

$8(x-4) + 32(y-4) + 16(z-0) = 0$

$\boxed{8(x-4) + 32(y-4) + 16z = 0}$

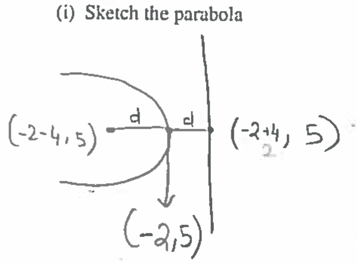
QUESTION 10. (6 points) Consider the parabola $-16(x+2) = (y-5)^2$.

(i) Sketch the parabola

$4d = -16$

$d = -4$

& before x so its left



(ii) Find the equation of the directrix line

$X = -2 + 4$

$\boxed{X = 2}$

(iii) Find the focus point.

Focus $= (-2-4, 5)$

$(-6, 5)$

QUESTION 2. a) **(4 points)** Does the line $L_1 : x = 5t - 20, y = -t + 3, z = 3t - 27$ ($t \in R$) intersect the line $L_2 : x = -2w + 20, y = -4w - 5, z = 2w - 3$ ($w \in R$)? If yes find the intersection point Q.

$L_1: \begin{cases} x = 5t - 20 \\ y = -t + 3 \\ z = 3t - 27 \end{cases}$ $L_2: \begin{cases} x = -2w + 20 \\ y = -4w - 5 \\ z = 2w - 3 \end{cases}$

$5t - 20 = -2w + 20 \Rightarrow 5t + 2w = 40$
$-t + 3 = -4w - 5 \Rightarrow -t + 4w = -8$

$t = 8 \quad w = 0$

check for z:
$z = 3t - 27 = 3(8) - 27 = -3$
$z = 2w - 3 = 2(0) - 3 = -3$
} they are equal $\Rightarrow$ L_1 and L_2 intersect

The point of intersection
$x = 2w + 20 = 2(0) + 20 = 20$
$y = -4w - 5 = -4(0) - 5 = -5$
$z = 2w - 3 = 2(0) - 3 = -3$

point Q intersection is (20, -5, -3)

✓

b) **(2 points)** Are the lines in (a) perpendicular? Explain Yes.

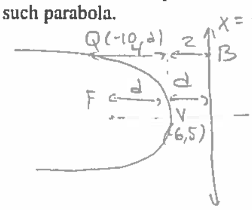
$D_1 = \langle 5, -1, 3 \rangle \quad D_2 = \langle -2, -4, 2 \rangle$

$D_1 \cdot D_2 = 5(-2) - 1(-4) + 3(2) = 0$ ✓

dot product $= 0 \Rightarrow$ They are perpendicular.

QUESTION 3. Given $x = -4$ is the directrix of of a parabola that has the point $(-6, 5)$ as its vertex point.

a) **(2 points)** Roughly, sketch such parabola.



$|d| = 2$

b) **(4 points)** Find the equation of the parabola

$4d(x - x_0) = (y - y_0)^2$
$-4(2)(x + 6) = (y - 5)^2$
$\boxed{-8(x + 6) = (y - 5)^2}$ ✓

c) **(2 points)** Find the focus of the parabola, say F.

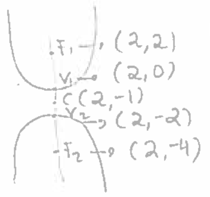
$\boxed{F(-8, 5)}$ ✓

d) **(2 points)** Given $Q = (-10, b)$ is a point on the curve of the parabola. Find $|QF|$ (HINT: You should know how to do this QUICKLY!, you do not need the value of b)

$|QL| = |QB| = |QF| = \boxed{6}$ ✓

QUESTION 6. Consider the hyperbola $(y+1)^2 - \frac{(x-2)^2}{8} = 1$.

a) **(2 points)** Draw the hyperbola, roughly



$|CF_1| = \sqrt{1+8} = 3$

b) **(2 points)** Find the hyperbola-constant K.

$(\frac{K}{2})^2 = 1$

$\frac{K}{2} = 1 \Rightarrow \boxed{K = 2}$ ✓

c) **(3 points)** Find the two vertices of the hyperbola.

$\boxed{V_1\ (2,0)}$

$\boxed{V_2\ (2,-2)}$ ✓

d) **(3 points)** Find the foci of the hyperbola.

$\boxed{F_1\ (2,2)}$

$\boxed{F_2\ (2,-4)}$ ✓

QUESTION 7. Given two lines $L_1: x = t+1, y = 2t+4, z = -5t+3$ and $L_2: x = 2w+7, y = 4w+16, z = -10w-27$.

(i) **(3 points)** Find the symmetric equation of L_1.

$\boxed{x-1 = \frac{y-4}{2} = \frac{-z+3}{5}}$ ✓

(ii) **(3 points)** Is D_1 parallel to D_2? (note that D_1 is the directional vector of L_1 and D_2 is the directional vector of L_2) Show the work

$D_1 <1,2,-5>$

$D_2 <2,4,-10>$

$D_1 = c\,D_2$

$<1,2,-5> = c<2,4,-10>$

$c = \frac{1}{2}$ ✓

$D_1 = \frac{1}{2} D_2 \Rightarrow$ They are parallel

(iii) **(2 points)** Is L_1 parallel to L_2? Explain (show the work)

Take $t=0 \rightarrow (1,4,3)$

check if $(1,4,3) \in L_2$ ✓

$1 = 2w+7 \Rightarrow w = -3$

$4 = 4w+16 \Rightarrow w = -3$

$3 = -10w-27 \Rightarrow w = -3$

$\Rightarrow$ it $\in$ to L_2.

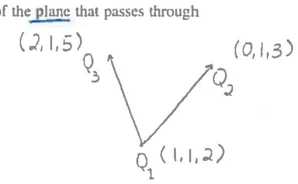
$\Rightarrow L_1$ and L_2 intersect and they are NOT parallel. They are collinear (same line/ on top of each other)

Quiz V MTH 111, Spring 2019

Ayman Badawi

15/15

7/7 **QUESTION 1.** Let $Q_1 = (1,1,2), Q_2 = (0,1,3), Q_3 = (2,1,5)$. Find the equation of the plane that passes through $Q_1, Q_2, Q3$.



$N \perp$ Plane

$N = \overrightarrow{Q_1Q_2} \times \overrightarrow{Q_1Q_3}$

$\overrightarrow{Q_1Q_2} = \langle 0-1, 1-1, 3-2 \rangle = \langle -1, 0, 1 \rangle$ ✓1

$\overrightarrow{Q_1Q_3} = \langle 2-1, 1-1, 5-2 \rangle = \langle 1, 0, 3 \rangle$ ✓1

$\overrightarrow{Q_1Q_2} \times \overrightarrow{Q_1Q_3} = \begin{vmatrix} i & j & k \\ -1 & 0 & 1 \\ 1 & 0 & 3 \end{vmatrix} = (\ 0\ )i - (-3-1)j + (\ 0\ )k = 4j = \langle 0, 4, 0 \rangle$ ✓3

choose Q_1 & a random point

$w = (x, y, z)$

$Q_1 = (1, 1, 2)$

$\overrightarrow{Q_1w} = (x-1, y-1, z-2)$

$N \cdot \overrightarrow{Q_1w} = \langle 0, 4, 0 \rangle \cdot \langle x-1, y-1, z-2 \rangle = 0$

$0(x-1) + 4(y-1) + 0(z-2) = 0$

$4(y-1) = 0$ ✓2

Haya

QUESTION 2. (i) **(6 points)** Does the line $L : x = 2t + 1, y = 5t - 1, z = -2t + 3$ lie entirely inside the plane $x + 2y + z = 23$? If not, does it intersect the plane? If yes, then find the intersection point.

it doesn't lie entirely on the plane but intersects it at $(5.2, 9.5, -1.2)$

$L : \left.\begin{aligned} x &= 2t+1 \\ y &= 5t-1 \\ z &= -2t+3 \end{aligned}\right\} t \in \mathbb{R}$

$P \Rightarrow x + 2y + z = 23$

$P(L) \Rightarrow (2t+1) + 2(5t-1) + (-2t+3) = 23 \leftarrow 1$

$\cancel{2t} + 1 + 10t - 2 - \cancel{2t} + 3 = 23$

$10t + 2 = 23$

$10t = 21$

$t = 21/10$

$= 2.1$

$x = 2(2.1) + 1 = 5.2$

$2) \rightarrow y = 5(2.1) - 1 = 9.5$

$z = -2(2.1) + 3 = -1.2$

$Q = (5.2, 9.5, -1.2)$

the point of intersection is $(5.2, 9.5, -1.2) \leftarrow 3$

(ii) **(4 points)** Given $N = < -2, 3, 2 >$ is perpendicular to the plane P and the point $(-1, 4, 2)$ lies inside the plane P. Find the equation of the plane P.

$N_x(x - P_x) + N_y(y - P_y) + N_z(z - P_z) = 0$

$-2(x+1) + 3(y-4) + 2(z-2) = 0 \Leftrightarrow$ plane.

1

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QUESTION 9. (5 points). Can we draw the entire line $L: x = 2t, y = -3t + 1, z = 11t + 4$ inside the plane $2x - 6y - 2z = 20$? EXPLAIN

$N_{plane} \cdot D_{line}$ must $= 0$

$N = \langle 2, -6, -2 \rangle$

$D = \langle 2, -3, 11 \rangle$

$N \cdot D = 4 + 18 - 22$

$= 0$ //

yes the line can be entirely drawn on the plane because the dot product of the normal and directional vector is 0

NO

take a point on L and check if the point lies in the plane or not

2

Name ________, ID ________

Quiz 4 ~~HW 9~~: MTH 111, Spring 2018

Ayman Badawi

$\frac{15}{15}$

QUESTION 1. a) Find the equation of the plane that contains the points $Q_1 = (0, 1, 1), Q_2 = (0, 2, 3), Q_3 = (1, 3, 2)$.

$\overrightarrow{Q_1Q_2}$: $(0,2,3) - (0,1,1) \longrightarrow \langle 0,1,2 \rangle$

$\overrightarrow{Q_1Q_3}$: $(1,3,2) - (0,1,1) \longrightarrow \langle 1,2,1 \rangle$

$\overrightarrow{Q_1Q_2} \cdot \overrightarrow{Q_1Q_3} = \langle N \rangle = i[(1 \times 1) - (2 \cdot 2)] - j[(0 \cdot 1) - (2 \cdot 1)] + K[(0 \cdot 2) - (1 \cdot 1)]$

$$\begin{matrix} i & j & K \\ 0 & 1 & 2 \\ 1 & 2 & 1 \end{matrix}$$

$-3i + 2j - K$

$\langle N \rangle$ $\langle -3, 2, -1 \rangle$

5

equation: $\boxed{-3x + 2(y-2) - 1(z-1) = 0}$

c) Given a plane $P: 5x - 7y + z = 21$ Can we draw the vector $V = \langle -4, -3, -1 \rangle$ inside the plane P? explain

3

$N \langle 5, -7, 1 \rangle$ $\quad N \cdot V = 0 = \perp \longrightarrow$ so inside plane

$V \langle -4, -3, -1 \rangle$ $\quad (5 \cdot -4) + (-7 \cdot -3) + (1 \cdot -1) = -20 + 21 - 1$

$= 0$

Exam II: MTH 111, Spring 2018

Ayman Badawi

Points =

55

QUESTION 2. (i) **(3 points)** What can you say about the line $L: x = 2t+1, y = t-1, z = -2t+3$ and the plane $x + 2y + z = 16$? (i.e., Doe L lie inside the plane? Does L intersect the plane exactly in one point? or neither?

L: $x = 2t+1$
$y = t-1$
$z = -2t+3$

P: $x + 2y + z = 16$

$(2t+1) + 2(t-1) - 2t + 3 = 16$
$2t + 1 + 2t - 2 - 2t + 3 = 16$
$2t = 14 \Rightarrow t = 14/2 \Rightarrow \boxed{t = 7}$

x: $2(7)+1 = 15$
y: $7 - 1 = 6$
z: $-2(7)+3 = -11$

$\boxed{Q: \text{intersection point}: (15, 6, -11)}$

(ii) **(4 points)** Given $N = <-2, 3, 2>$ is perpendicular to the plane P and the point $(-1, 4, 2)$ lies inside the plane P. Find the equation of the plane P.

$N = <-2, 3, 2> \perp P$ at $Q(-1, 4, 2)$

Find eqn → Directional vector
→ point Q

P: $-2(x+1) + 3(y-4) + 2(z-2) = 0$

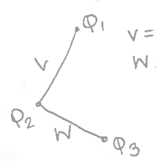
P: $-2x - 2 + 3y - 12 + 2z - 4 = 0$

$\boxed{P: -2x + 3y + 2z = 18}$

(iii) **(6 points)** Find the equation of the plane that contains the points $Q_1 = (4, 4, 0)$, $Q_2 = (0, 2, 6)$ and $Q_3 = (4, 0, 8)$.

Eqn of plane → directional vector and point Q_1

$Q_1: (4, 4, 0)$
$Q_2: (0, 2, 6)$
$Q_3: (4, 0, 8)$



$V = Q_1Q_2 = <4, 2, -6>$
$W = Q_3Q_2 = <4, -2, 2>$

$$V \times W = \begin{vmatrix} i & j & k \\ 4 & 2 & -6 \\ 4 & -2 & 2 \end{vmatrix} = \begin{vmatrix} 2 & -6 \\ -2 & 2 \end{vmatrix}, -\begin{vmatrix} 4 & -6 \\ 4 & 2 \end{vmatrix}, \begin{vmatrix} 4 & 2 \\ 4 & -2 \end{vmatrix}$$

$= <4 - 12, -(8 + 24), -8 - 8>$
$= <-8, -32, -16>$

P: $-8(x-4) - 32(y-4) - 16(z+0) = 0$

P: $-8x + 32 - 32y + 128 - 16z = 0$

$\boxed{P: -8x - 32y - 16z = -160}$

Exam II: MTH 111, Fall 2017

Ayman Badawi

Points = $\frac{47}{47}$

Haya Alshamsi

QUESTION 2. (i) (3 points) Can we draw the vector $v = <3, -5, 2>$ inside the plane $x - 4y - 11z = 7$? explain

$V = <3, -5, 2>$ $\quad N \cdot V = 3(1) - 5(-4) + 2(-11)$

$N = <1, -4, -11>$ $\quad N \cdot V = 3 + 20 - 22 = 1 \neq 0$

NO. The two vectors are not perpendicular, hence v can't be drawn inside the plane.

(ii) (4 points) Given $N = <4, 6, 2>$ is perpendicular to the plane P and the point $(4, 1, 1)$ lies inside the plane P. Find the equation of the plane P.

$N = <4, 6, 2>$
$<a, b, c>$
$Q(4, 1, 1)$
$Q(x_0, y_0, z_0)$

$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$

$4(x - 4) + 6(y - 1) + 2(z - 1) = 0$

$4x - 16 + 6y - 6 + 2z - 2 = 0$

$\boxed{4x + 6y + 2z = 24}$

(iii) (6 points)Find the equation of the plane that contains the points $Q_1 = (1, 1, 4)$, $Q_2 = (2, 3, 6)$ and $Q_3 = (1, 1, 8)$.

$Q_1(1, 1, 4)$
$Q_2(2, 3, 6)$
$Q_3(1, 1, 8)$

$\overrightarrow{Q_1Q_2} = <1, 2, 2>$

$\overrightarrow{Q_1Q_3} = <0, 0, 4>$

$\vec{N} = \overrightarrow{Q_1Q_2} \times \overrightarrow{Q_1Q_3} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 2 \\ 0 & 0 & 4 \end{vmatrix}$

$\vec{N} = 8\hat{i} - 4\hat{j} + 0\hat{k}$

$\boxed{\vec{N} = <8, -4, 0>}$

$\cancel{8(x-1) - 4(y+4) + 0(z)}$

$8(x - 1) - 4(y - 1) + 0(z - 4) = 0$

$8x - 8 - 4y + 4 = 0$

$\boxed{8x - 4y = 4}$

$\boxed{2x - y = 1}$

QUESTION 3. (i) **(4 points)** The line $L: x = 2w, y = -w+1, z = 3$ intersects the plane $4x + 7y + z = 12$ in a point Q. Find Q.

$$L: \begin{cases} x = 2w \\ y = -w+1 \quad ; w \in \mathbb{R} \\ z = 3 \end{cases}$$

$P: 4x + 7y + z = 12$

$4(2w) + 7(-w+1) + 3 = 12$

$8w - 7w + 7 + 3 = 12$

$w + 10 = 12$

$\boxed{w = 2}$ $\rightarrow$ The plane and the line intersect when $w = 2$

$\Rightarrow$ $\boxed{Q(4, -1, 3)}$

Exam I: MTH 111, Spring 2017

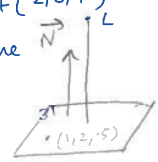
Ayman Badawi

Points = $\frac{58}{58}$

QUESTION 1. (4 points) Given that the line $L = 2 + t, y = -3t, z = 1 + 2t$ is perpendicular to a plane, say P. If the point $(1, 2, -5)$ lies in the plane P, find the equation of the plane P.

The parametric eqn can be written as $L: t\langle 1,-3,2\rangle + (2,0,1)$

since $L \perp$ to plane & pt $(1,2,-5)$ lies on the plane



$1(x-1) + -3(y-2) + 2(z+5) = 0$

$x - 1 - 3y + 6 + 2z + 10 = 0$

$x - 3y + 2z + 15 = 0$

(iii) Let $Q_1 = (1, 1, 0)$, $Q_2 = (0, -1, 2)$ and $Q_3 = (2, 2, 2)$.

a. **(5 points)** Find the equation of the plane that contains Q_1, Q_2, Q_3.

$\overrightarrow{Q_1Q_2}$ $<-1,-2,2>$ $\overrightarrow{Q_1Q_3}$ $<1,1,2>$

$$N = |Q_1Q_2 \times Q_1Q_3| = \begin{vmatrix} i & j & k \\ -1 & -2 & 2 \\ 1 & 1 & 2 \end{vmatrix} = <-6, 4, 1>$$

$$P: \ -6(x-2) + 4(y-2) + 1(z-2) = 0$$

b. **(2 points)** Find the area of the triangle that has Q_1, Q_2, Q_3 as vertices.

$$A = \frac{1}{2}\left|\overrightarrow{Q_1Q_2} \times \overrightarrow{Q_1Q_3}\right| = \frac{\sqrt{6^2+4^2+1^2}}{2} = \frac{\sqrt{53}}{2} \text{ units}^2$$

(iv) **(4 points)** Given $L: x = t+1, y = 8, z = 4t+1$ lies entirely inside the plane $P: ax + 2y + z = b$ Find the values of a, b. $D <1, 0, 4>$ $N <a, 2, 1>$

$N \cdot D = 0$

$a + 4 = 0$

$\boxed{a = -4}$

$-4(t+1) + 2(8) + 4t + 1 = b$

$-4t - 4 + 16 + 4t + 1 = b$

$\boxed{b = 13}$

Name ____________, ID ________ $t\langle 1,-3,2\rangle+(2,0,1)$

Exam I: MTH 111, Spring 2017

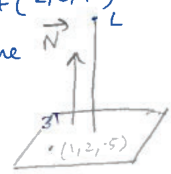
Ayman Badawi

$$\text{Points} = \frac{58}{58}$$

QUESTION 1. (4 points) Given that the line $L = 2 + t, y = -3t, z = 1 + 2t$ is perpendicular to a plane, say P. If the point $(1, 2, -5)$ lies in the plane P, find the equation of the plane P.

The parametric eqn can be written as $L: t\langle 1,-3,2\rangle + (2,0,1)$

since $L \perp$ to plane & pt $(1,2,-5)$ lies on the plane



$$1(x-1) + -3(y-2) + 2(z+5) = 0$$

$$x - 1 - 3y + 6 + 2z + 10 = 0$$

$$x - 3y + 2z + 15 = 0$$

QUESTION 2. (5 points) The two planes $P_1 : 2x - y + z = 6$ and $P_2 : -x + y + 4z = 4$ intersect in a line L. Find a parametric equations of L.

$P_1 : 2x - y + z = 6 \quad \langle 2,-1,1\rangle \rightarrow \vec{N_1}$

$P_2 : -x + y + 4z = 4 \quad \langle -1,1,4\rangle \rightarrow \vec{N_2}$

$$\vec{N_1} \times \vec{N_2} = \begin{vmatrix} \hat{i} & \hat{j} & k \\ 2 & -1 & 1 \\ -1 & 1 & 4 \end{vmatrix} = \hat{i}(-4-1) - \hat{j}(8+1) + \hat{k}(2-1) = -5\hat{i} - 9\hat{j} + \hat{k} \rightarrow \langle -5,-9,1\rangle$$

Assume $z = 0$

$$\begin{array}{r} 2x - y = 6 \\ + \quad -x + y = 4 \\ \hline x = 10 \end{array}$$

$-10 + y = 4$

$y = 14$

pt $(10,14,0)$

$L : t\langle -5,-9,1\rangle + (10,14,0)$

$: \langle -5t, -9t, t\rangle + (10,14,0)$

$x = -5t + 10$; $y = -9t + 14$; $z = t$

Name NADIN ELSHIRBINI, ID [illegible]

Quiz 5: MTH 111, Spring 2018

Ayman Badawi

$\frac{15}{15}$

QUESTION 1. a) The Plane $P: 2x + y - z = 16$ intersects the line $L: x = 3t, y = -2t + 4, z = -t - 2$ at a point Q find Q.

$2(3t) - 2t + 4 + t + 2 = 16$

$6t - 2t + 4 + t + 2 = 16$

$t = 2$ 1/2

$\boxed{Q(6,0,-4)}$ 1/2

$x = 3(2) = 6$

$y = -2(2) + 4 = 0$

$z = -2 - 2 = -4$

4/4

c) The two planes $P_1: 2x + y - z = 6$ and $P_2: 4x - y + z = 12$ intersect in a line L. Find a parametric equations of L.

$N_1: \langle 2,1,-1\rangle$, $N_2 \langle 4,-1,1\rangle$

$D = N_1 \times N_2 = \begin{vmatrix} i & j & k \\ 2 & 1 & -1 \\ 4 & -1 & 1 \end{vmatrix} = \langle 0,-6,-6\rangle$ 1/2

$$\boxed{L: \begin{cases} x = 3 \\ y = -6t \\ z = -6t \end{cases} ;\ t \in \mathbb{R}}$$

take $z = 0$

$2x + y = 6$

$4x - y = 12$

$x = 3 \quad y = 0$

$\Rightarrow Q(3,0,0)$ 1/2

5/5

QUESTION 2. Find $f'(x)$ and do not simplify

a) $f(x) = 3x^2(x+2)^2 + 2018x - 2017$

$f'(x) = 6x(x+2)^2 + 6x^2(x+2) + 2018$ 2/2

Product formula

or $f(x) = 3x^2(x^2 + 4x + 4) + 2018x - 2017$

$= 3x^4 + 12x^3 + 12x^2 + 2018x - 2017$

so $f'(x) = 12x^3 + 36x^2 + 24x + 2018$

b) $f(x) = 8\sqrt{x} + \frac{6}{x^3} + 2x^2$

$f'(x) = \frac{4}{\sqrt{x}} - \frac{18}{x^4} + 4x$ 2/2

c) If $f(x) = 18\sqrt{x} + 7x + 1$, find $f'(9)$

$f'(x) = \frac{9}{\sqrt{x}} + 7 \Big|_{x=9} = 10$ 2/2

. Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: abadawi@aus.edu, www.ayman-badawi.com

2.1.2 Exam I Review from previous semesters

3.10 Exam1-Review from previous semesters

Name NADIN ELSHIRBINI, ID 72754

Exam I: MTH 111, Spring 2018

Ayman Badawi

Nadin El Shirbini

Points = $\frac{80}{80}$

QUESTION 1. a) (3 points) Are the points $q_1 = (1, 2, -2)$, $q_2 = (3, 3, 1)$, and $q_3 = (5, 4, 4)$ co-linear? Show the work

$\overrightarrow{Q_1Q_2} = <2, 1, 3>$

$\overrightarrow{Q_1Q_3} = <4, 2, 6>$

$$\overrightarrow{Q_1Q_2} \times \overrightarrow{Q_1Q_3} = \begin{vmatrix} i & j & k \\ 2 & 1 & 3 \\ 4 & 2 & 6 \end{vmatrix} = < \begin{vmatrix} 1 & 3 \\ 2 & 6 \end{vmatrix}, \begin{vmatrix} 2 & 3 \\ 4 & 6 \end{vmatrix}, \begin{vmatrix} 2 & 1 \\ 4 & 2 \end{vmatrix} > = <0, 0, 0>$$

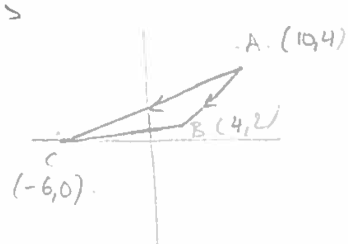
cross product is zero $\Rightarrow$ they are colinear

b) (3 points) Given $A = (10, 4)$, $B = (4, 2)$, and $C = (-6, 0)$ are the vertices of a triangle. Roughly, sketch the triangle ABC. Find the area of the triangle ABC.

$\overrightarrow{AB} = <-6, -2>$

$\overrightarrow{AC} = <-16, -4>$

$$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} i & j & k \\ -6 & -2 & 0 \\ -16 & -4 & 0 \end{vmatrix} = <0, 0, -8>$$



$$A_{\Delta ABC} = \frac{1}{2} |\overrightarrow{AB} \times \overrightarrow{AC}| = \frac{1}{2}\sqrt{(-8)^2} = \boxed{4 \text{ units}^2}$$

c) (3 points) Find a vector F that is perpendicular to both vectors $V =< 2, -1, 4 >$ and $W =< 0, 4, 2 >$

$$\vec{F} = \vec{V} \times \vec{W} = \begin{vmatrix} i & j & k \\ 2 & -1 & 4 \\ 0 & 4 & 2 \end{vmatrix} = \boxed{<-18, -4, 8>}$$

d) (2 points) Let V, W as in (c). Find a vector F that is perpendicular to both V and W such that $|F| = 2$.(hint: Just think a little)

$$|F| = \sqrt{18^2 + 4^2 + 8^2} = 2\sqrt{101}$$

$$(2)\frac{1}{|F|} \cdot F = \frac{2}{2\sqrt{101}} \cdot F = \frac{1}{\sqrt{101}} \cdot F = \boxed{\frac{1}{\sqrt{101}} <-18, -4, 8>}$$

$$\boxed{F < \frac{-18}{\sqrt{101}}, \frac{-4}{\sqrt{101}}, \frac{8}{\sqrt{101}} >}$$

check if $|F| = 2$:

$$|F| = \sqrt{\left(\frac{-18}{\sqrt{101}}\right)^2 + \left(\frac{-4}{\sqrt{101}}\right)^2 + \left(\frac{8}{\sqrt{101}}\right)^2} = 2$$

QUESTION 2. a) **(4 points)** Does the line $L_1 : x = 5t - 20, y = -t + 3, z = 3t - 27$ ($t \in R$) intersect the line $L_2 : x = -2w + 20, y = -4w - 5, z = 2w - 3$ ($w \in R$)? If yes find the intersection point Q.

$$L_1 : \begin{cases} x = 5t - 20 \\ y = -t + 3 \\ z = 3t - 27 \end{cases} \qquad L_2 : \begin{cases} x = -2w + 20 \\ y = -4w - 5 \\ z = 2w - 3 \end{cases}$$

$$5t - 20 = -2w + 20 \Rightarrow 5t + 2w = 40$$
$$-t + 3 = -4w - 5 \Rightarrow -t + 4w = -8$$
$$t = 8 \quad w = 0$$

check for z:

$z = 3t - 27 = 3(8) - 27 = -3$
$z = 2w - 3 = 2(0) - 3 = -3$
} they are equal $\Rightarrow$ L_1 and L_2 intersect

The point of intersection

$x = 2w + 20 = 2(0) + 20 = 20$
$y = -4w - 5 = -4(0) - 5 = -5$
$z = 2w - 3 = 2(0) - 3 = -3$

point of intersection is $(20, -5, -3)$

b) **(2 points)** Are the lines in (a) perpendicular? Explain Yes.

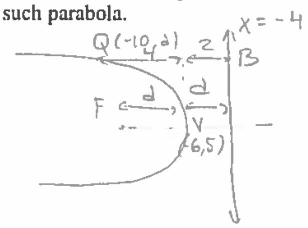
$D_1 = \langle 5, -1, 3 \rangle \qquad D_2 = \langle -2, -4, 2 \rangle$

$D_1 \cdot D_2 = 5(-2) - 1(-4) + 3(2) = 0$

dot product $= 0 \Rightarrow$ They are perpendicular.

QUESTION 3. Given $x = -4$ is the directrix of of a parabola that has the point $(-6, 5)$ as its vertex point.

a) **(2 points)** Roughly, sketch such parabola.



$|d| = 2$

b) **(4 points)** Find the equation of the parabola

$$4d(x - x_0) = (y - y_0)^2$$
$$-4(2)(x + 6) = (y - 5)^2$$
$$\boxed{-8(x + 6) = (y - 5)^2}$$

c) **(2 points)** Find the focus of the parabola, say F.

$\boxed{F(-8, 5)}$

d) **(2 points)** Given $Q = (-10, b)$ is a point on the curve of the parabola. Find $|QF|$ (HINT: You should know how to do this QUICKLY!, you do not need the value of b)

$|QL| = |QB| = |QF| = \boxed{6}$

QUESTION 4. Given $y = x^2 - 6x - 1$ is an equation of a parabola.

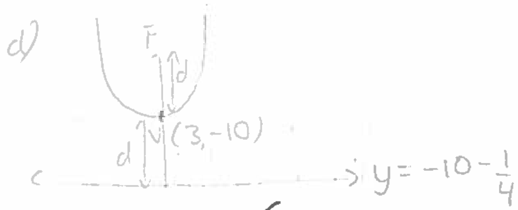
a)**(3 points)** Write the equation in the standard form.

$y = (x-3)^2 - 9 - 1$

$y = (x-3)^2 - 10$

$\boxed{(y+10) = (x-3)^2}$

$4d = 1 \Rightarrow d = 1/4$



b) **(2 points)** Find the equation of the directrix line.

$\boxed{y = -10 - \frac{1}{4} = -\frac{41}{4}}$

c)**(2 points)** Find the focus, say F

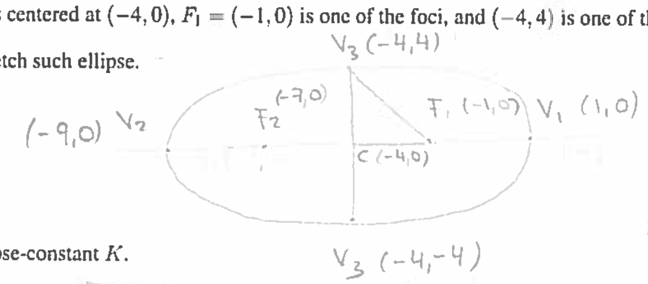
$\boxed{F(3, -10 + \frac{1}{4})} \rightarrow \boxed{F(3, -\frac{39}{4})}$

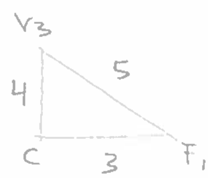
d)**(2 points)** Roughly, sketch the graph of such parabola.

(see picture)

QUESTION 5. An ellipse is centered at $(-4, 0)$, $F_1 = (-1, 0)$ is one of the foci, and $(-4, 4)$ is one of the vertices.

(i) **(2 points)** Roughly, sketch such ellipse.





(ii) **(3 points)** Find the ellipse-constant K.

$|V_3F_1| = \frac{K}{2} = 5 \Rightarrow \boxed{K = 10}$

(iii) **(2 points)** Find the second foci of the ellipse.

$\boxed{F_2(-7, 0)}$

(iv) **(3 points)** Find the remaining three vertices of the ellipse

$\boxed{V_3(-4,-4)}$

$\boxed{V_1(1,0)}$

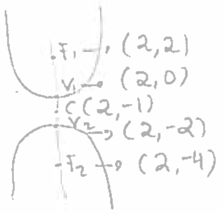
$\boxed{V_2(-9,0)}$

(v) **(3 points)** Find the equation of the ellipse.

$$\boxed{\frac{(x+4)^2}{25} + \frac{y^2}{16} = 1}$$

QUESTION 6. Consider the hyperbola $(y+1)^2 - \frac{(x-2)^2}{8} = 1$.

a) **(2 points)** Draw the hyperbola, roughly



$|CF_1| = \sqrt{1+8} = 3$

b) **(2 points)** Find the hyperbola-constant K.

$\left(\frac{K}{2}\right)^2 = 1$

$\frac{K}{2} = 1 \Rightarrow \boxed{K=2}$ ✓

c) **(3 points)** Find the two vertices of the hyperbola.

$\boxed{V_1\ (2,0)}$

$\boxed{V_2\ (2,-2)}$ ✓

d) **(3 points)** Find the foci of the hyperbola.

$\boxed{F_1\ (2,2)}$

$\boxed{F_2\ (2,-4)}$ ✓

QUESTION 7. Given two lines $L_1: x = t+1, y = 2t+4, z = -5t+3$ and $L_2: x = 2w+7, y = 4w+16, z = -10w-27$.

(i) **(3 points)** Find the symmetric equation of L_1.

$\boxed{x-1 = \frac{y-4}{2} = \frac{-z+3}{5}}$ ✓

(ii) **(3 points)** Is D_1 parallel to D_2? (note that D_1 is the directional vector of L_1 and D_2 is the directional vector of L_2) Show the work

$D_1 < 1,2,-5 >$
$D_2 < 2,4,-10 >$

$D_1 = c D_2$
$<1,2,-5> = c <2,4,-10>$
$c = \frac{1}{2}$ ✓

$D_1 = \frac{1}{2} D_2 \Rightarrow$ They are parallel

(iii) **(2 points)** Is L_1 parallel to L_2? Explain (show the work)

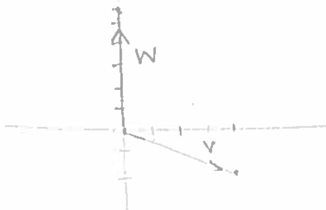
Take $t=0 \rightarrow (1,4,3)$
check if $(1,4,3) \in L_2$

$1 = 2w+7 \Rightarrow w = -3$
$4 = 4w+16 \Rightarrow w = -3$
$3 = -10w-27 \Rightarrow w = -3$

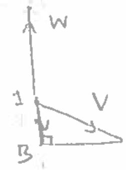
$\Rightarrow$ it $\in$ to L_2. ✓
$\Rightarrow L_1$ and L_2 intersect and they are NOT parallel. They are collinear (same line/on top of each other)

QUESTION 8. Let (0, 0) be the initial point of the two vectors $V = <4, -2>$, and $w = <0, 6>$.

a) **(2 points)** Draw V and W in the xy-plane.



b)



$proj_W^V = \overrightarrow{1B}$

c)

$proj_V^W = \overrightarrow{1B}$

b) **(2 points)** Use the picture that you draw in (a) in order to draw $Proj_W^V$

c)**(2 points)** Use the picture that you draw in (a) in order to draw $Proj_v^w$

d) **(4 points)** Find $Proj_w^v$ and find its length.

$$proj_W^V = \frac{V \cdot W}{|W|^2} \cdot W = \frac{-12}{36} \cdot W = -\frac{1}{3} <0, 6> = \boxed{<0, -2>}$$

$$\boxed{|proj_W^V| = \sqrt{2^2} = 2}$$

e)**(3 points)** Find the angle between V and W

$$\cos\theta = \frac{V \cdot W}{|V||W|} = \frac{-12}{(6)(2\sqrt{5})} = -\frac{\sqrt{5}}{5}$$

$$\theta = \cos^{-1}\left(-\frac{\sqrt{5}}{5}\right) = \boxed{116.565°}$$

Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: abadawi@aus.edu, www.ayman-badawi.com

Name Krstin Raed, ID 78656

MTH 111 Math for Architects Spring 2019, 1–6

Exam I: MTH 111, Spring 2019

$F = v \times w$

Ayman Badawi

Points = $\frac{87}{87}$

QUESTION 1. b) **(4 points)** Given $A = (6, 10)$, $B = (-7, 3)$, and $C = (-4, -2)$ are the vertices of a triangle. Find the area of the triangle ABC.

Area of the triangle $ABC = \frac{1}{2}|AB \times AC|$

$AB = \langle -13, -7 \rangle$ (B-A)

$AC = \langle -10, -12 \rangle$ (C-A)

$$AB \times AC = \begin{vmatrix} i & j & k \\ -13 & -7 & 0 \\ -10 & -12 & 0 \end{vmatrix} = 0i - 0j + \underline{86k} = \underline{86}$$

Area of $\Delta ABC = \frac{1}{2} 86 = \boxed{43 \text{ units}^2}$

c) **(3 points)** Find a vector F that is perpendicular to both vectors $V = \langle 2, 6, -3 \rangle$ and $W = \langle 5, -4, 1 \rangle$ such that $|F| = 111$.

$$F = v \times w = \begin{vmatrix} i & j & k \\ 2 & 6 & -3 \\ 5 & -4 & 1 \end{vmatrix} = -6i - 17j - 38k$$

$|F| = 111 = \frac{111}{|F|} F$

$= \boxed{\frac{111}{42} \langle -6, -17, -38 \rangle}$

QUESTION 2. a) **(4 points)** The line $L_1 : x = -2t - 3, y = -3t + 3, z = 4t - 2$ $(t \in R)$ intersects the line $L_2 : x = 2w - 13, y = 4w - 15, z = 4w - 6$ $(w \in R)$ in a point Q. Find Q.

$L_1 : x = -2t - 3$
$y = -3t + 3$
$z = 4t - 2$

$L_2 : x = 2w - 13$
$y = 4w - 15$
$z = 4w - 6$

use substitution method

find pt of intersection: $-2t - 3 = 2w - 13$

$\frac{-2t}{-2} = \frac{2w - 13 + 3}{-2}$

$\boxed{t = -w + 5}$

Sub in second equ

$-3(-w+5)+3 = 4w-15$

$3w - 15 + 3 = 4w - 15$

$4w - 3w = -15 + 15 + 3$

$\boxed{1w = 3}$

$t = -3 + 5$

$\boxed{t = 2}$

• now sub in each line to get intersection pt

$-2(2) - 3 = 2(3) - 13$

$\boxed{-7 = -7}$

$-3(2) + 3 = 4(3) - 15$

$\boxed{-3 = -3}$

$4(2) - 2 = 4(3) - 6$

$\boxed{6 = 6}$

$\boxed{\text{Intersection pt} = Q = (-7, -3, 6)}$

b) **(2 points)** Are the lines in (a) perpendicular? Explain

$D_1 = \langle -2, -3, 4 \rangle$

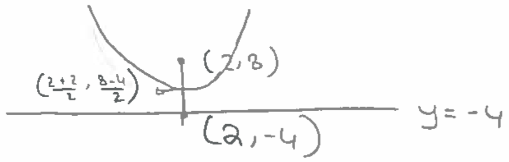
$D_2 = \langle 2, 4, 4 \rangle$

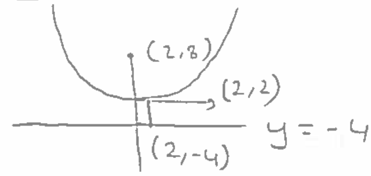
$D_1 \cdot D_2 = (-2 \times 2) + (-3 \times 4) + (4 \times 4)$
$= 0$

So they are perpendicular because their dot product is zero & they intersect

QUESTION 3. Given $y = -4$ is the directrix of a parabola that has the point $F = (2, 8)$ as its focus point.

a) **(2 points)** Roughly, sketch such parabola.



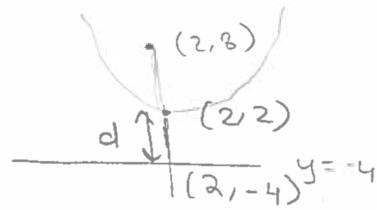


d = 6 & its up

b)**(4 points)** Find the equation of the parabola

$$4d(y-2) = (x-2)^2$$
$$4(6)(y-2) = (x-2)^2$$
$$24(y-2) = (x-2)^2$$

$\boxed{d=6}$



d = (-4-2)
-6

c) **(2 points)** Find the vertex of the parabola, say V.

$$V = (2,2)$$

QUESTION 4. Given $y = 4x^2 + 24x - 3$ is an equation of a parabola.

a)**(3 points)** Write the equation in the standard form.

$$y = 4x^2 + 24x - 3$$
$$y = 4(x^2 + 6x) - 3$$
$$y = 4((x+3)^2 - 9) - 3$$
$$y = 4(x+3)^2 - 36 - 3$$
$$y = 4(x+3)^2 - 39$$
$$\frac{1}{4}(y+39) = \frac{4(x+3)^2}{4}$$
$$\frac{1}{4}(y+39) = (x+3)^2$$

$$4d = \frac{1}{4}$$
$$d = \frac{1}{4 \times 4}$$
$$\boxed{d = \frac{1}{16}}$$

so +

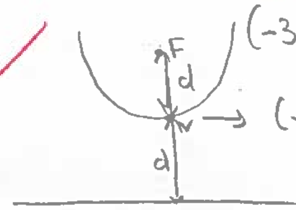
b) **(2 points)** Find the equation of the directrix line.

$$y = -\frac{625}{16}$$

c)**(2 points)** Find the focus, say F

$$F = \left(-3, -39 + \frac{1}{16}\right) = \left(-3, -\frac{623}{16}\right)$$

d)**(2 points)** Roughly, sketch the graph of such parabola.



$\left(-3, -39 + \frac{1}{16}\right)$

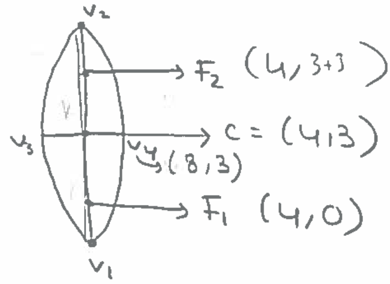
$(-3, -39)$

$\left(-3, -39 - \frac{1}{16}\right)$

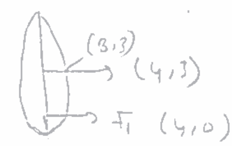
$\left(-3, -\frac{625}{16}\right)$

QUESTION 5. An ellipse is centered at $(4,3)$, $F_1 = (4,0)$ is one of the foci, and $(8,3)$ is one of the vertices.

(i) **(2 points)** Roughly, sketch such ellipse.



x does not change



$CF^2 = \left(\frac{k}{2}\right)^2 - b^2$

$3^2 = \left(\frac{k}{2}\right)^2 - 4^2$

$25 = \left(\frac{k}{2}\right)^2$

$\boxed{CF = 3}$

$\boxed{b = 4}$

(ii) **(3 points)** Find the ellipse-constant K.

$$CF^2 = \left(\frac{k}{2}\right)^2 - b^2$$

$$3^2 = \left(\frac{k}{2}\right)^2 - 4^2$$

$$\boxed{k = 10}$$

(iii) **(2 points)** Find the second foci of the ellipse.

$$F_2 = (4, 3+3)$$

$$(4, 6)$$

(iv) **(3 points)** Find the remaining three vertices of the ellipse

$$V_1 = \left(4, 3 - \frac{10}{2}\right) \quad \boxed{(4, -2)}$$

$$V_2 \left(4, 3 + \frac{10}{2}\right) \quad (4, 8)$$

$$V_3 \ (0, 3)$$

(v) **(3 points)** Find the equation of the ellipse.

$$\frac{(y-3)^2}{\left(\frac{10}{2}\right)^2} + \frac{(x-4)^2}{4^2} = 1$$

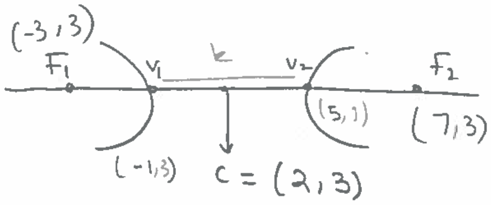
$$\frac{(y-3)^2}{25} + \frac{(x-4)^2}{16} = 1$$

QUESTION 6. Consider the hyperbola $\frac{(x-2)^2}{9} - \frac{(y-3)^2}{16} = 1$.

$\left(\frac{k}{2}\right)^2$ $\qquad b^2$

a) **(2 points)** Draw the hyperbola, roughly.

under x so right left



b) **(2 points)** Find the hyperbola-constant K.

$\left(\frac{k}{2}\right)^2 = 9$

$\frac{k}{2} = \sqrt{9}$

$k = 3 \times 2$

$\boxed{k = 6}$

c) **(3 points)** Find the two vertices of the hyperbola.

$V_2 = (2+3, 3)$
$(5,3)$

$V_1 = (2-3, 3)$
$(-1, 3)$

d) **(3 points)** Find the foci of the hyperbola.

$F_1 = (2-5, 3) \quad (-3, 3)$

$F_2 = (2+5, 3) \quad (7, 3)$

$CF^2 = \left(\frac{k}{2}\right)^2 + b^2$

$CF^2 = 9 + 16$

$= 25$

$\boxed{CF = 5}$

QUESTION 7. **(4 points)** Given two lines $L_1 : x = t + 1, y = 2t + 4, z = -5t + 3$ ($t \in R$) and $L_2 : x = 2w - 1, y = 4w + 1, z = -10w + 13$ ($w \in R$). Is L_1 parallel to L_2? Explain (show the work)

· 2 lines are // if they have cst & they do not intersect

$L_1 :\ x = t + 1$
$y = 2t + 4$
$z = -5t + 3$

$L_2 :\ x = 2w - 1$
$y = 4w + 1$
$z = -10w + 13$

$D_1 \langle 1, 2, -5 \rangle$ $\qquad D_2 \langle 2, 4, -10 \rangle$

$1 = c2$
$2 = c4$
$-5 = c(-10)$

$c = \frac{1}{2}$
$c = \frac{1}{2}$
$c = \frac{1}{2}$

they have a cst

$L_1 // L_2$

take $t = 0$

$1 = 2w - 1$
$4 = 4w + 1$
$3 = -10w + 13$

$2w = 2$
$\boxed{w = 1}$
$4 - 1 = 4w$
$3 = 4w$
$\boxed{w = \frac{3}{4}}$
$3 - 13 = -10w$
$-10 = -10w$

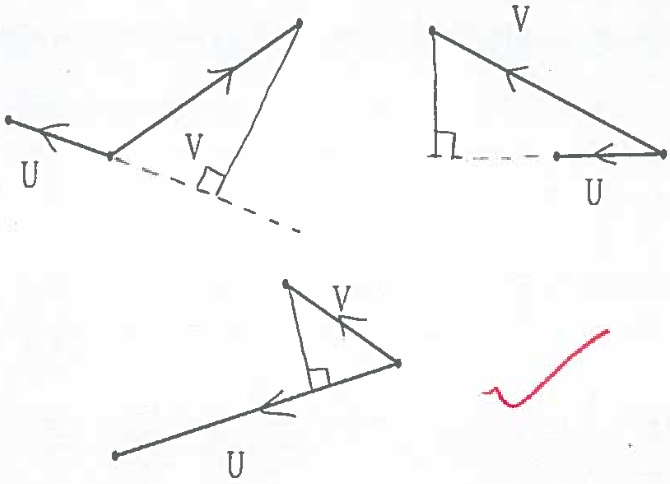
they do not intersect

$2w = 2$
$w = 1$
$4w = 4 - 1$
$w = \frac{3}{4}$
$10w = 13 - 3$
$10w \quad 10$
$w = 1$

QUESTION 8. (6 points)

$\text{proj}_U V$

Stare at the below. Then find Projection of V over U



QUESTION 9. (4 points) Find the equation of the plane that contains the points $Q_1 = (4,4,0)$, $Q_2 = (0,2,6)$ and $Q_3 = (4,0,8)$.

$$N = \overrightarrow{Q_1Q_2} \times \overrightarrow{Q_1Q_3}$$

$$\langle -4, -2, 6\rangle \times \langle 0, -4, 8\rangle$$

$$\begin{vmatrix} i & j & k \\ -4 & -2 & 6 \\ 0 & -4 & 8 \end{vmatrix} = 8i + 32j + 16k$$

$$\langle 8, 32, 16\rangle$$

choose a pt

$Q_1 = (4,4,0)$

$8(x-4) + 32(y-4) + 16(z-0) = 0$

$$\boxed{8(x-4) + 32(y-4) + 16z = 0}$$

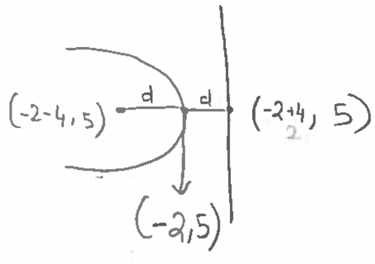
QUESTION 10. (6 points) Consider the parabola $-16(x+2) = (y-5)^2$.

(i) Sketch the parabola

$4d = -16$

$d = -4$

& before x so its left



(ii) Find the equation of the directrix line

$X = -2 + 4$

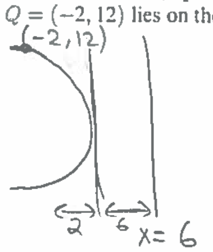
$$\boxed{X = 2}$$

(iii) Find the focus point.

Focus $= (-2-4, 5)$

$(-6, 5)$

QUESTION 11. **(4 points)** Given that $x = 6$ is the directrix line of a parabola that has F as its focus point. If the point $Q = (-2, 12)$ lies on the parabola. Find $|QF|$ (i.e., the distance between Q and F).



$|QF| = |QL| = 8$

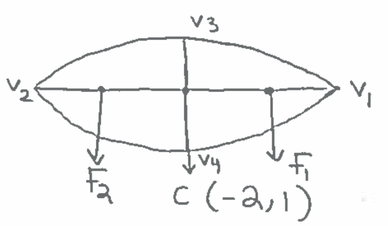
QUESTION 12. **(6 points)** Consider the ellipse $\frac{(y-1)^2}{9} + \frac{(x+2)^2}{25} = 1.$

$9 \to b^2$

$25 \to$ big # so its $\left(\frac{k}{2}\right)^2$

so the shape is

(i) Sketch (roughly)



(ii) Find the foci of the ellipse

$CF^2 = \left(\frac{k}{2}\right)^2 - b^2$

$= 25 - 9$

$= 16$

$CF^2 = 16$

so $\boxed{CF = 4}$

so $F_1\ (-2+4\ ,\ 1)$

$(2, 1)$

$F_2\ (-2-4, 1)$

$(-6, 1)$

(iii) Find all four vertices of the ellipse.

$\left(\frac{k}{2}\right)^2 = 25$ $\boxed{\pm\frac{k}{2}}$

$\boxed{\frac{k}{2} = 5}$ $\boxed{\pm b}$

$b^2 = 9$ $\boxed{b = 3}$

$V_1 = (-2+5\ ,1)$

$(3, 1)$

$V_2 = (-2-5, 1)$

$(-7, 1)$

$V_3 = (-2, 1+3)$ $(-2, 4)$

$V_4 = (-2, 1-3)$ $(-2, -2)$

QUESTION 13. **(4 points)** Given $Q = (1, 6, 4)$ is not on the line $L : x = t + 1, y = 2t + 4, z = -5t + 3$ $(t \in R)$. Find $|QL|$.

$D = \langle 1, 2, -5 \rangle$

$I = \langle 1, 4, 3 \rangle$

$|QL| = \frac{|D \times IQ|}{|D|} = \frac{\sqrt{12^2+1^2+2^2}}{\sqrt{1^2+2^2+5^2}}$

$= \frac{\sqrt{149}}{\sqrt{30}}$

$IQ = \langle 0, 2, 1 \rangle$

$Q-I$

$IQ \times D = \begin{vmatrix} i & j & k \\ 0 & 2 & 1 \\ 1 & 2 & -5 \end{vmatrix}$

$= -12i + 1j - 2k$

Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: abadawi@aus.edu, www.ayman-badawi.com

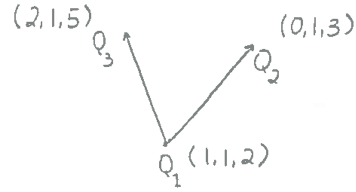
3.5 Questions with Solutions on Planes in 3 D from previous semesters

Quiz V MTH 111, Spring 2019

Ayman Badawi

15/15 :)

7/7 **QUESTION 1.** Let $Q_1 = (1,1,2), Q_2 = (0,1,3), Q_3 = (2,1,5)$. Find the equation of the plane that passes through $Q_1, Q_2, Q3$.



$N \perp$ Plane

$N = \overrightarrow{Q_1Q_2} \times \overrightarrow{Q_1Q_3}$

$\overrightarrow{Q_1Q_2} = \langle 0-1, 1-1, 3-2 \rangle = \langle -1, 0, 1 \rangle$ ✓ 1

$\overrightarrow{Q_1Q_3} = \langle 2-1, 1-1, 5-2 \rangle = \langle 1, 0, 3 \rangle$ ✓ 1

$\overrightarrow{Q_1Q_2} \times \overrightarrow{Q_1Q_3} = \begin{vmatrix} i & j & k \\ -1 & 0 & 1 \\ 1 & 0 & 3 \end{vmatrix} = (0)i - (-3-1)j + (0)k = 4j = \langle 0, 4, 0 \rangle$ ✓ 3

choose Q_1 & a random point

$w = (x, y, z)$

$Q_1 = (1, 1, 2)$

$\overrightarrow{Q_1w} = (x-1, y-1, z-2)$

$N \cdot \overrightarrow{Q_1w} = \langle 0, 4, 0 \rangle \cdot \langle x-1, y-1, z-2 \rangle = 0$

$0(x-1) + 4(y-1) + 0(z-2) = 0$

$4(y-1) = 0$ ✓ 2

Haya

QUESTION 2. (i) **(6 points)** Does the line $L: x = 2t+1, y = 5t-1, z = -2t+3$ lie entirely inside the plane $x + 2y + z = 23$? If not, does it intersect the plane? If yes, then find the intersection point.

$L: \left.\begin{array}{l} x = 2t+1 \\ y = 5t-1 \\ z = -2t+3 \end{array}\right\} t \in R$

$P \Rightarrow x + 2y + z = 23$

it doesnt lie interiorly on the plane but intersects it at $(5.2, 9.5, -1.2)$

$P(L) \Rightarrow (2t+1) + 2(5t-1) + (-2t+3) = 23 \leftarrow$ 1

$\cancel{2t} + 1 + 10t - 2 - \cancel{2t} + 3 = 23$

$10t + 2 = 23$

$10t = 21$

$t = 21/10$

$= 2.1$

$x = 2(2.1) + 1 = 5.2$

2) $\to$ $y = 5(2.1) - 1 = 9.5$

$z = -2(2.1) + 3 = -1.2$

$Q = (5.2, 9.5, -1.2)$

the point of intersection is $(5.2, 9.5, -1.2) \leftarrow$ 3

(ii) **(4 points)** Given $N = < -2, 3, 2 >$ is perpendicular to the plane P and the point $(-1, 4, 2)$ lies inside the plane P. Find the equation of the plane P.

$N_x(x - P_x) + N_y(y - P_y) + N_z(z - P_z) = 0$

$-2(x+1) + 3(y-4) + 2(z-2) = 0 \Leftrightarrow$ plane.

1

QUESTION 9. (5 points). Can we draw the entire line $L: x = 2t, y = -3t + 1, z = 11t + 4$ inside the plane $2x - 6y - 2z = 20$? EXPLAIN

take a point on L and check if the point lies in the plane or not

$N_{plane} \cdot D_{line}$ must $= 0$

$N = \langle 2, -6, -2 \rangle$

$D = \langle 2, -3, 11 \rangle$

$N \cdot D = 4 + 18 - 22$

$= 0$ ✓

NO

yes the line can be entirely drawn on the plane because the dot product of the normal and directional vector is 0

2

Name________, ID ________

Quiz4 ~~HW 9~~: MTH 111, Spring 2018

Ayman Badawi

$\frac{15}{15}$

QUESTION 1. a) Find the equation of the plane that contains the points $Q_1 = (0, 1, 1), Q_2 = (0, 2, 3), Q_3 = (1, 3, 2)$.

$\overrightarrow{Q_1Q_2}$: $(0,2,3) - (0,1,1) \to \langle 0,1,2 \rangle$

$\overrightarrow{Q_1Q_3}$: $(1,3,2) - (0,1,1) \to \langle 1,2,1 \rangle$

$\overrightarrow{QQ_2} \cdot \overrightarrow{Q_1Q_3} = \langle N \rangle = i[(1\times1)-(2\cdot2)] - j[(0\cdot1)-(2\cdot1)] + K[(0\cdot2)-(1\cdot1)]$

$$\begin{array}{ccc} i & j & K \\ 0 & 1 & 2 \\ 1 & 2 & 1 \end{array}$$

$-3i + 2j - K$

$\langle N \rangle$ $\langle -3, 2, -1 \rangle$

5

equation: $\boxed{-3x + 2(y-2) - 1(z-1) = 0}$

c) Given a plane $P : 5x - 7y + z = 21$ Can we draw the vector $V = \langle -4, -3, -1 \rangle$ inside the plane P? explain

3

$N \langle 5, -7, 1 \rangle$ $N \cdot V = 0 = \perp \longrightarrow$ so inside plane

$V \langle -4, -3, -1 \rangle$ $(5 \cdot -4) + (-7 \cdot -3) + (1 \cdot -1) = -20 + 21 - 1$

$= 0$

d) Find the distance between the point (0, -10, 5) and the plane P ...

Exam II: MTH 111, Spring 2018

Ayman Badawi

Points = ~~47~~

55

QUESTION 2. (i) **(3 points)** What can you say about the line $L: x = 2t + 1, y = t - 1, z = -2t + 3$ and the plane $x + 2y + z = 16$? (i.e., Doe L lie inside the plane? Does L intersect the plane exactly in one point? or neither?

L: $x = 2t + 1$
$y = t - 1$
$z = -2t + 3$

P: $x + 2y + z = 16$

$(2t + 1) + 2(t - 1) - 2t + 3 = 16$

$2t + 1 + 2t - 2 - 2t + 3 = 16$

$2t = 14 \Rightarrow t = 14/2 \Rightarrow \boxed{t = 7}$

x: $2(7) + 1 = 15$
y: $7 - 1 = 6$
z: $-2(7) + 3 = -11$

$\boxed{\text{Q: intersection point} : (15, 6, -11)}$

(ii) **(4 points)** Given $N = < -2, 3, 2 >$ is perpendicular to the plane P and the point $(-1, 4, 2)$ lies inside the plane P. Find the equation of the plane P.

Find eqn → Directional vector, point Q

$N = <-2, 3, 2> \perp P$ at $Q(-1, 4, 2)$

P: $-2(x + 1) + 3(y - 4) + 2(z - 2) = 0$

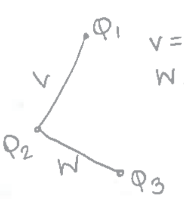
P: $-2x - 2 + 3y - 12 + 2z - 4 = 0$

$\boxed{P: -2x + 3y + 2z = 18}$

(iii) **(6 points)** Find the equation of the plane that contains the points $Q_1 = (4, 4, 0)$, $Q_2 = (0, 2, 6)$ and $Q_3 = (4, 0, 8)$.

Eqn of Plane → directional vector and point Q_1

$Q_1: (4, 4, 0)$
$Q_2: (0, 2, 6)$
$Q_3: (4, 0, 8)$



$V = Q_1Q_2 = <4, 2, -6>$

$W = Q_3Q_2 = <4, -2, 2>$

$$V \times W = \begin{vmatrix} i & j & k \\ 4 & 2 & -6 \\ 4 & -2 & 2 \end{vmatrix} = \left< \begin{vmatrix} 2 & -6 \\ -2 & 2 \end{vmatrix}, -\begin{vmatrix} 4 & -6 \\ 4 & 2 \end{vmatrix}, \begin{vmatrix} 4 & 2 \\ 4 & -2 \end{vmatrix} \right>$$

$= <4 - 12, -(8 + 24), -8 - 8>$

$= <-8, -32, -16>$

P: $-8(x - 4) - 32(y - 4) - 16(z + 0) = 0$

P: $-8x + 32 - 32y + 128 - 16z = 0$

$\boxed{P: -8x - 32y - 16z = -160}$

Exam II: MTH 111, Fall 2017

Ayman Badawi

Points = $\frac{47}{47}$

Haya Alshamsi

QUESTION 2. (i) **(3 points)** Can we draw the vector $v = <3, -5, 2>$ inside the plane $x - 4y - 11z = 7$? explain

$V = <3, -5, 2>$ $\qquad N \cdot V = 3(1) - 5(-4) + 2(-11)$

$N = <1, -4, -11>$ $\qquad N \cdot V = 3 + 20 - 22 = 1 \neq 0$

NO. The two vectors are not perpendicular, hence V can't be drawn inside the plane.

(ii) **(4 points)** Given $N = <4, 6, 2>$ is perpendicular to the plane P and the point $(4, 1, 1)$ lies inside the plane P. Find the equation of the plane P.

$N = <4, 6, 2>$
$<a, b, c>$
$Q(4, 1, 1)$
$Q(x_0, y_0, z_0)$

$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$

$4(x - 4) + 6(y - 1) + 2(z - 1) = 0$

$4x - 16 + 6y - 6 + 2z - 2 = 0$

$\boxed{4x + 6y + 2z = 24}$

(iii) **(6 points)** Find the equation of the plane that contains the points $Q_1 = (1, 1, 4)$, $Q_2 = (2, 3, 6)$ and $Q_3 = (1, 1, 8)$.

$Q_1(1, 1, 4)$
$Q_2(2, 3, 6)$
$Q_3(1, 1, 8)$

$\overrightarrow{Q_1Q_2} = <1, 2, 2>$

$\overrightarrow{Q_1Q_3} = <0, 0, 4>$

$\vec{N} = \overrightarrow{Q_1Q_2} \times \overrightarrow{Q_1Q_3} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 2 \\ 0 & 0 & 4 \end{vmatrix}$

$\vec{N} = 8\hat{i} - 4\hat{j} + 0\hat{k}$

$\boxed{\vec{N} = <8, -4, 0>}$

$\cancel{8(x-1) - 4(y+4) + 0(z)}$

$8(x - 1) - 4(y - 1) + 0(z - 4) = 0$

$8x - 8 - 4y + 4 = 0$

$\boxed{8x - 4y = 4}$

$\boxed{2x - y = 1}$

QUESTION 3. (i) **(4 points)** The line $L: x = 2w, y = -w + 1, z = 3$ intersects the plane $4x + 7y + z = 12$ in a point Q. Find Q.

$$L: \begin{cases} x = 2w \\ y = -w + 1 \quad ; w \in \mathbb{R} \\ z = 3 \end{cases}$$

$P: 4x + 7y + z = 12$

$4(2w) + 7(-w + 1) + 3 = 12$

$8w - 7w + 7 + 3 = 12$

$w + 10 = 12$

$\boxed{w = 2}$ $\rightarrow$ The plane and the line intersect when $w = 2$

$\Rightarrow$ $\boxed{Q(4, -1, 3)}$

Name________________________, ID ____________ $t\langle 1,-3,2\rangle + (2,0,1)$

Exam I: MTH 111, Spring 2017

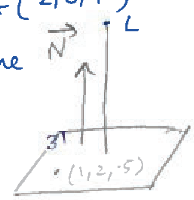
Ayman Badawi

$$\text{Points} = \frac{58}{58}$$

QUESTION 1. (4 points) Given that the line $L = 2 + t, y = -3t, z = 1 + 2t$ is perpendicular to a plane, say P. If the point $(1, 2, -5)$ lies in the plane P, find the equation of the plane P.

The parametric eqn can be written as $L : t\langle 1,-3,2\rangle + (2,0,1)$

since $L \perp$ to plane & pt $(1,2,-5)$ lies on the plane



$1(x-1) + -3(y-2) + 2(z+5) = 0$

$x - 1 - 3y + 6 + 2z + 10 = 0$

$x - 3y + 2z + 15 = 0$

(iii) Let $Q_1 = (1,1,0)$, $Q_2 = (0,-1,2)$ and $Q_3 = (2,2,2)$.

a. **(5 points)** Find the equation of the plane that contains Q_1, Q_2, Q_3.

$\overrightarrow{Q_1Q_2} < -1, -2, 2 >$ $\quad \overrightarrow{Q_1Q_3} < 1, 1, 2 >$

$$N = |Q_1Q_2 \times Q_1Q_3| = \begin{vmatrix} i & j & k \\ -1 & -2 & 2 \\ 1 & 1 & 2 \end{vmatrix} = < -6, 4, 1 >$$

$$P: -6(x-2) + 4(y-2) + 1(z-2) = 0$$

b. **(2 points)** Find the area of the triangle that has Q_1, Q_2, Q_3 as vertices.

$$A = \frac{1}{2} |\overrightarrow{Q_1Q_2} \times \overrightarrow{Q_1Q_3}| = \frac{\sqrt{6^2+4^2+1^2}}{2} = \frac{\sqrt{53}}{2} \text{ units}^2$$

(iv) **(4 points)** Given $L: x = t+1, y = 8, z = 4t+1$ lies entirely inside the plane $P: ax + 2y + z = b$ Find the values of a, b. $D < 1, 0, 4 >$ $\quad N < a, 2, 1 >$

$N \cdot D = 0$

$a + 4 = 0$

$\boxed{a = -4}$

$-4(t+1) + 2(8) + 4t + 1 = b$

$-4t - 4 + 16 + 4t + 1 = b$

$\boxed{b = 13}$

2.1.3 Exam II Review from previous semesters

QUESTION 11. (9 points).

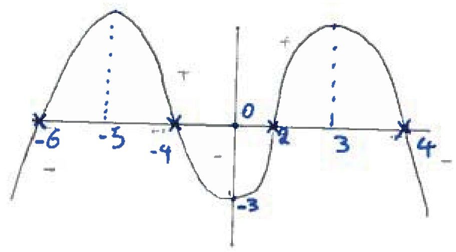


Figure 2. Question: You are looking at the curve of $f'(x)$.

(i) Find all x values where $f(x)$ is maximum.

$x: \boxed{-4, 4}$

(ii) Find all x values where $f(x)$ is minimum.

$x: \boxed{-6, 2}$

(iii) For what values of x does $f(x)$ increase?

$x \in: (-6,-4) \cup (2,4)$

(iv) For what values of x does $f(x)$ decrease?

$x \in: (-\infty,-6) \cup (-4,2) \cup (4,+\infty)$

(v) For what values of x do the slopes of tangent lines are positive?

$x \in (-6,-4) \cup (2,4)$

(vi) For what values of x do the slopes of normal lines are negative?

$x \in (-6,-4) \cup (2,4)$

Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: abadawi@aus.edu, www.ayman-badawi.com

(iii) For what values of x does $f(x)$ have a local minimum value?

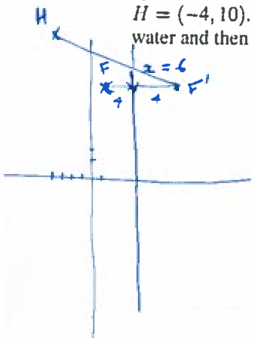
$f(x)$ has a local minimum value for $x = 4$;

(iv) For what values of x does $f(x)$ have a local maximum value?

according to the sign of $f(x)$, $f(x)$ does not have a local maximum.

⊖ -4 ⊖ 4 ⊕

QUESTION 5. (5 points) There is a fire-station located at the point $F = (2, 8)$. A house is on fire and it is located at $H = (-4, 10)$. There is a river that is located at $x = 6$. The fire-men want to find a point Q on the river in order to get water and then travel to the House such that $|FQ| + |QH|$ is minimum. Find Q.



WE FLIP F ABOUT $x = 6$

$F' = (10;8)$ AT EQUAL DISTANCE TO $x = 6$ THAN F. (4 UNIT)

HF' POSSESSES THE FOLLOWING EQUATION:

$y = mx + b$; $m = \frac{\Delta y}{\Delta x} = \frac{10+4}{8-10} = -7$

$10 = -7(-4) + b$; $b = -18$

$\boxed{y = -7x - 18}$ NOW WE NEED TO FIND Q: INTERSECTION pt. BETWEEN [HF'] and $x = 6$, THUS $Q = (6;36)$

PLEASE TURN OVER

QUESTION 6. (4 points) Find the equation of the tangent line to the curve of $f(x) = 12\sqrt{x} - 5x + 1$ at the point $(4, 5)$.

$f(x) = 12 \times x^{1/2} - 5x + 1$

$f'(x) = 12 \times \frac{1}{2} x^{-1/2} - 5 = 6x^{-\frac{1}{2}} - 5$

$x = 4$; $f'(4) = 6(4)^{-1/2} - 5 = -2$

THUS ; in $y = mx + b$, $m = -2$.

$5 = -2(4) + b$; $b = 13$

EQUATION OF TANGENT LINE AT $(4;5)$ IS AS FOLLOWS: $y = -2x + 13$.

QUESTION 7. (5 points) Imagine that you want to construct a box that has a square base, say of length x (and hence it has width x), and with height $12 - x$ so that the volume is maximum. What is the value of x? (note that Volume = length X width X Height)

VOLUME $= L \times W \times H$

$= x \times x \times (12 - x) = x^2(12 - x)$

$V = x^2(12 - x)$

$V' = 2x(12 - x) + x^2(-1) = 24x - 2x^2 - x^2$

$= 24x - \cancel{2}x^2$

$V' = 0$; $24x - \cancel{2}x^2 = 0$; $x = 0$ CANCELED $(x \neq 0)$

$x(24 - \cancel{2}x) = 0$ OR $x = 24$

$V'' = 24 - 2x$

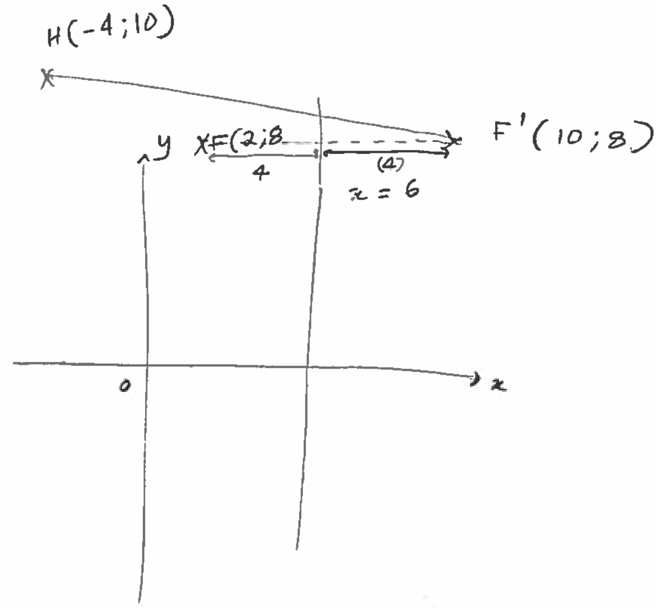
$V''(24) = -24 < 0$ SO V is MAXIMAL at $x = 24$.

QUESTION 5

F (2; 8)

H (−4; 10)

$x = 6$.



H (−4; 10)

F' (10; 8)

$y = mx + b$

$$m = \frac{\Delta y}{\Delta x} = \frac{10-8}{-4-10} = \frac{2}{-14} = -\frac{1}{7}$$

$$10 = -\frac{1}{7}(-4) + b$$

$$10 = \frac{4}{7} + b \qquad b = \boxed{\frac{66}{7}}$$

$$y = -\frac{1}{7}x + \frac{66}{7}$$

COORDINATES OF Q: $y = -\frac{1}{7} \times 6 + \frac{66}{7}$

$$= \frac{60}{7}$$

$$Q = \left(6\ ;\ \frac{60}{7}\right)$$

FOR $|FQ| + |QH|$ TO BE MINIMAL.

QUESTION 8. (6 points) Let $f(x) = -e^x + e^{10}x + 4$

(i) For what values of x does $f(x)$ increase?

$f(x)$ INCREASES if $f'(x) > 0$.

$[a(b^{E(x)})]' = a(b^{E(x)}) \times E'_x \times \ln$

$f(x) = -e^x + e^{10}x + 4$

$f'(x) = -e^x \times (1) \times (1) + [(e^{10} \times 0 \times 1)(x) + (1)(e^{10})] + 0$

$= -e^x + e^{10}$

$f'(x) = 0$; $-e^x + e^{10} = 0$

$-e^x = -e^{10}$

$\ln e^x = \ln e^{10}$

$x = 10$

$\oplus$ 10 $\ominus$ sign of $f'(x$

$f(x)$ increases for $x \in (-\infty ; 10)$.

(ii) For what values of x does $f(x)$ decrease?

for $x \in (10 ; +\infty)$, $f(x)$ decreases.

(iii) For what values of x does $f(x)$ have a maximum value?

looking at the sketch, $f(x)$ clearly has a maximum at $x = 10$;

~~f(10)~~ =

QUESTION 9. (6 points) Find y' and do not simplify

(i) $y = ln\left[\frac{(x+2)^3}{3x+7}\right] = 3\ln(x+2) - \ln(3x+7)$

$y' = \frac{3}{\ln(e)} \times \frac{1}{x+2} - \frac{1}{1} \times \frac{3}{3x+7}$

$\boxed{y' = \frac{3}{x+2} - \frac{3}{3x+7}}$

$[a \log_b(k(x))]' = \frac{a}{\ln(b)} \times \frac{k'(x)}{k(x)}$

(ii) $y = \underbrace{(7x+3)}_{1}\underbrace{e^{(2x^2-5x)}}_{2} + 10x$

(1) $7x+3$

(1)' 7

(2) $e^{(2x^2-5x)}$

(2)' $e^{(2x^2-5x)} \times (4x-5)$

$y' = 7e^{(2x^2-5x)} + (7x+3)(e^{(2x^2-5x)} \cdot (4x-5))$
$+10$

(iii) $y = ln((6x+2)^3(-7x+4)^7) = 3\ln(6x+2) + 7\ln(-7x+4)$

$y' = \frac{3}{1} \times \frac{6}{6x+2} + \frac{7}{1} \times \frac{-7}{-7x+4} \boxed{= \frac{18}{(6x+2)} + \frac{-49}{(-7x+4)}}$

Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: abadawi@aus.edu, www.ayman-badawi.com

Rania Hegab

Exam II: MTH 111, Spring 2018

Ayman Badawi

Points = $\frac{}{47}$

$\frac{55}{55}$ Excellent !!

QUESTION 1. (8 points) Find y' and DO NOT SIMPLIFY

(i) $y = 6e^{(3x^2+6x+1)}$

$\boxed{y' = 6e^{(3x^2+6x+1)} \cdot (6x+6)}$ ✓

(ii) $y = (2x+3)\sqrt{7x+2}$

$y = (2x+3)(7x+2)^{\frac{1}{2}}$

$y' = (1)'(2) + (2)'(1)$

$\boxed{y' = 1(7x+2)^{\frac{1}{2}} + \frac{1}{2}(7x+2)^{-\frac{1}{2}}(2x+3)}$ ✓

(iii) $y = ln[\frac{(3x+2)^3(2x+7)^2}{(7x+12)^4}]$

$y = 3\ln(3x+2) + 2\ln(2x+7) - 4\ln(7x+12)$

$y' = \frac{3(3)}{3x+2} + \frac{2(2)}{2x+7} - \frac{4(7)}{7x+12}$

$\boxed{y' = \frac{9}{3x+2} + \frac{4}{2x+7} - \frac{28}{7x+12}}$ ✓

(iv) $y = 2(3x^2+5x)^{12}$

$\boxed{y = 24(3x^2+5x)^{11} \cdot (6x^1+5)}$ ✓

QUESTION 2. (i) **(3 points)** What can you say about the line $L: x = 2t+1, y = t-1, z = -2t+3$ and the plane $x + 2y + z = 16$? (i.e., Doe L lie inside the plane? Does L intersect the plane exactly in one point? or neither?)

$L: x = 2t+1$
$y = t-1$
$z = -2t+3$

$P: x + 2y + z = 16$

$(2t+1) + 2(t-1) - 2t + 3 = 16$

$2t + 1 + 2t - 2 - 2t + 3 = 16$

$2t = 14 \Rightarrow t = 14/2 \Rightarrow \boxed{t = 7}$ ✓

$x: 2(7)+1 = 15$
$y: 7-1 = 6$
$z: -2(7)+3 = -11$

$\boxed{\text{Q: intersection point} : (15, 6, -11)}$

(ii) **(4 points)** Given $N = <-2, 3, 2>$ is perpendicular to the plane P and the point $(-1, 4, 2)$ lies inside the plane P. Find the equation of the plane P.

Find eqn → Directional vector
→ point Q

$N = <-2, 3, 2> \perp P$ at $Q(-1, 4, 2)$

$P: -2(x+1) + 3(y-4) + 2(z-2) = 0$

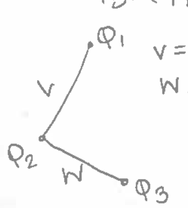
$P: -2x - 2 + 3y - 12 + 2z - 4 = 0$

$\boxed{P: -2x + 3y + 2z = 18}$ ✓

(iii) **(6 points)** Find the equation of the plane that contains the points $Q_1 = (4, 4, 0)$, $Q_2 = (0, 2, 6)$ and $Q_3 = (4, 0, 8)$.

Eqn of plane → directional vector and point Q_1

$Q_1: (4, 4, 0)$
$Q_2: (0, 2, 6)$
$Q_3: (4, 0, 8)$



$V = Q_1Q_2 = <4, 2, -6>$
$W = Q_3Q_2 = <4, -2, 2>$

$V \times W = \begin{vmatrix} i & j & k \\ 4 & 2 & -6 \\ 4 & -2 & 2 \end{vmatrix} = \begin{vmatrix} 2 & -6 \\ -2 & 2 \end{vmatrix}, -\begin{vmatrix} 4 & -6 \\ 4 & 2 \end{vmatrix}, \begin{vmatrix} 4 & 2 \\ 4 & -2 \end{vmatrix}$

$= <4-12, -(8+24), -8-8>$

$= <-8, -32, -16>$

$P: -8(x-4) - 32(y-4) - 16(z+0) = 0$

$P: -8x + 32 - 32y + 128 - 16z = 0$

$\boxed{P: -8x - 32y - 16z = -160}$ ✓

QUESTION 4. (7 points) Let $f(x) = -x^3 + 6x^2 + 15x + 1$.

(i) For what values of x does $f(x)$ increase?

$f'(x) = -3x^2 + 12x^1 + 15$

$\boxed{\begin{matrix} x = 5 \\ x = -1 \end{matrix}}$

$f(x)$ increases $\rightarrow (-1, 5)$

(ii) For what values of x does $f(x)$ decrease?

$f(x)$ decreases $\rightarrow (-\infty, -1) \cup (5, +\infty)$

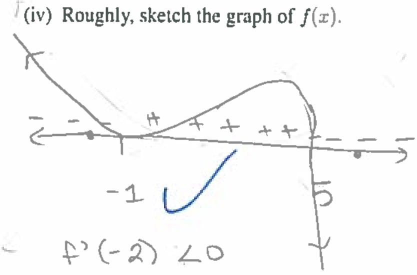
(iii) Find all minimum, maximum points of $f(x)$.

min at $x = -1 \rightarrow (-1, -7)$

max at $x = 5 \rightarrow (5, -43) \rightarrow 101$

$-(5)^3 + 6(25) + 15(5) + 1 = 101$

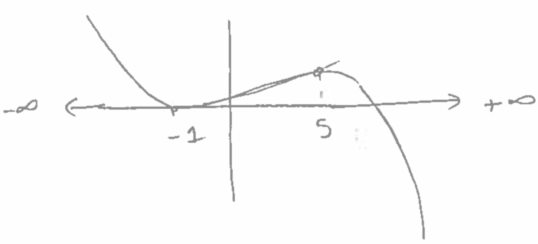
(iv) Roughly, sketch the graph of $f(x)$.



$f'(-2) < 0$

$f'(0) > 0$

$f'(6) < 0$



QUESTION 5. (4 points) Let $f(x) = 2xe^{(x-1)} + ln(2x-1) + 4$. Find the equation of the tangent line to the curve of $f(x)$ at $x = 1$.

$F(x) = 2xe^{(x-1)} + \ln(2x-1) + 4$

$Q: (1, F(1)) = (1, 6)$

$f(1) = 2(1)e^{(1-1)} + \ln(2(1)-1) + 4 = 6$

$f'(x) = (1)'(2) + (2)'(1) + \frac{\log(2x-1)}{\log(10)} + 0$

$f'(x) = 2e^{(x-1)} + e^{(x-1)}(1)(2x) + \log(2x-1) \cdot \frac{1}{\log_{10}}$

$f'(x) = 2e^{(x-1)} + 2xe^{(x-1)} + \frac{2}{\log(10)} \Rightarrow f'(1) = 6$

$y = mx + b$

$6 = 6(1) + b$

$6 = 6 + b$

$6 - 6 = b$

$b = 0$

$\boxed{y = 6x}$

QUESTION 6. (7 points) Consider $f(x) = 4 - \sqrt{x}$, $k(x) = -2$. Find the length and the width of the largest rectangle that you can draw between $f(x)$ and $k(x)$, see picture.

Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: `abadawi@aus.edu`, `www.ayman-badawi.com`

$\rightarrow A = l \cdot w$

$A = m(6 - \sqrt{m})$

$A = m(6 - m^{1/2})$

$A = 6m - m^{3/2}$

$\rightarrow A' = 6 - \frac{3}{2}m^{1/2}$

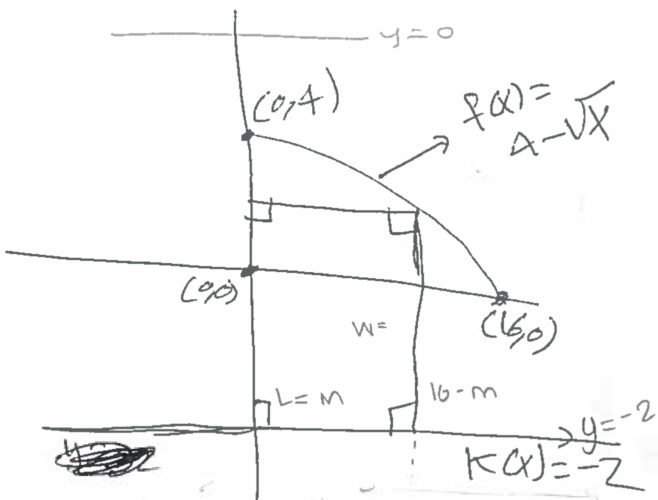
$\rightarrow 0 = 6 - \frac{3}{2}m^{1/2}$

$6 = \frac{3}{2}m^{1/2}$

$\frac{6}{3/2} = \frac{3/2}{3/2}m^{1/2}$

$\sqrt{4} = \sqrt{m^{1/2}}$

$\boxed{M = 16}$



$L = m$

$W = 4 - \sqrt{m} + 2 = 6 - \sqrt{m}$

$L = m = \boxed{16}$

$W = 6 - \sqrt{m} = 6 - \sqrt{16} = \boxed{2}$

$\rightarrow A'' = -\frac{3}{4}m^{-1/2}$

$A''(16) = -\frac{3}{4}(16)^{-1/2} < 0 \rightarrow \boxed{\text{max.}}$

Name Haya Sujaa, ID g00082558

MTH 111 Math for the Architects Spring 2018, 1–4

Exam II: MTH 111, Spring 2018

Ayman Badawi

Points = $\frac{62}{62}$

QUESTION 1. (12 points) Find y' and DO NOT SIMPLIFY

(i) $y = 4e^{(2x^2-4x)} + 2x - 5$

$y' = 4e^{(2x^2-4x)} \cdot (4x-4) + 2$

(ii) $y = (5x^2 + 3x)\sqrt{5x+10}$

$y = (5x^2+3)(5x+10)^{1/2}$

$y' = \left[(5x^2+3) \cdot \frac{1}{2}(5x+10)^{-1/2} \cdot 5\right] + \left[(5x+10)^{1/2} \cdot (10x)\right]$

(iii) $y = ln[(2x^5 + 4x^3 - 3x)(2x+7)^5]$

$y = \ln(2x^5+4x^3-3x) + \ln(2x+7)^5$

$y' = \frac{10x^4 + 12x^2 - 3}{2x^5+4x^3-3x} + \frac{10}{2x+7}$

(iv) $y = 3(e^{(3x+2)} + 7x^4 + 5x + 2)^4$

$y' = 12(e^{(3x+2)} + 7x^4 + 5x + 2)^3 \cdot (3e^{(3x+2)} + 28x^3 + 5)$

(v) **(4 points)** Can we draw the vector $V = < 1, -2, -6 >$ inside $P : 5x + 7y - 3z = 19$? explain

$V \cdot N$ must $= 0$

$\langle 1, -2, -6 \rangle$ | $V \cdot N = 5 + -14 + 18 = 9$
$\langle 5, 7, -3 \rangle$ |

$\therefore$ NO you cannot draw V on the Plane because the Dot product of V and the Normal is not 0

QUESTION 3. (7 points) Let $f(x) = e^{(x^2+2x+1)} + 3$.

(i) For what values of x does $f(x)$ increase?

$f'(x) = e^{(x^2+2x+1)} \cdot (2x+2)$

$e^{(x^2+2x+1)} \cdot (2x+2) = 0$

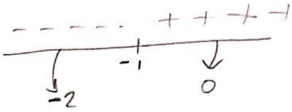
$\frac{}{(2x+2)} \quad \frac{}{(2x+2)}$

$e^{(x^2+2x+1)} = 0$

$\ln e^{(x^2+2x+1)} = 0$

$x^2 + 2x + 1 = 0$

(1) → $x = -1$ ← critical value



← (2)

$f'(-2) = -$

$f'(0) = +$

$\therefore$ $f(x)$ increases from ← (3)

$(-1, +\infty)$

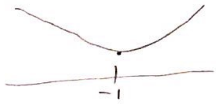
(ii) For what values of x does $f(x)$ decrease?

$f(x)$ is decreasing from $(-\infty, -1)$

(iii) Find all local minimum, maximum points of $f(x)$ (just find the x-values where local min. and local max exist).

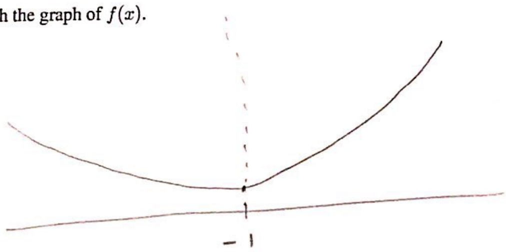
[No local or absolute maximum]

[local and absolute minimum at $x = -1$
point $(-1, 4)$]



$f(-1) = 4$

(iv) Roughly, sketch the graph of $f(x)$.



 Haya

QUESTION 4. (5 points) Let $f(x) = ln(5x-4)+4$. Find the equation of the tangent line to the curve of $f(x)$ at $x = 1$.

$f(1) = \ln(5-4)+4$
$= 4$
point $= (1, 4)$

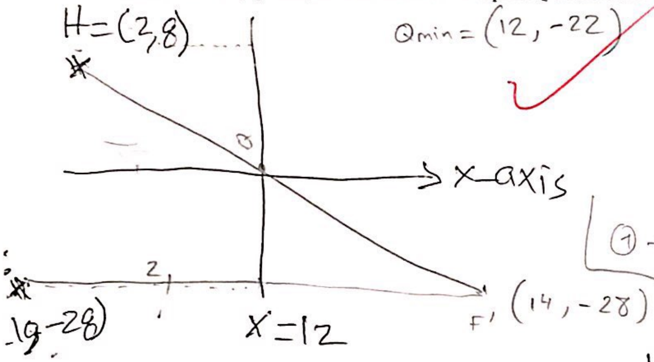
$f'(x) = \frac{5}{5x-4}$

(1) → $f'(1) = m = \frac{5}{1}$

$y = 5x + b$
$4 = 5(1) + b$
(2) → $b = 4 - 5$
$= -1$

(3) → $\boxed{y = 5x - 1}$ is the equation of the tangent line.

QUESTION 5. (7 points) Given H and F. Find a point Q on the line $x = 12$ such that $|HQ| + |FQ|$ is minimum.



$Q_{min} = (12, -22)$

(H=(2,8),
F=(10,-28))

$m = \frac{-28-8}{14-2} = -3$

(1) → $y = -3x + b$
$8 = -3(2) + b$
$b = 8 + 6$
$= 14$
$y = -3x + 14$

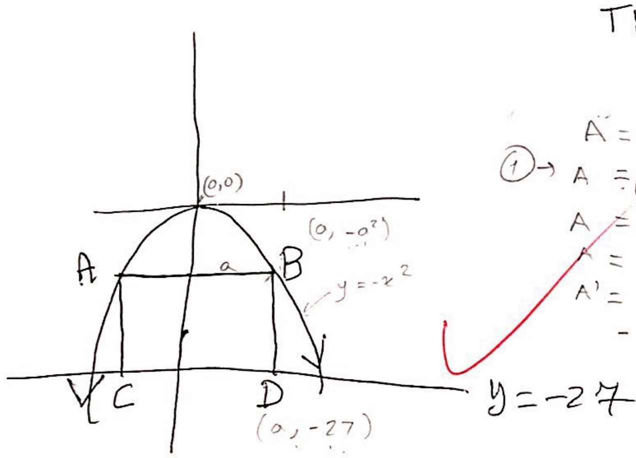
|AC|

$Q = (12, y)$
$y = -3(12) + 14$
$= -22$

$Q_{min} = (12, -22)$

QUESTION 6. (7 points) Consider the following picture. Find $|AB|$ and $|\cancel{CD}|$ so that the area is MAXIMUM.

The curve is $y = -x^2$,
the line $y = -27$



$A = |BD||AB|$

(1) → $A = [-a^2 - (-27)] \cdot 2a$
$A = (-a^2 + 27) \cdot 2a$
$A = -2a^3 + 54a$
$A' = -6a^2 + 54$
$-6a^2 + 54 = 0$
$-6a^2 = -54$
$a^2 = -54/-6$
$a = \sqrt{9}$
$a = +3$

$A'' = -12a$
(2) → $A''(3) = - \therefore$ max.

$A = (-3^2 + 27)(6)$
$= 108$ units^2

$|AB| = 2a$
$= 6$ units

$|BD| = (-a^2 + 27)$
$= 18$ units

Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: abadawi@aus.edu, www.ayman-badawi.com

Final Exam, MTH 111, Fall 2016

Ayman Badawi

QUESTION 1. (8 points)

(i) $\int (x^2+4)^2\,dx =$

$\int (x^2+4)(x^2+4)\,dx$.

$\int x^4+4x^2+4x^2+16$

$\int x^4+8x^2+16\,dx$

$\int x^4+8x^2+16\,dx$.

$\boxed{= \frac{x^5}{5}+\frac{8x^3}{3}+16x+C.}$

(ii) $\int (x+1)(x^2+2x+1)^{10}\,dx$

Power formula $a\,f'(x)\,(f(x))^{n-1}$ $\quad n-1=10$

$a\,f'(x) = x+1$ $\quad n=11.$

$f'(x) = 2x+2$ $\quad 11a(2x+2) = x+1$

$22a\,\cancel{(x+1)} = \cancel{(x+1)}$

$22a = 1$

$a = 1/22.$

$\boxed{\frac{1}{22}(x^2+2x+1)^{11}+C}$

(iii) $\int (x+1)e^{(2x^2+4x)}\,dx =$

$a e^{f(x)} \rightarrow a\,f'(x)\,e^{f(x)}$ ~~the~~

$f'(x) = (2\times 2)x+4$

$= 4x+4.$

$(4x+4)a = (x+1)$

$4\cancel{(x+1)}a = \cancel{(x+1)}$

$4a=1$

$a=\frac{1}{4}$ $\longrightarrow$ $\boxed{\frac{1}{4}e^{2x^2+4x}+C}$

(iv) $\int \frac{6x+6}{3x^2+6x-7}\,dx =$

$\frac{a}{\ln b} \times \frac{f'(x)}{f(x)} = \frac{a\,f'(x)}{f(x)}$

$f'(x) \rightarrow (3\times 2)x+6 = 6x+6$ hence $a=1$

$\boxed{\ln|(3x^2+6x-7)|+C}$

QUESTION 2. (8 points). Find y' and do not simplify

(i) $y = \frac{1+x^2+x^3}{x^{12}}$

$y = (1+x^2+x^3)(x^{-12})$

$y' = (2x+3x^2)(x^{-12}) + (1+x^2+x^3)(-12x^{-13}) \rightarrow \boxed{y' = -12x^{-13}-10x^{-11}-9x^{-10}}$

(ii) $y = e^{(6x^2+7x+1)} + 10x^2 - x + 23$

$y' = e^{6x^2+7x+1} \times (6\times 2)x+7) + (10\times 2)x - 1$

$\boxed{y' = (12x+7)e^{6x^2+7x+1} + 20x - 1}$

(iii) $y = (21+3x-4x^3)^{10}$

$y' = 10(21+3x-4x^3)^9(3-(4\times 3)x^2) \longrightarrow y' = 10(21+3x-4x^3)^9(3-12x^2)$

(iv) $y = ln[(4x+3)^6(-5x+30)^8]$

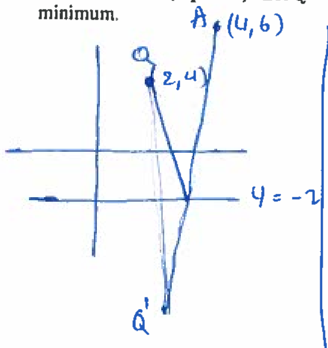
$y = \ln(4x+3)^6 + \ln(-5x+30)^8$

$y = 6\ln(4x+3) + 8\ln(-5x+30)$

$y' = \frac{6\times 4}{4x+3} + \frac{8\times -5}{-5x+30}$

$y' = \frac{24}{4x+3} + \frac{-40}{-5x+30}$

QUESTION 3. (4 points). Let $Q = (2,4)$, $A = (4,6)$. Find a point B on the line $y = -2$ such that $|QB| + |AB|$ is minimum.



$Q' \rightarrow 4-(-2) = 4+2 = 6$

$-2-6 = -8.$

$\rightarrow (2, -8)$

Equation of a line.

$(4,6)\ (2,-8)$

$$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$$

$$\frac{y+8}{6+8} = \frac{x-2}{4-2}$$

$$\rightarrow \frac{y+8}{14} \times \frac{x-2}{2}$$

$2(y+8) = (x-2)14$

$2(-2+8) = (x-2)14$

$\frac{12}{14} + 2 = x$

$x = 20/7$

Point co-ordinates

$\frac{20}{7}$

$\left(\frac{20}{7}, -2\right)$

QUESTION 4. (4 points). For what values of x does the tangent line to the curve $y = 4e^{(3x)} - 26x + 2$ have slope equal 10?

$y' = 10$

$y' = (4e^{3x} \times 3) - 26$

$= 12e^{3x} - 26$

$10 = 12e^{3x} - 26$

$\frac{10+26}{12} = e^{3x}$

$3 = e^{3x} \rightarrow \log_e 3 = 3x \rightarrow \ln 3 = 3x.$

$\frac{\ln 3}{3} = x = 0.366$

QUESTION 5. (6 points). The plane $P_1 : 2x + 2y - z = 2$ intersects the plane $P_2 : -x + y + 2z = 7$ in a line L. Find a parametric equations of L.

$N_1 = \langle 2, 2, -1 \rangle$

$N_2 = \langle -1, 1, 2 \rangle$

$N_1 \times N_2$

$$\begin{vmatrix} 2 & 2 & -1 \\ -1 & 1 & 2 \end{vmatrix} \rightarrow \begin{vmatrix} 2 & -1 \\ 1 & 2 \end{vmatrix} i - \begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix} j + \begin{vmatrix} 2 & 2 \\ -1 & 1 \end{vmatrix} k$$

$[(2\times2) - (-1\times1)]\, i - [(2\times2) - (-1\times-1)]\, j + [(2\times1) - (2\times-1)]\, k$

① $5i - 3j + 4k \rightarrow \langle 5, -3, 4 \rangle$

② Take $z = 0$.

$2x + 2y = 2$

$(-x + y = 7)\, 2$

$-2x + 2y = 14$

$2x + 2y = 2$

$-2x + 2y = 14$

$4y = 16$

$y = \frac{16}{4}$

$= 4$

$2x + 2(4) = 2$

$2x + 8 = 2$

$2x = 2 - 8$

$x = \frac{-6}{2}$

$x = -3$

$(-3, 4, 0)$

③ Parametric equation is

$(-3, 4, 0) + t\langle 5, -3, 4 \rangle$

$x = -3 + 5t$

$y = 4 - 3t$

$z = 4t$

QUESTION 9. (3 points).

$$\int_{-6}^{4} f(x) = 7 - 10 = \underline{\underline{-3}}$$

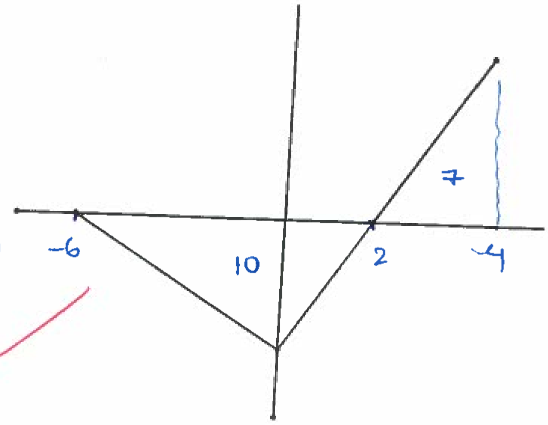


Figure 1. Question: The area of the region that is determined by the curve of f(x) between x = -6 and x = 2 is 10, and the area of the region determined by the curve of f(x) between x = 2 and x = 4 is 7. Find $\int_{-6}^{4} f(x)\,dx$

QUESTION 10. (6 points).

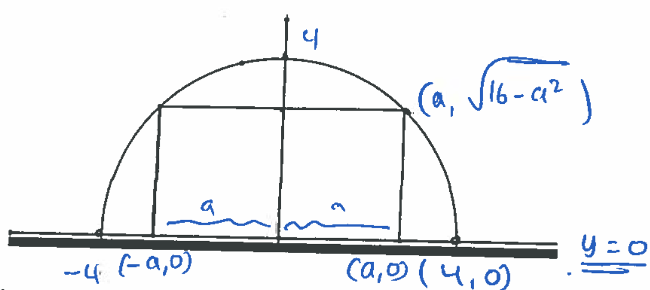


Figure 2. Question: We want to construct a rectangle with maximum area inside the semicircle $y = \sqrt{16-x^2}$ (see picture). Find the area of such rectangle

$A = 2a \times (\sqrt{16-a^2} - 0)$

$A = 2a(\sqrt{16-a^2})$

$A = (2a)(16-a^2)^{1/2}$

Product formula.

$A' = 2(16-a^2)^{1/2} + \cancel{2}a \times \frac{1}{\cancel{2}}(16-a^2)^{-1/2}(-2a)$

$0 = 2(16-a^2)^{1/2} - 2a^2(16-a^2)^{-1/2}$

$\cancel{2}(16-a^2)^{1/2} = \cancel{2}a^2(16-a^2)^{-1/2}$

$(16-a^2)^{1/2} = a^2(16-a^2)^{-1/2}$

$(16-a^2)^{1/2} = \dfrac{a^2}{(16-a^2)^{1/2}}$

$(16-a^2)^{1/2}(16-a^2)^{1/2} = a^2$

$16 - a^2 = a^2$

$16 = 2a^2$

$\pm\sqrt{\frac{16}{2}} = a$

$\pm\sqrt{8} = a$

a is alway +ve

hence

$a = 2\sqrt{2}$.

$A = (2\sqrt{2})2 \times \sqrt{16-(2\sqrt{2})^2}$

$= 4\sqrt{2} \times 2\sqrt{2}$

$= \underline{\underline{16}}$

QUESTION 11. Let $y = -x^3 + 12x + 2$

(i) Find all x values where $f(x)$ is maximum.

$x = 2$

$y' = -3x^2 + 12$

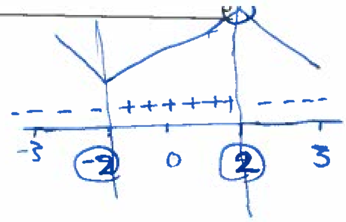
$0 = -3x^2 + 12$

$-12 = -3x^2$

$\pm\sqrt{\frac{12}{3}} = x$

$\pm\sqrt{4} = x$

$\pm 2 = x$



(ii) Find all x values where $f(x)$ is minimum.

$x = -2$

(iii) For what values of x does $f(x)$ increase?

$(-2, 2)$

(iv) For what values of x does $f(x)$ decrease?

$(-\infty, -2) \cup (2, \infty)$

(v) For what values of x do the slopes of tangent lines are positive?

$(-2, 2)$

(vi) What is the equation of the normal line to the curve of f(x) at the point (1, 13)?

$y' = -3x^2 + 12$

$= -3(1)^2 + 12$

$= 9$

negative reciprocal $= \frac{-1}{9}$

$y = mx + c$

$13 = \frac{-1}{9}(1) + c$

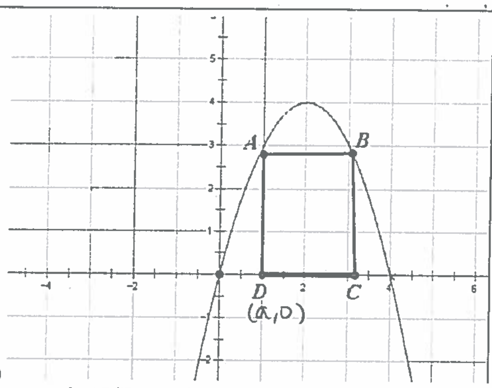
$13 + \frac{1}{9} = c \quad c = \frac{118}{9}$

$y = mx + c$

$\boxed{y = -\frac{1}{9}x + \frac{118}{9}}$

Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: abadawi@aus.edu, www.ayman-badawi.com



$y = -x^2 + 4x$

$D(a,0) \to A = (a, -a^2+4a)$

$L = -a^2 + 4a$

$|OD| = |CF| = a$

$|CD| = W = 4 - 2a$

QUESTION 9. (8 points)

We want to construct a rectangle ABCD (see picture) of maximum area between the x-axis and the curve $y = -x^2 + 4x$. Find the length and the width of such rectangle. (Hint: Note that the curve intersects x-axis at $x = 0$ and at $x = 4$. Let O be the origin (0, 0) and F be (4, 0). Then |OD| = |CF|)

$A = W \times L$

$L = -a^2 + 4a$, $W = 4 - 2a \to A = (4-2a)(-a^2+4a)$

$A = -4a^2 + 16a + 2a^3 - 8a^2 = 2a^3 - 12a^2 + 16a$

$A' = 6a^2 - 24a + 16 = 0 \to 2(3a^2 - 12a + 8) = 0$

$\to 3a^2 - 12a + 8 = 0 \to a = \dfrac{12 \pm \sqrt{48}}{2(3)}$

$A'' = 12a - 24 \to a = \dfrac{12+\sqrt{48}}{6} \to A'' > 0$

$a = \dfrac{12-\sqrt{48}}{6} \to A'' < 0 \to$ Area max

when $a = \dfrac{12-\sqrt{48}}{6}$

$\boxed{L = \dfrac{8}{3}}$, $\boxed{W = \dfrac{4\sqrt{3}}{3}}$ $\to$ $\boxed{A = \dfrac{32\sqrt{3}}{9} \text{ units}^2}$

Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: abadawi@aus.edu, www.ayman-badawi.com

 Hoya

QUESTION 4. (5 points) Let $f(x) = ln(5x-4)+4$. Find the equation of the tangent line to the curve of $f(x)$ at $x = 1$.

$f(1) = \ln(5-4)+4$
$= 4$
Point $= (1, 4)$

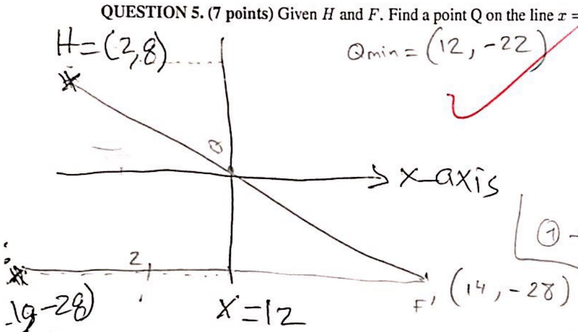
$f'(x) = \frac{5}{5x-4}$

① → $f'(1) = m = \frac{5}{1}$

$y = 5x + b$
$4 = 5(1) + b$
② → $b = 4 - 5$
$= -1$

③ → $\boxed{y = 5x - 1}$ is the equation of the tangent line.

QUESTION 5. (7 points) Given H and F. Find a point Q on the line $x = 12$ such that $|HQ| + |FQ|$ is minimum.



Qmin = (12, -22)

(H = (2,8),
F = (10, -28))

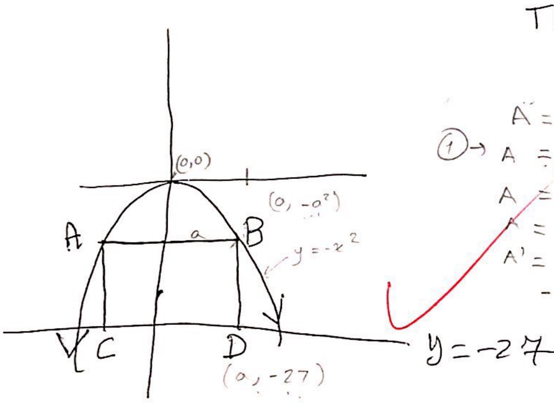
$m = \frac{-28-8}{14-2} = -3$

① → $y = -3x + b$
$8 = -3(2) + b$
$b = 8 + 6$
$= 14$
$y = -3x + 14$

|AC|

$Q = (12, y)$
$y = -3(12) + 14$
$= -22$
$Q_{min} = (12, -22)$

QUESTION 6. (7 points) Consider the following picture. Find $|AB|$ and ~~$|CD|$~~ so that the area is MAXIMUM.



The curve is $y = -x^2$,
the line $y = -27$

$A = |BD||AB|$

① → $A = [-a^2 - (-27)] \cdot 2a$
$A = (-a^2 + 27) \cdot 2a$
$A = -2a^3 + 54a$
$A' = -6a^2 + 54$
$-6a^2 + 54 = 0$
$-6a^2 = -54$
$a^2 = -54/-6$
$a = \sqrt{9}$
$a = +3$
$A'' = -12a$
② → $A''(3) = - \therefore$ max.

$A = (-3^2 + 27)(6)$
$= 108$ units^2

$|AB| = 2a$
$= 6$ units

$|BD| = (-a^2 + 27)$
$= 18$ units

Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: abadawi@aus.edu, www.ayman-badawi.com

2.1.4 Final Exam Review

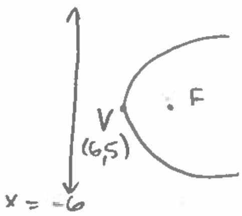
Final Exam: MTH 111, Fall 2017

Ayman Badawi

$$\text{Points} = \frac{81}{82}$$

QUESTION 1. (6 points) Given $x = -6$ is the directrix of of a parabola that has the point $(6, 5)$ as its vertex point.

a) Find the equation of the parabola



$|VL| = |-6-6| = |-12| = 12$

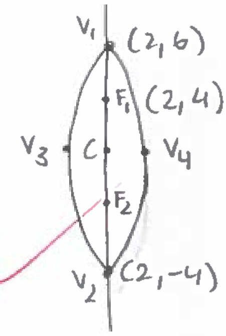
$4(12)(x-6) = (y-5)^2 \Rightarrow 48(x-6) = (y-5)^2$

b) Find the focus of the parabola.

$|VF| = 12 \rightarrow F(18, 5)$

QUESTION 2. (8 points) Given $(2, -4), (2, 6)$ are the vertices of the major axis of an ellipse (recall major axis is the longer axis) and $(2, 4)$ is one of the foci.

(i) Find the vertices of the minor axis (shorter axis). (you may want to draw such ellipse (roughly)).



$|V_1V_2| = K = |6+4| = 10 \rightarrow \frac{K}{2} = 5 = |V_1C|$

$C = (2, 1) \rightarrow |F_1C| = |4-1| = 3 \rightarrow b^2 = \left(\frac{K}{2}\right)^2 - |F_1C|^2$

$b^2 = 5^2 - 3^2 = 16 \rightarrow V_3(18, 1), V_4(-14, 1)$

(ii) Find the ellipse-constant K.

$K = 10$

(iii) Find the second foci of the ellipse.

$F_2(2, -2)$

(iv) Find the equation of the ellipse.

$$\frac{(x-2)^2}{16} + \frac{(y-1)^2}{25} = 1$$

QUESTION 3. (5 points) Given $y = 3x^2 + 12x + 9$ is an equation of a parabola. Write the equation of the parabola in the standard form and find the equation of its directrix.

$y = 3x^2 + 12x + 9 \rightarrow y = 3(x^2 + 4x + 3) \rightarrow y = 3\left[(x+2)^2 - 4 + 3\right]$

$y = 3(x+2)^2 - 1(3) \rightarrow \frac{1}{3}(y+3) = (x+2)^2$

$4d = \frac{1}{3} \rightarrow d = \frac{1}{12}$

$V = (-2, -3) \rightarrow$ directrix $\rightarrow x = -2 - \frac{1}{12} \Rightarrow \boxed{\frac{-25}{12} = x}$

$y = -3 - \frac{1}{12}$

1

QUESTION 4. a) **(4 points)** Given two lines $L_1 : x = 2t, y = -2t + 3, z = -t + 1$ and $L_2 : x = -4w - 12, y = 4w + 15, z = 2w + 7$. Is L_1 parallel to L_2? EXPLAIN clearly.

$L_1: x = 2t,\ y = -2t+3,\ z = -t+1 \quad t \in \mathbb{R}$

$L_2: x = -4w-12,\ y = 4w+15,\ z = 2w+7 \quad w \in \mathbb{R}$

$\rightarrow D_1 = \langle 2, -2, -1 \rangle,\ D_2 = \langle -4, 4, 2 \rangle$

$D_2 = C D_1 \rightarrow C = -2 \rightarrow D_1 \parallel D_2$

intersection: $L_1 \rightarrow t = 0 \rightarrow Q(0,3,1)$

Q lies on V_2

$L_2 \rightarrow x: 0 = -4w - 12 \rightarrow w = -3$ / $z: 1 = 2w + 7 \rightarrow w = -3$

$y: 3 = 4w + 15 \rightarrow w = -3$ / $\rightarrow \boxed{L_1 \text{ not } \parallel L_2 \text{ they overlap}}$

b)**(4 points)** Let L be the line L_1 as in (a). Given that the point $Q = (2, 3, 4)$ does not lie on L. Find $|QL|$ (distance between Q and L).

$I = (0,3,1),\ Q(2,3,4) \rightarrow \vec{IQ} = \langle 2, 0, 3 \rangle$

$|QL| = \dfrac{|\vec{IQ} \times D_1|}{|D_1|}, \quad \vec{IQ} \times D = \begin{vmatrix} i & j & k \\ 2 & 0 & 3 \\ 2 & -2 & -1 \end{vmatrix} = \langle 6, 8, -4 \rangle$

$\rightarrow |QL| = \dfrac{\sqrt{6^2 + 8^2 + (-4)^2}}{\sqrt{2^2 + (-2)^2 + (-1)^2}} = \boxed{\dfrac{2\sqrt{29}}{3}}$

c) **(6 points)** Convince me that $q_1 = (1, 4, 2), q_2 = (2, 1, -1)$, and $q_3 = (3, 5, 2)$ are not co-linear. Then find the area of the triangle with vertices q_1, q_2, q_3.

$\vec{q_1 Q_2} = \langle 1, -3, -3 \rangle$

$\vec{Q_1 Q_3} = \langle 2, 1, 0 \rangle$

$\rightarrow \vec{Q_1Q_2} \times \vec{Q_1 Q_3} = \begin{vmatrix} i & j & k \\ 1 & -3 & -3 \\ 2 & 1 & 0 \end{vmatrix} = \langle 3, -6, 7 \rangle \Rightarrow \boxed{\vec{Q_1Q_3} \text{ not } \parallel \vec{Q_1Q_2} \Rightarrow \text{not colinear}}$

$A_\Delta = \frac{1}{2} |\vec{Q_1Q_2} \times \vec{Q_1Q_3}|$

$= \frac{1}{2}\sqrt{3^2 + (-6)^2 + 7^2}$

$= \dfrac{\sqrt{94}}{2} \text{ units}^2$

d)**(6 points)** The two planes $P_1 : 2x + y + 2z = 2$ and $P_2 : -x + y - z = 5$ intersects in a line L. Find a parametric equations of L.

$N_1 = \langle 2, 1, 2 \rangle,\ N_2 = \langle -1, 1, -1 \rangle$

$D = N_1 \times N_2 = \begin{vmatrix} i & j & k \\ 2 & 1 & 2 \\ -1 & 1 & -1 \end{vmatrix} = \langle -3, 0, 3 \rangle$

$\rightarrow$ let $z = 0 \rightarrow 2x + y = 2$, $\ -1 \times [-x + y = 5] \rightarrow$

$2x + \cancel{y} = 2$

$x - \cancel{y} = -5$

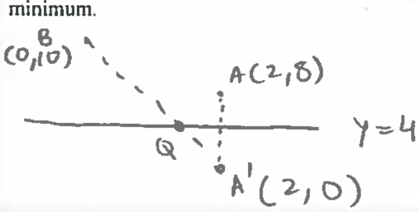
$3x = -3 \rightarrow \boxed{x = -1} \rightarrow 2(-1) + y + 2(0) = 2$

$\boxed{y = 4}$

$Q = (-1, 4, 0)$

$\rightarrow \boxed{L_1: \ x = -3t - 1,\ y = 4,\ z = 3t \quad t \in \mathbb{R}}$

QUESTION 5. (6 points) Let $A = (2, 8)$, $B = (0, 10)$. Find a point Q on the line $y = 4$ such that $|BQ| + |QA|$ is minimum.



$|AL| = |8 - 4| = 4$

$\rightarrow m = \dfrac{y_1 - y_2}{x_1 - x_2} = \dfrac{10 - 0}{0 - 2} = \dfrac{10}{-2} = -5$

$y = -5x + b \rightarrow 10 = -5(0) + b \rightarrow b = 10$

$y = -5x + 10 \rightarrow 4 = -5x + 10 \rightarrow 4 - 10 = -5x \rightarrow x = \dfrac{+6}{+5}$

$\boxed{Q = \left(\frac{6}{5}, 4\right)}$

QUESTION 6. (9 points)

(i) Given $f'(1) = 2$ and $y = f(x^2 + 2x - 7)$. Then $y'(2) =$

$y' = [f'(x^2+2x-7)][2x+2] = [f'(2^2+2(2)-7)][2(2)+2] =$

$[f'(1)][6] = 6(2) = \boxed{12}$

(ii) Let $f(x) = -6e^{(x^3+6x-7)}$. Then $f'(x) =$

$f(x) = -6e^{(x^3+6x-7)} \rightarrow f'(x) = -6(3x^2+6)(e^{x^3+6x-7})$

$f(x) = \ln(5x-9)^3 + \ln(2x-3)^7 = 3\ln(5x-9) + 7\ln(2x-3)$.

(iii) Let $f(x) = \ln((5x-9)^3(2x-3)^7)$. Then $f'(x) =$ $\boxed{f'(x) = \frac{3(5)}{5x-9} + \frac{7(2)}{2x-3}}$

QUESTION 7. (10 points)

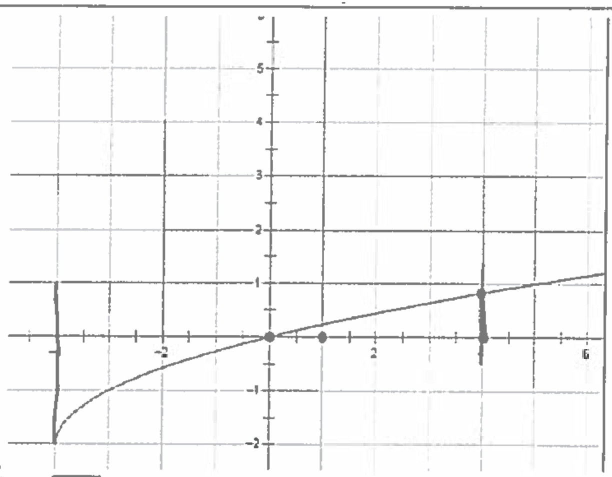
$\int \frac{x+1}{x^2+2x+1}\,dx = \int (x+1)(x^2+2x+1)^{-1}\,dx = \boxed{\frac{1}{2}\ln|(x^2+2x+1)| + C}$

(ii) $\int \frac{e^x+3}{(e^x+3x+1)^2}\,dx = \int (e^x+3)(e^x+3x+1)^{-2}\,dx = \boxed{\frac{(e^x+3x+1)^{-1}}{-1} + C}$

(iii) $\int x^5(x+1)^2\,dx = \int x^5(x^2+2x+1)\,dx = \int x^7 + 2x^6 + x^5\,dx =$

$\int x^7\,dx + 2\int x^6\,dx + \int x^5\,dx = \boxed{\frac{x^8}{8} + \frac{2x^7}{7} + \frac{x^6}{6} + C}$

(iv) $\int 10(2x+7)^{11}\,dx = 5\int 2(2x+7)^{11}\,dx \Rightarrow \boxed{\frac{5(2x+7)^{12}}{12} + C}$



$y = \sqrt{x+4} - 2$

$2 = \sqrt{x+4}$

$4 = x+4$

$x = 0$

QUESTION 8.

Stare at $f(x) = \sqrt{x+4} - 2$ where $-4 \le x \le 4$. Then

a) **(6 points)** Find the area of the region bounded by the curve of $f(x)$, x-axis, and $-4 \le x \le 4$.

$$-\int_{-4}^{0} \sqrt{x+4} - 2\,dx + \int_{0}^{4} \sqrt{x+4} - 2\,dx = -\left(\frac{2}{3}(x+4)^{3/2} - 2x\Big|_{-4}^{0}\right) + \left(\frac{2}{3}(x+4)^{3/2} - 2x\Big|_{0}^{4}\right) = -\left(\frac{16}{3} - 8\right) + \left[\left(\frac{2}{3}(8)^{3/2} - 8\right) - \frac{16}{3}\right]$$

$$\boxed{\text{Area} \approx 4.42 \text{ unit}^2}$$

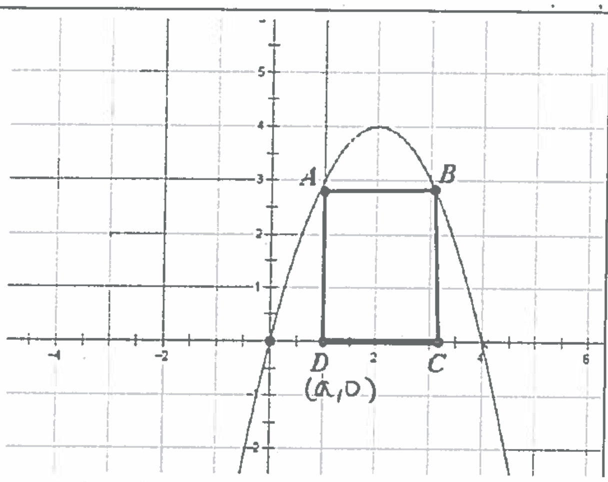
b) **(4 points)** Imagine that the region between x = 0 and x = 4 is rotated about x-axis 360 degrees. What is the volume of the object?

$$\pi \int_{0}^{4} (\sqrt{x+4} - 2)^2 dx \rightarrow \pi \int_{0}^{4} (x+4) - 4\sqrt{x+4} + 4\,dx$$

$$\Rightarrow \pi \left[\int_{0}^{4} x+8\,dx - 4\int_{0}^{4} \sqrt{x+4}\,dx\right] \Rightarrow \pi\left[\left(\frac{x^2}{2} + 8x\Big|_{0}^{4}\right) - 4\left(\frac{2(x+4)^{3/2}}{3}\Big|_{0}^{4}\right)\right]$$

$$\Rightarrow \pi\left[(40 - 0) - 4\left(\frac{2(8)^{3/2}}{3} - \frac{2(4)^{3/2}}{3}\right)\right]$$

$$\boxed{\text{Volume} \approx 0.99\pi \text{ units}^3}$$



$Y = -x^2 + 4x$

$D(a,0) \to A = (a, -a^2+4a)$

$L = -a^2+4a$

$|OD| = |CF| = a$

$|CD| = W = 4-2a$

QUESTION 9. (8 points)

We want to construct a rectangle ABCD (see picture) of maximum area between the x-axis and the curve $y = -x^2 + 4x$. Find the length and the width of such rectangle. (Hint: Note that the curve intersects x-axis at $x = 0$ and at $x = 4$. Let O be the origin (0, 0) and F be (4, 0). Then |OD| = |CF|)

$A = W \times L$

$L = -a^2+4a \;,\; W = 4-2a \to A = (4-2a)(-a^2+4a)$

$A = -4a^2 + 16a + 2a^3 - 8a^2 = 2a^3 - 12a^2 + 16a$

$A' = 6a^2 - 24a + 16 = 0 \to 2(3a^2 - 12a + 8) = 0$

$\to 3a^2 - 12a + 8 = 0 \to a = \dfrac{12 \pm \sqrt{48}}{2(3)}$

$A'' = 12a - 24 \to a = \dfrac{12+\sqrt{48}}{6} \to A'' > 0$

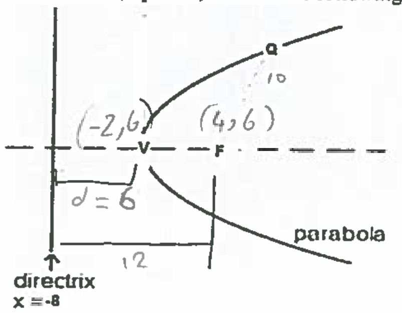
$a = \dfrac{12-\sqrt{48}}{6} \to A'' < 0 \to$ Area max when $a = \dfrac{12-\sqrt{48}}{6}$

$\boxed{L = \dfrac{8}{3}} \;,\; \boxed{W = \dfrac{4\sqrt{3}}{3}} \to \boxed{A = \dfrac{32\sqrt{3}}{9} \text{ units}^2}$

Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: abadawi@aus.edu, www.ayman-badawi.com

QUESTION 3. (4 points) Stare at the following graph.



Given $F = (4, 6)$, the directrix line, L is $x = -8$, and $|QF| = 10$.

(i) Find $|QL| = |QF| = 10$

(ii) Find $v = (-2, 6)$

(iii) Find the equation of the parabola

$24(x+2) = (y-6)^2$

QUESTION 4. (6 points). Find y' and do not simplify

(i) $y = ln[(4x+3)^{10}(-5x+30)^3]$

$y = \ln(4x+3)^{10} + \ln(-5x+30)^3$

$y = 10\ln(4x+3) + 3\ln(-5x+30)$

$y' = \frac{10 \cdot 4}{4x+3} + \frac{3 \cdot -5}{-5x+30}$

$y' = \frac{40}{(4x+3)} + \frac{-15}{(-5x+30)}$

(ii) $y = e^{(6x^3+x^2-1)} + 10x^2 - x + 23$

$y' = \left[\left[e^{(6x^3+x^2-1)} \cdot (18x^2+2x)\right] + 20x - 1\right]$

(iii) $y = (21 + 5x - 6x^3)^7$

$y' = 7(21+5x-6x^3)^6 \cdot (5-18x^2)$

QUESTION 5. (6 points).

(i) Find $\int xe^{(x^2+1)}\,dx$

$u = x^2+1$

$u' = 2x$

$\frac{1}{2}\left(e^{(x^2+1)}\right) + C$

(ii) Find $\int \frac{e^{2x}+1}{(e^{2x}+2x-5)^3}\,dx$

$\int (e^{2x}+1)(e^{2x}+2x-5)^{-3}\,dx$

$u = e^{2x} + 2x - 5$

$u' = 2e^{2x} + 2$

$\frac{1}{2}\int 2(e^{2x}+1)(e^{2x}+2x-5)^{-3}\,dx$

$\frac{1}{2} \cdot \frac{1}{-2}(e^{2x}+2x-5)^{-2} + C$

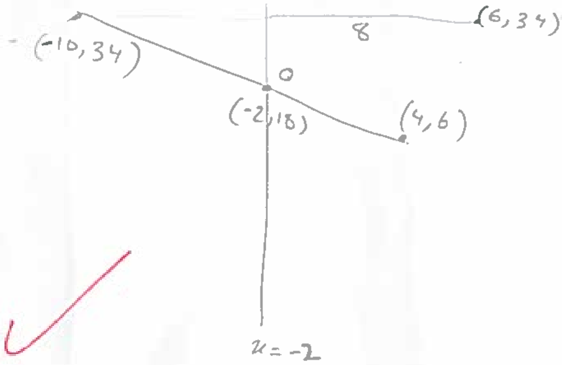
(iii) Find $\int (6x+3)(x^2+x-5)^{11}\,dx$

$u = x^2 + x - 5$

$u' = 2x + 1$

$3 \cdot \frac{1}{12}(x^2+x-5)^{12} + C$

QUESTION 6. (5 points). Let $H = (4,6)$, $F = (6,34)$. Find a point Q on the line $x = -2$ such that $|HQ| + |FQ|$ is minimum.



$y = mx + b$

$m = \frac{6-34}{4+10} = -2$

$6 = -2(4) + b$

$b = 14$

$y = -2x + 14$

$y = -2(-2) + 14$
$= 18$

$Q = (-2, 18)$

QUESTION 7. (4 points). For what values of x does the tangent line to the curve $y = ln(4x+1) + 7x + 2$ have slope equal 8?

$y' = 8$

$y' = \frac{4}{4x+1} + 7 = 8$

$\frac{4}{4x+1} = 1$

$4 = 4x + 1$

$4x = 4 - 1$

$x = 3/4$

check $\frac{4}{4(\frac{3}{4})+1} + 7 =$

$1 + 7 = 8$ //

the T line has slope 8 at $x = \frac{3}{4}$

QUESTION 8. (6 points). The plane $P_1 : x + 2y - 3z = 2$ intersects the plane $P_2 : -x + 5y + z = 19$ in a line L. Find a parametric equations of L.

① → $N_1 \times N_2 = D$

$N_1 = \langle 1, 2, -3 \rangle$

$N_2 = \langle -1, 5, 1 \rangle$

$D = (2+15)i - (1-3)j + (5+2)k$
$= \langle 17, 2, 7 \rangle$

② → $z = 0$

$x + 2y = 2$

$-x + 5y = 19$

$x = \frac{\begin{vmatrix} 2 & 2 \\ 19 & 5 \end{vmatrix}}{\begin{vmatrix} 1 & 2 \\ -1 & 5 \end{vmatrix}}$ $\quad y = \frac{\begin{vmatrix} 1 & 2 \\ -1 & 19 \end{vmatrix}}{\begin{vmatrix} 1 & 2 \\ -1 & 5 \end{vmatrix}}$

$x = \frac{-28}{7}$ $\quad y = \frac{21}{7}$

$x = -4$ $\quad y = 3$

③ → $(-4, 3, 0)$

$D = \langle 17, 2, 7 \rangle$

$L:\ \left.\begin{aligned} x &= 17t - 4 \\ y &= 2t + 3 \\ z &= 7t \end{aligned}\right\}\ t \in \mathbb{R}$

QUESTION 9. (5 points). Can we draw the entire line $L : x = 2t, y = -3t + 1, z = 11t + 4$ inside the plane $2x - 6y - 2z = 20$? EXPLAIN

$N_{plane} \cdot D_{line}$ must $= 0$

$N = \langle 2, -6, -2 \rangle$

$D = \langle 2, -3, 11 \rangle$

$N \cdot D = 4 + 18 - 22$
$= 0$ //

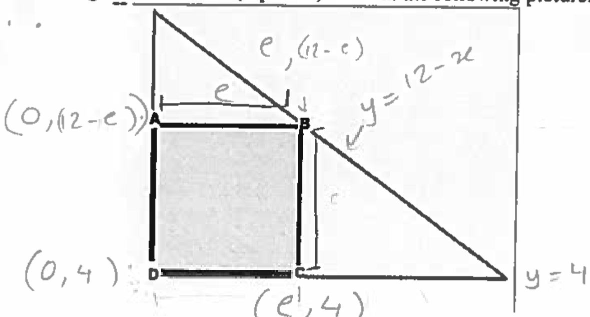
take a point on L and check if the point lies in the plane or not

NO

yes the line can be entirely drawn on the plane because the dot product of the normal and directional vector is 0

2

QUESTION 10. (8 points) Stare at the following picture.



We want to construct a rectangle ABCD of largest area as in the picture above. Note that A and D lie on the y-axis, D and C lie on the line y = 4 (note that y = 4 intersects the y-axis at D), and B lies on the line y = 12 - x. Find |DC| and |BC|.

$|BC| = (12-e) - 4$

$|DC| = e$

$A = |BC| \cdot |DC|$

$= [(12-e)-4] \cdot e$

$= (-e+8)e$

$= -e^2 + 8e$

$A' = -2e + 8$

① → $-2e + 8 = 0$

$e = 4$

② → $|BC| = (12-4) - 4$

$= 8 - 4$

$= 4$ units

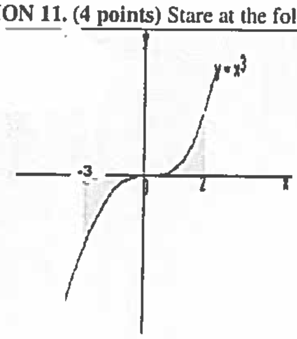
$|DC| = e$

$= 4$ units

Area $= 4 \times 4$

$= 16$ units^2

QUESTION 11. (4 points) Stare at the following picture.



Find the area of the shaded region. Note that $y = f(x) = x^3$ and x is between -3 and 2.

$A = \left[\int_{-3}^{0} x^3\,dx\right] + \int_{0}^{2} x^3\,dx$

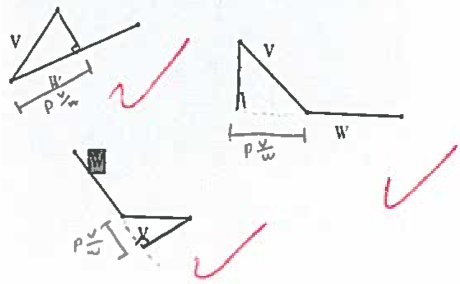
$= \left[\int_{-3}^{0} \frac{1}{4}x^4\right] + \int_{0}^{2} \frac{1}{4}x^4$

$= \left[\left[\frac{1}{4}0^4\right] - \left[\frac{1}{4}(-3)^4\right]\right] + \left[\left[\frac{1}{4}(2)^4\right] - \left[\frac{1}{4}(0)^4\right]\right]$

$= [0 + 20.25] + [4 - 0]$

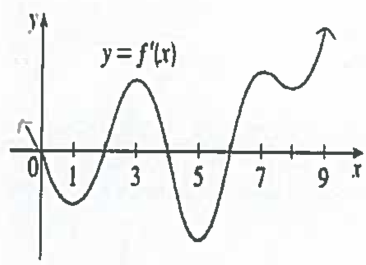
$= 24.25$ units^2

QUESTION 12. (4.5 points) Stare at the following picture.



Draw the projection of V over W.

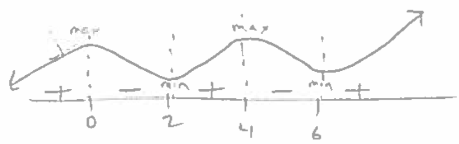
QUESTION 13. (7.5 points) Stare at the following graph of $y = f'(x)$.



critical values = 0, 2, 4, 6

↗ = (−∞, 0), (2, 4), (6, +∞)

↘ = (0, 2), (4, 6)



(i) At what value(s) of x does $f(x)$ have local max.?

at $x = 0$ and $x = 4$

(ii) At what value(s) of x does $f(x)$ have local min.?

at $x = 2$ and $x = 6$

(iii) For what values of x does $f(x)$ increase?

$(-\infty, 0) \cup (2,4) \cup (6, +\infty)$

(iv) For what values of x does $f(x)$ decrease?

$(0,2) \cup (4,6)$

(v) For what values of x will the normal lines have positive slope.

Normal line will have a + slope whe the tangent line has − slope

∴ when the function x is decreasing ∴ $(0,2) \cup (4,6)$

QUESTION 14. (5 points) Given $L_1 : x = 2t, y = t + 1, z = 3t$ is perpendicular to $L_2 : x = 4w + 6, y = -2w, z = aw + 1$ and they intersect at a point Q. Find the value of a and find the point Q.

$L_1 : \left.\begin{array}{l} x = 2t \\ y = t+1 \\ z = 3t \end{array}\right\} t \in \mathbb{R}$ $\quad L_2 : \left.\begin{array}{l} x = 4w+6 \\ y = -2w \\ z = aw+1 \end{array}\right\} w \in \mathbb{R}$

$Q = (2, 2, 3)$

$a = -2$

$x = x \quad y = y$

$2t = 4w + 6 \quad | \quad t + 1 = -2w, \quad t + 2w = -1$

$2t - 4w = 6$

$t + 2w = -1$

$t = \dfrac{\begin{vmatrix} 6 & -4 \\ -1 & 2 \end{vmatrix}}{\begin{vmatrix} 2 & -4 \\ 1 & 2 \end{vmatrix}} \quad w = \dfrac{\begin{vmatrix} 2 & 6 \\ 1 & -1 \end{vmatrix}}{\begin{vmatrix} 2 & -4 \\ 1 & 2 \end{vmatrix}}$

$t = 1$

$w = -1$

$x = 2$

$y = 2$

$z = 3$

$z = aw + 1$

$3 = a(-1) + 1$

$3 - 1 = a(-1)$

$2 = a \cdot -1$

$a = \frac{2}{-1} \quad a = -2$

Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: abadawi@aus.edu, www.ayman-badawi.com

Final Exam: MTH 111, Spring 2018

Ayman Badawi

Points = $\frac{\quad}{100}$

QUESTION 1. (9 points) Find y' and DO NOT SIMPLIFY

(i) $y = (x+1)e^{(3x+2)}$

$y' = e^{3x+2} + (3x+3)e^{3x+2} = e^{3x+2}(3x+4)$

(ii) $y = ln[(3x-2)^4(2x+1)^7]$

$y' = \frac{12}{3x-2} + \frac{14}{2x+1}$

(iii) $y = (7x+2)^9$

$y' = 63(7x+2)^8$

QUESTION 2. (i) **(6 points)** Does the line line $L_1 : x = t+1, y = t-1, z = 7$ intersect the line $L_2 : x = -w+4, y = w-2, z = 2w+3$? If yes, then find the intersection point. Is L_1 perpendicular to L_2?

$D_1 < 1,1,0 >$ $D_2 < -1,1,2 >$

$D_1 \neq c D_2 \Rightarrow$ $\boxed{L_1 \text{ and } L_2 \text{ intersect}}$

$D_1 \times D_2 = < 2,-2,2 >$

$\Rightarrow L_1$ not $\perp L_2$.

-2

$t+1 = -w+4 \rightarrow t+w = 3$

$t-1 = w-2 \rightarrow t-w = -1$

$t = 1 \quad w = 2$

using $t=1$:

$x = 1+1 = 2$

$y = 1-1 = 0$

$z = 7$

or

Using $w=2$:

$x = -2+4 = 2$

$y = 2-2 = 0$

$z = 2(2)+3 = 7$

$\boxed{\text{point of intersection } (2,0,7)}$

(ii) **(4 points)** Convince me that $L_1 : x = t, y = 10, z = -t+4$ is parallel to $L_2 : x = 4w+1, y = 7, z = -4w+2$

$D_1 = < 1,0,-1 >$ $D_2 = < 4,0,-4 >$

$D_2 = 4D_1$

$t=0 \rightarrow (0,10,4)$

$$\left.\begin{array}{l} x:\ 0 = 4w+1 \rightarrow w = \frac{-1}{4} \\ z:\ 4 = -4w+2 \rightarrow w = \frac{-1}{4} \\ y:\ w = 0 \end{array}\right\}$$ diff. $w \Rightarrow L_1$ and L_2 are parallel.

(iii) Let $Q_1 = (1,1,0)$, $Q_2 = (0,-1,2)$ and $Q_3 = (2,2,2)$.

a. **(5 points)** Find the equation of the plane that contains Q_1, Q_2, Q_3.

$\overrightarrow{Q_1Q_2} <-1,-2,2> \quad \overrightarrow{Q_1Q_3} <1,1,2>$

$N = |Q_1Q_2 \times Q_1Q_3| = \begin{vmatrix} i & j & k \\ -1 & -2 & 2 \\ 1 & 1 & 2 \end{vmatrix} = <-6,4,1>$

$P: \ -6(x-2) + 4(y-2) + 1(z-2) = 0$

b. **(2 points)** Find the area of the triangle that has Q_1, Q_2, Q_3 as vertices.

$A = \frac{1}{2} |\overrightarrow{Q_1Q_2} \times \overrightarrow{Q_1Q_3}| = \frac{\sqrt{6^2+4^2+1^2}}{2} = \frac{\sqrt{53}}{2}$ units^2

(iv) **(4 points)** Given $L: x = t+1, y = 8, z = 4t+1$ lies entirely inside the plane $P: ax + 2y + z = b$ Find the values of a, b.

$D <1,0,4> \quad N <a,2,1>$

$N \cdot D = 0$

$a + 4 = 0$

$\boxed{a = -4}$

$-4(t+1) + 2(8) + 4t + 1 = b$

$-4t - 4 + 16 + 4t + 1 = b$

$\boxed{b = 13}$

(v) **(4 points)**item Find the distance between the point $(1,-1,1)$ and the line $L: x = t+1, y = 2t+3, z = -2t+10$

$Q(1,-1,1) \quad I(1,3,10)$

$V = \overrightarrow{IQ} = <0,-4,-9> \quad D <1,2,-2>$

$V \times D = \begin{vmatrix} i & j & k \\ 0 & -4 & -9 \\ 1 & 2 & -2 \end{vmatrix} = <26,-9,4>$

$d = \frac{|V \times D|}{|D|} = \frac{\sqrt{26^2+9^2+4^2}}{\sqrt{1^2+2^2+2^2}} = \frac{\sqrt{773}}{3}$ units

(vi) **(3 points)** For what values of x will the tangent line to the curve $f(x) = e^x - 4x + 2$ be horizontal? (Hint: Note that horizontal lines have slope 0)

$f'(x) = e^x - 4$

$0 = e^x - 4$

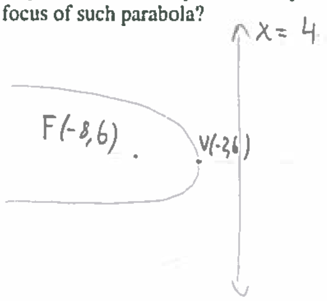
$e^x = 4$

$\ln e^x = \ln 4$

$x \ln e = \ln 4$

$\boxed{x = \ln 4}$

(vii) **(5 points)** Find the equation of a parabola that has x = 4 as its directrix line and (-2, 6) as its vertex. What is the focus of such parabola?



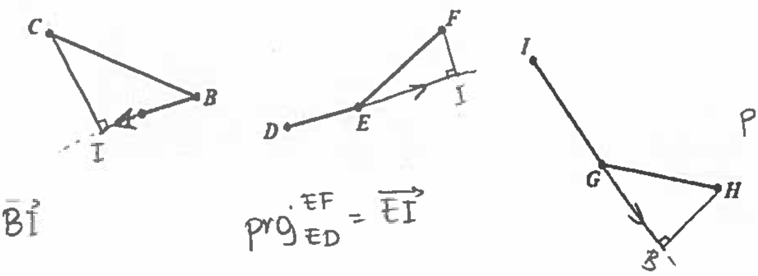
$d = |-2-4| = 6$

$-4d(x-x_0) = (y-y_0)^2$

$\boxed{-24(x+2) = (y-6)^2}$

$\boxed{F(-8,6)}$

(viii) **(6 points)**



$\text{proj}_{BA}^{BC} = \vec{BI}$ $\quad$ $\text{proj}_{ED}^{EF} = \vec{EI}$ $\quad$ $\text{proj}_{GI}^{GH} = \vec{GB}$

Use the pictures above
1. Draw the projection vector of BC over BA
2. Draw the projection vector of EF over ED
3. Draw the projection vector of GH over GI

(ix) Let $f(x) = (x^2 - 6x + 5)^4$.

a. **(3 points)** Find $f'(x)$. Then find the sign of $f'(x)$.

$f'(x) = 4(2x-6)(x^2-6x+5)^3$

$0 = 4(2x-6)(x^2-6x+5)^3$

$2x-6=0 \qquad x^2-6x+5=0$

$x=3 \qquad x=5 \quad x=1$

x	$-\infty$		1		3		5		$+\infty$
$f'(x)$		$-$	0	$+$	0	$-$	0	$+$	
$f(x)$		↘		↗		↘		↗	

$f'(x)$ negative for $(-\infty, 1) \cup (3,5)$

$f'(x)$ positive for $(1,3) \cup (5,+\infty)$

b. **(2 points)** For what values of x does $f(x)$ increase?

$(1,3) \cup (5,+\infty)$

c. **(2 points)** For what values of x does $f(x)$ decrease?

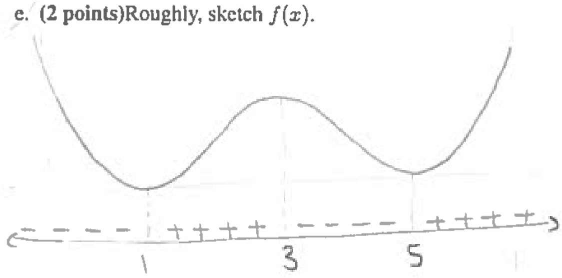
$(-\infty, 1) \cup (3,5)$

d. **(2 points)** Find all local min (max) points of $f(x)$ if possible

min at $x=1$ and $x=5$ $\qquad$ MIN : $(1,0)$ and $(5,0)$

max at $x=3$ $\qquad$ MAX : $(3,256)$

e. **(2 points)** Roughly, sketch $f(x)$.



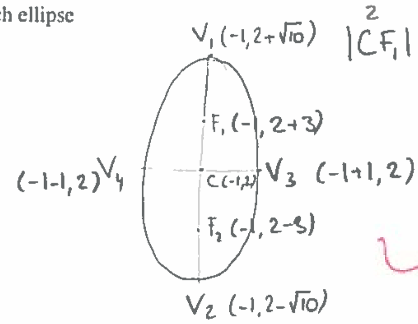
(x) Consider the ellipse $(x+1)^2 + \frac{(y-2)^2}{10} = 1$

$C(-1,2)$

$\frac{k}{2} = \sqrt{10}$

$|CF_1| = \sqrt{10-1} = 3$

a. **(2 points)** Roughly, draw such ellipse



b. **(2 points)** Find the foci

$F_1(-1,5)$

$F_2(-1,-1)$

c. **(2 points)** Find the ellipse constant

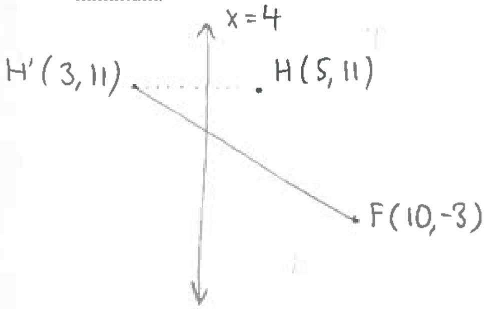
$k = 2\sqrt{10}$

d. **(2 points)** Find all four vertices

$V_1(-1, 2+\sqrt{10})$ $V_3(0,2)$

$V_2(-1, 2-\sqrt{10})$ $V_4(-2,2)$

(xi) **(6 points)** Let $H = (5, 11)$ and $F = (10, -3)$. Find a point Q on the vertical line $x = 4$ such that $|HQ| + |QF|$ is minimum.



$m = \frac{-3-11}{10-3} = -2$

$11 = -2(3) + b$

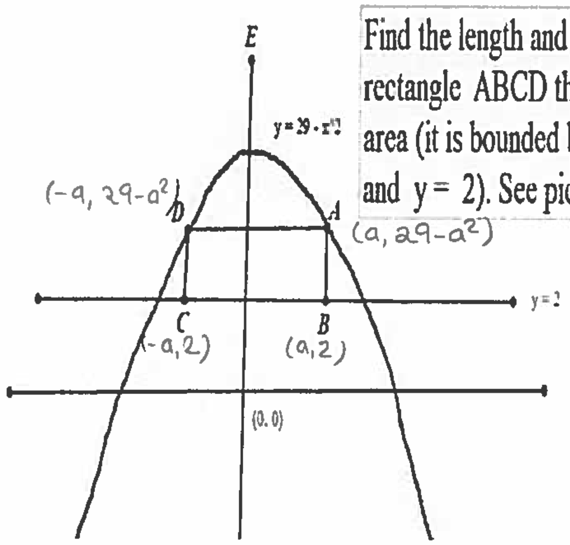
$b = 17$

$y = -2x + 17$

$y = -2(4) + 17 = 9$

$Q(4,9)$

(xii) (8 points)



Find the length and the width of the rectangle ABCD that has maximum area (it is bounded by y = 29 - x^2 and y = 2). See picture

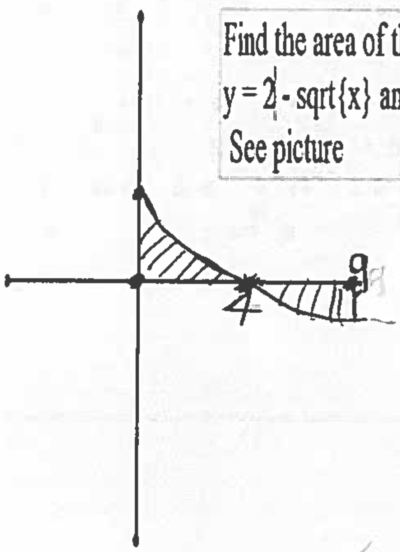
$W = |BC| = 2a$
$L = |AB| = 29 - a^2 - 2 = 27 - a^2$

$A = LW = 2a(27 - a^2)$
$A = 54a - 2a^3$
$A' = 54 - 6a^2$
$0 = 54 - 6a^2$
$54 = 6a^2 \Rightarrow a = 3$

$A'' = -12a \big|_{a=3} < 0 \Rightarrow$ max. at $a = 3$

$W = 2a = 6$ units
$L = 27 - a^2 = 18$ units

(xiii) (6 points)



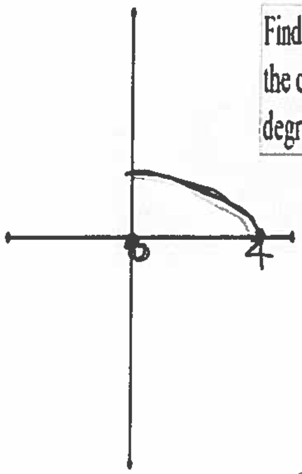
Find the area of the shaded region that is bounded by y = 2 - sqrt{x} and x-axis, where x is between 0 and 9. See picture

$A = \int_0^9 2 - \sqrt{x}\, dx = \int_0^4 2 - \sqrt{x}\, dx - \int_4^9 2 - \sqrt{x}\, dx$

$= \left[2x - \frac{2}{3} x^{\frac{3}{2}} \Big|_0^4 \right] - \left[2x - \frac{2}{3} x^{\frac{3}{2}} \Big|_4^9 \right]$

$= \frac{8}{3} - \left(0 - \frac{8}{3}\right) = \frac{8}{3} + \frac{8}{3} = \frac{16}{3}$ unit^2

(xiv) (4 points)



Find the volume of the solid object that is obtained by rotating the curve of y = sqrt{4 - x}, where x is between 0 and 4, 360 degrees about the x-axix

$$V = \pi \int_0^4 \left(\sqrt{4-x}\right)^2 dx = \pi \int_0^4 4-x \, dx$$

$$= \pi \left(4x - \frac{x^2}{2} \Big|_0^4 \right) = \pi (8-0)$$

$$= 8\pi \text{ units}^3$$

(xv) (3 points) $\frac{1}{6}\int 6x^2(2x^3+7)^9\,dx$

$$\frac{(2x^3+7)^{10}}{60} + C$$

(xvi) (3 points) $\frac{1}{2}\int \frac{2(x+1)}{x^2+2x+3}\,dx$

$$\frac{\ln|x^2+2x+3|}{2} + C$$

(xvii) (3 points) $\frac{1}{4}\int 4(x+5)e^{(2x^2+20x+1)}\,dx$

$$\frac{1}{4} e^{2x^2+20x+1} + C$$

Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: abadawi@aus.edu, www.ayman-badawi.com

2.2 Worked out Solutions for all Assessment Tools

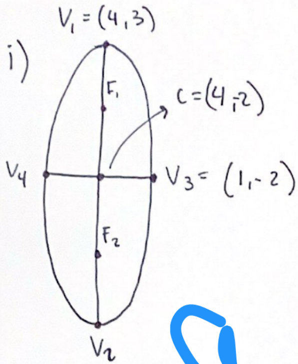
2.2.1 Solution for Quiz I

16/15

Dana Abodahab 00091595

Question 1

i)



$b>a \Rightarrow$ vertical

$b^2=\left(\frac{k}{2}\right)^2$ $\quad |b|=|(-2-3)|$

$(5)^2=\left(\frac{k}{2}\right)^2$ $\quad b=5$

ii) $\boxed{\therefore k=10}$

$|CF^2|=|a^2-b^2|$

$CF^2=9-25$

$CF^2=16$

$\boxed{\therefore CF=4}$

iii) $F_1=(4,-2+4)=(4,2)$

$F_2=(4,-2-4)=(4,-6)$

iv) $$\frac{(x-x_0)^2}{a^2}+\frac{(y-y_0)^2}{b^2}=1$$

$$\frac{(x-4)^2}{9}+\frac{(y+2)^2}{25}=1$$

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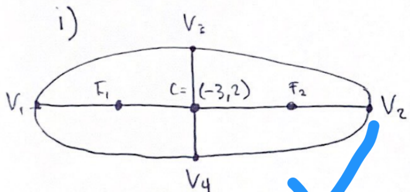
QUESTION 2

i)



$C=(-3,2)$

$CF^2=a^2-b^2$

$CF^2=9-4$

$CF^2=5$

$\boxed{\therefore CF=\sqrt{5}}$

$$\frac{(x+3)^2}{9}+\frac{(y-2)^2}{4}=1$$

$a^2=\left(\frac{k}{2}\right)^2$ $\quad b^2$

$\therefore k=3\times 2=6$

ii) $\boxed{k=6}$

iii)

$F_1=(-3+\sqrt{5},2)$

$F_2=(-3-\sqrt{5},2)$

iv) $b=\frac{1}{2}$ minor axis

$\therefore b=2$

$V_3=(-3,2+2)=(-3,4)$

$V_4=(-3,2-2)=(-3,0)$

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2.2.2 Solution for Quiz II

15/15

Quiz 2.

Q1) $y = x^2 - 6x + 10.$

standard form

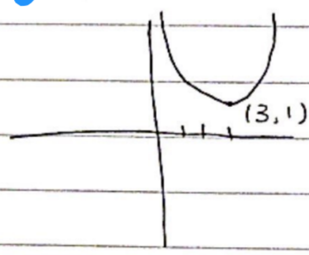
i. $y = x^2 - 6x + 10$

$\cancel{y = x(x-6)+10}$ $\cancel{y = x(x-6)+10}$

$y = x^2 - 2 \cdot x \cdot 3 + 3^2 + 1$

$y = (x-3)^2 + 1$

ii) Sketch



iii) Focus

$(3, 1 + 1/4) = (3, \frac{5}{4})$

8/8

iv) Vertex

$(3, 1)$

v) Directrix

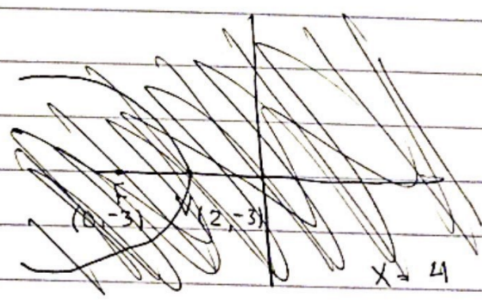
$y = \frac{3}{4}$

Q2) $-8(x-2) = (y+3)^2$

i) Sketch

$4d = -8$

$\boxed{d = -2}$



7/7

ii) Focus

$(2-2, -3)$

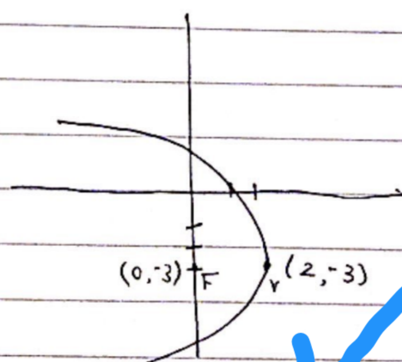
$(0, -3)$

iii) Vertex

$(2, -3)$

iv) Directrix.

$x = 4.$



2.2.3 Solution for Quiz III

Jude Al Jundi
900091801

Quiz Three

15/15

$8x^2 - 32x - y^2 + 6y = -15$

$(8x^2 - 32x) + (-y^2 + 6y) = -15$

$8(x^2 - 4x) - (y^2 - 6y) = -15$

$8[(x+(-2))^2 - (-2)^2] - [(y+(-3))^2 - (-3)^2] = -15$

$8[(x-2)^2 - 4] - [(y-3)^2 - 9] = -15$

$8(x-2)^2 - 32 - (y-3)^2 + 9 = -15$

$8(x-2)^2 - 23 - (y-3)^2 = -15$

$8(x-2)^2 - (y-3)^2 = 8$

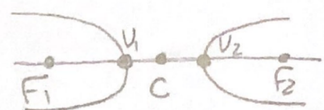
$\frac{(x-2)^2}{1} - \frac{(y-3)^2}{8} = 1$

a^2 b^2

standard form

$\frac{9}{9}$

Sketch:



$=(2,3)$

$a^2 = \left(\frac{k}{2}\right)^2$

$1 = \frac{k}{2}$

$k = 2$

$|V_1C| = \frac{k}{2} = 1$

$\rightarrow V_1 = (2-1, 3) = (1,3)$

$\rightarrow V_2 = (2+1, 3) = (3,3)$

$|F_1C| = \sqrt{a^2 + b^2} = \sqrt{1+8}$

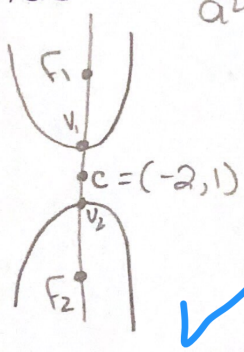
$= \sqrt{9} = 3$

$\rightarrow F_1 = (2-3, 3) = (-1, 3)$

$\rightarrow F_2 = (2+3, 3) = (5, 3)$

Question 2: $\frac{(y-1)^2}{9} - \frac{(x+2)^2}{16} = 1$

Sketch:



$a^2 \leftarrow 9$

$a^2 = \left(\frac{k}{2}\right)^2$

$9 = \left(\frac{k}{2}\right)^2$

$3 = \frac{k}{2}$

$k = 6$

$\frac{k}{2} = 3$

$|V_1C| = 3$

$\rightarrow V_1 = (-2, 1+3) = (-2, 4)$

$\rightarrow V_2 = (-2, 1-3) = (-2, -2)$

$|F_1C| = \sqrt{a^2 + b^2}$

$= \sqrt{9 + 16} = \sqrt{25} = 5$

$\rightarrow F_1 = (-2, 1+5) = (-2, 6)$

$\rightarrow F_2 = (-2, 1-5) = (-2, -4)$

$\frac{6}{6}$

2.2.4 Solution for Quiz IV

Quiz 4

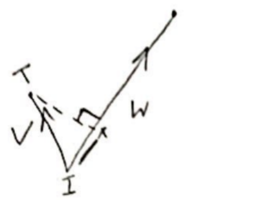
Omar El-Sebaie
(b00091963)

12/15

Q1:

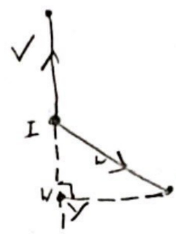
3/3

①



$Proj_w^v = \vec{IX}$

②



$Proj_v^w = IY$

Q2.

4/4

L_1: $x = -t + 2$
$y = 3t + 4$
$z = 2t - 3$

$-t + 2 = 10w - 8$
$3t + 4 = 2w + 2$

$(10 + t = 10) \times 3$
$= 30w + 3t = 30$
$2w - 3t = 2$
$32w = 32$

$\boxed{w = 1}$
$\boxed{t = 0}$

L_2: $x = 10w - 8$
$y = 2w + 2$
$z = 2w - 5$

$\boxed{\begin{array}{l} z = 2(0) - 3 = -3 \\ z = 2(1) - 5 = -3 \end{array}}$

L_1 and L_2 intersects

2/2

check if $L_1 \perp L_2$

$D_1 \cdot D_2 = (-1(10) + 3(2) + 2(2))$
$-10 + 6 + 4 = 0$

L_1 is perpendicular to L_2

Q3.

6/6

$\boxed{D_1 = \langle -4, 1, 5 \rangle}$
$\boxed{D_2 = \langle 8, -2, -10 \rangle}$

L_1 $x = -4t + 2$
$y = t + 4$
$z = 5t - 3$

$D_1 c = D_2$
$-4c = 8$
$1c = -2$
$5c = -10$

$\boxed{c = -2}$

$\boxed{D_1 \parallel D_2}$

$-4t + 2 = 8w - 6$
$t + 4 = -2w + 6$
$8w + 4t = 8$
$2w + t = 2 \times 4$
$8w + 4t = 8$
$8w + 4t = 8$

L_1

L_2. $x = 8w - 6$
$y = -2w + 6$
$z = -10w + 9$

$t = \frac{-x + 2}{4}$
$t = y - 4$
$t = \frac{z + 3}{5}$

Select point L_1 assuming $t = 0$
$Q(2, 4, -3)$ check in @ L_2
$2 = 8w - 6 \Rightarrow w = 1$ $\therefore Q$ is not in L_2
$4 = -2w + 6 \Rightarrow w = 1$ L_1 is parallel to L_2
$-3 = -10w + 9 \Rightarrow w = 12/10$

ii) symmetric equation to L_2

$x = -4t+2 \rightarrow \frac{-x+2}{4}$

$y = t+4 \rightarrow y-4 = t$

$z = 5t+3 \rightarrow \frac{z+3}{5} = t$

$$\frac{-x+2}{4} = y-4 = \frac{z+3}{5}$$

2.2.5 Solution for Quiz V

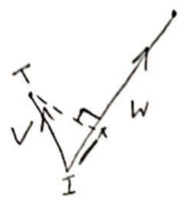
Quiz 4

Omar El-Sebaie
(b00091963)

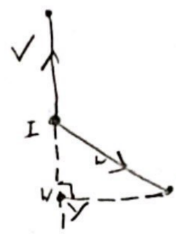
$\frac{12}{15}$

Q1:

$\frac{3}{3}$

① 

$Proj_w^v = \vec{IX}$

② 

$Proj_v^w = IY$

Q2.

$\frac{4}{4}$

$L_1: x = -t+2$
$y = 3t+4$
$z = 2t-3$

$-t+2 = 10w-8$
$3t+4 = 2w+2$

$(10+t = 10) \times 3$
$= 30w + 3t = 30$
$2w - 3t = 2$
$32w = 32$

$\boxed{w = 1}$
$\boxed{t = 0}$

$L_2: x = 10w-8$
$y = 2w+2$
$z = 2w-5$

$\boxed{z = 2(0)-3 = -3}$
$\boxed{z = 2(1)-5 = -3}$

L_1 and L_2 intersects

$\frac{2}{2}$

check if $L_1 \perp L_2$

L_1 is perpendicular to L_2

$D_1 \cdot D_2 = (-1(10) + 3(2) + 2(2))$
$-10 + 6 + 4 = 0$

Q3.

$\frac{6}{6}$

$\boxed{D_1 = \langle -4, 1, 5 \rangle}$
$\boxed{D_2 = \langle 8, -2, -10 \rangle}$

$\boxed{D_1 \parallel D_2}$

$L_1 \quad x = -4t+2$
$y = t+4$
$z = 5t-3$

$D_1 c = D_2$
$-4c = 8$
$1c = -2$
$5c = -10$

$\boxed{c = -2}$

$-4t + 2 = 8w - 6$
$t + 4 = -2w + 6$
$8w + 4t = 8$
$2w + t = 2 \times 4$
$8w + 4t = 8$
$8w + 4t = 8$

L_1

$L_2. \quad x = 8w-6$
$y = -2w+6$
$z = -10w+9$

$t = \frac{-x+2}{4}$
$t = y - 4$
$t = \frac{z+3}{5}$

Select point L_1 assuming $t = 0$
$Q(2, 4, -3)$ check in L_2

$2 = 8w - 6 \Rightarrow w = 1$
$4 = -2w + 6 \Rightarrow w = 1$
$-3 = -10w + 9 \Rightarrow w = 12/10$

$\therefore Q$ is not in L_2
L_1 is parallel to L_2

ii) symmetric equation to L_2

$x = -4t+2 \rightarrow \frac{-x+2}{4}$

$y = t+4 \rightarrow y-4 = t$

$z = 5t+3 \rightarrow \frac{z+3}{5} = t$

$$\frac{-x+2}{4} = y-4 = \frac{z+3}{5}$$

Scanned with CamScanner

2.2.6 Solution for Quiz VI

Quiz 6

15/15

3

1) i- $y = 7x^2 + 10\sqrt{x} + \sin(9x)$

$y = 14x + 5x^{-1/2} + 9\cos(9x)$

3

ii- $y = 2(x^3 + 7x + 3)^{11}$

$y' = 22(x^3 + 7x + 3)^{10} \quad (3x^2 + 7)$

iii- $y = \cos(7x)\ (\sin(5x) + 2)$

3

$\cancel{\frac{d}{dx}\left(f(x)\, g(x)\right) = f(x)\frac{d}{dx}(g(x)) + g(x)\frac{d}{dx}(f(x))}$

$\cancel{y' = 5\cos(5x)\cos(7x) - 7\sin(5x)\sin 7x}$

$y' = -7\sin 7x\,(\sin(5x + 2)) + 5\cos(5x)\,(\cos(7x))$

2) i- $f(x) = (x^2 - 6x - 7)^3$

$f'(x) = 3(x^2 - 6x - 7)^2 (2x - 6)$

$0 = (6x - 18)(x^2 - 6x - 7)^2$

2

$6x - 18 = 0$

$\frac{6x}{6} = \frac{18}{6}$

$\boxed{x = 3}$

$(x^2 - 6x - 7)^2 = 0$

$(x^2 - 6x - 7) = 0$

$(x - 7)(x + 1) = 0$

$\boxed{x = 7}$

$\boxed{x = -1}$

ii.

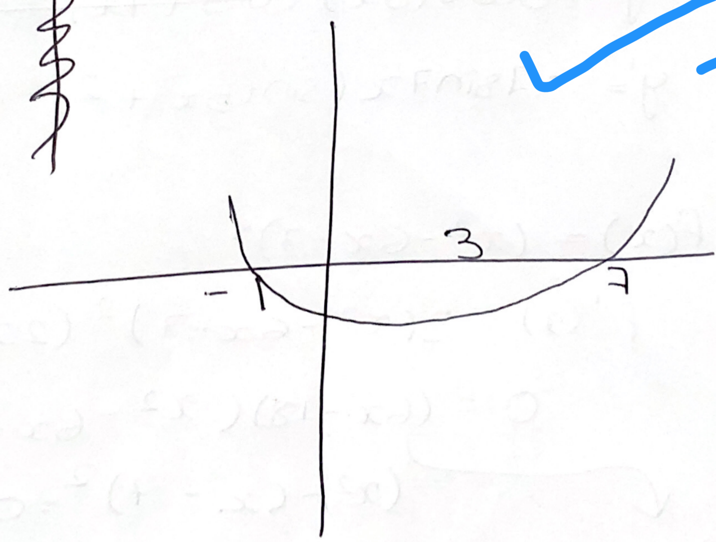
Critical values	$f(x) = (x^2-6x-7)^3$
7	$f(7) = (7^2-6(7)-7)^3$ $=0$
-1	$f(-1)\ ((-1)^2-6(-1)-7)^3$ $=0$
3	$f(3) = ((3)^2-6(3)-7)^3$ $=-4096$

Local min is -4096 and occurs when $x=3$

No local max.

✓ 2

iii -



✓ 2

2.2.7 Solution for Quiz VII

Quiz 7

Q1

(i) $y = \ln\left(\frac{x^2+3x}{5-2x}\right)$

$y' = \ln(x^2+3x) - \ln(5-2x)$

$= \frac{2x+3}{x^2+3x} - \frac{-2}{5-2x}$

(ii) $y = e^{(x^3+6x+3)}$

$y' = e^{(x^3+6x+3)} \cdot (3x^2+6)$

(iii) $y = \ln(7x^2+5x-3)\, e^x$

$y' = \frac{14x+5}{7x^2+5x-3} \cdot e^x + \ln(7x^2+5x-3) \cdot e^x$

Q2.

$f(x) = \ln(3x-9+e) + e^{(x-3)}$

$y = mx + c$

$m = f'(3) = \frac{3}{3x-9+e} + e^{(x-3)} = \frac{3}{9-9+e} + e^{(3-3)} = 2.1$

$y = 2.1x + c$

$y = \ln(3(3)-9+e) + e^{(3-3)} = 2$

$2 = 2.1x + c$ (at $(3,2)$)

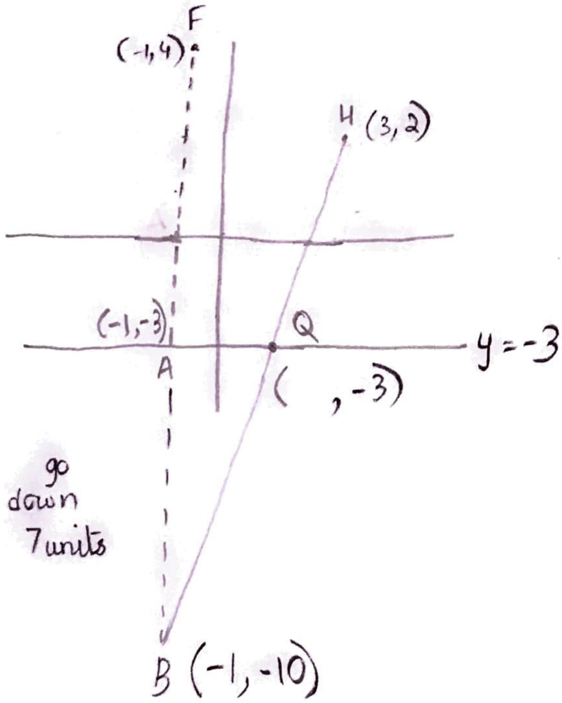
$2 = 2.1(3) + c$

$c = -4.3$

$\therefore$ equation of tangent line $\Rightarrow \boxed{y = 2.1x - 4.3}$

Q3 $H = (3,2)$ and $F = (-1,4)$, $y = -3$

$|HQ| + |QF|$ is minimum



$|FA| = 4 - (-3) = 7$

$|FA| = |AB| = 7$

$H = (3,2)$ and $B = (-1,-10)$

$y = mx + c$

$$m = \frac{\Delta y}{\Delta x} = \frac{-10-2}{-1-3} = \frac{-12}{-4} = 3$$

$\therefore y = 3x + c$

take $H = (3,2)$ to find c:

$2 = 3(3) + c$

$c = -7$

$\therefore Q \Rightarrow -3 = 3x - 7$

$$\frac{-3+7}{3} = x$$

$$x = \frac{4}{3}$$

$$\boxed{Q = \left(\frac{4}{3}, -3\right)}$$

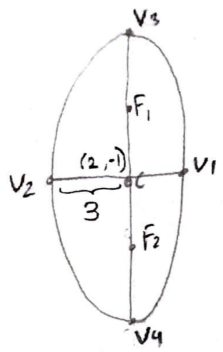
2.2.8 Solution for EXAM I

EXAM 1

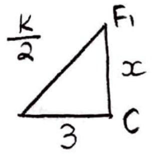
56/56

Q1

(i)

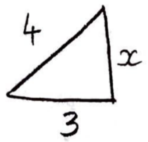


(ii)



$\left(\frac{K}{2}\right)^2 = 16$

$\frac{K}{2} = 4 \quad \therefore K = 8$



$x = \sqrt{4^2 - 3^2} = \sqrt{7}$

$\therefore F_1 = (2, -1 + \sqrt{7})$

$F_2 = (2, -1 - \sqrt{7})$

(iii) $a = \sqrt{9} = 3$

$b = \sqrt{16} = 4$

$\therefore V_1 = (5, -1) \quad V_3 = (2, 3)$

$V_2 = (-1, -1) \quad V_4 = (2, -5)$

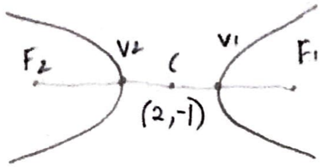
(iv) $\left(\frac{K}{2}\right)^2 = 16$

$\frac{K}{2} = 4$

$K = 8$

Q2.

(i)



9

(ii) $|CF_1| = \sqrt{a^2+b^2} = \sqrt{4+5} = 3$

$\therefore F_1 = (5, -1)$

$F_2 = (-1, -1)$

(iii) $4 = \left(\frac{K}{2}\right)^2$

$2 \times 2 = K$

$4 = K$

$|V_1C| = \frac{K}{2} = \frac{4}{2} = 2$

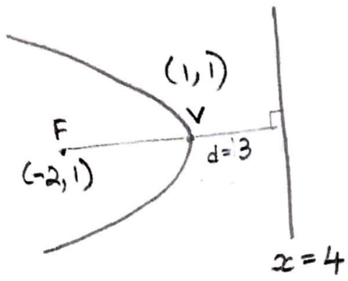
$\therefore V_1 = (4, -1)$

$V_2 = (0, -1)$

(iv) $4 = \left(\frac{K}{2}\right)^2$

$K = 4$

Q3



$$V = \left(\frac{-2+4}{2}, 1\right)$$

$$= (1, 1)$$

Equation : $-4d(x - x_0) = (y - y_0)^2$

$$-4(3)(x-1) = (y-1)^2$$

$$-12(x-1) = (y-1)^2$$

Q4 $N_1 = \langle 1, 1, 2 \rangle$

$N_2 = \langle 1, 1, -1 \rangle$

$$N_1 \times N_2 = \begin{vmatrix} i & j & k \\ 1 & 1 & 2 \\ 1 & 1 & -1 \end{vmatrix} = \left\langle \begin{vmatrix} 1 & 2 \\ 1 & -1 \end{vmatrix}, -\begin{vmatrix} 1 & 2 \\ 1 & -1 \end{vmatrix}, \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} \right\rangle$$

$$= \langle -1-2, -(-1-2), 1-1 \rangle$$

$$= \langle -3, 3, 0 \rangle$$

→ Choose point Q :

let $x = 0$

$(y + 2z = 3) \times (-1)$

$y - z = 6$

$$\Rightarrow \begin{array}{r} -y - 2z = -3 \\ y - z = 6 \\ \hline -3z = 3 \\ z = -1 \end{array}$$

$y = 5$

L : $Dt + Q$

$t\langle -3, 3, 0 \rangle + (0, 5, -1 \rangle$

$\langle -3t, 3t, 0 \rangle + (0, 5, -1 \rangle$

$\Rightarrow x = -3t$

$y = 3t + 5$

$z = -1$

Q5 $4(t+2) + (-2t+1) + (-t+3) = 10$

$4t+8-2t+1-t+3 = 10$

$t+12 = 10$

$t = -2$

$Q \Rightarrow x = (-2)+2 = 0$

$y = -2(-2)+1 = 5$

$z = -(-2)+3 = 5$

$Q = \underline{(0,5,5)} \Rightarrow$ Intersection point

9

Q6.(i) $\overrightarrow{Q_1Q_2} = \langle -3, 2, -1 \rangle$

$\overrightarrow{Q_1Q_3} = \langle -5, 4, 5 \rangle$

$$\overrightarrow{Q_1Q_2} \times \overrightarrow{Q_1Q_3} = \begin{vmatrix} i & j & k \\ -3 & 2 & -1 \\ -5 & 4 & 5 \end{vmatrix}$$

$$= \left\langle \begin{vmatrix} 2 & -1 \\ 4 & 5 \end{vmatrix}, -\begin{vmatrix} -3 & -1 \\ -5 & 5 \end{vmatrix}, \begin{vmatrix} -3 & 2 \\ -5 & 4 \end{vmatrix} \right\rangle$$

$$= \langle 10-(-4), -(-15-5), -12-(-10) \rangle$$

$$= \langle 14, 20, -2 \rangle$$

$\Rightarrow \langle 14, 20, -2 \rangle \neq \langle 0,0,0 \rangle \therefore Q_1, Q_2$ and Q_3 are not co linear.

(ii) Area of $\Delta Q_1Q_2Q_3 = \dfrac{|\overrightarrow{Q_1Q_2} \times \overrightarrow{Q_1Q_3}|}{2}$

$$= \frac{\sqrt{14^2 + 20^2 + 2^2}}{2}$$

$$= \frac{\sqrt{196 + 400 + 4}}{2} = 5\sqrt{6} \text{ units}^2$$

9

(iii) $N = \langle 14, 20, -2 \rangle$

$Q_1 = (1, 2, 3)$

eqn : $14(x-1) + 20(y-2) - 2(z-3) = 0$

Q7. $L_1 : x = 2t + 1$
$y = -t + 3$
$z = 4t + 1$

$L_2 : x = 4w - 3$
$y = -2w + 5$
$z = 8w - 7$

$D_1 = \langle 2, -1, 4 \rangle$, $D_2 = \langle 4, -2, 8 \rangle$

$D_1 \parallel D_2$?

$\Rightarrow D_1 = C D_2$

$\langle 2, -1, 4 \rangle = \langle 4C, -2C, 8C \rangle$

$$\left.\begin{array}{ll} 2 = 4C & \Rightarrow c = \frac{1}{2} \\ -1 = -2C & \Rightarrow c = \frac{1}{2} \\ 4 = 8C & \Rightarrow c = \frac{1}{2} \end{array}\right\} \text{c is same} \quad \therefore D_1 \parallel D_2$$

8

$\Rightarrow$ Let Q be point on $L_1 = (1, 3, 1)$

$$\left.\begin{array}{ll} 1 = 4w - 3 & \Rightarrow w = 1 \\ 3 = -2w + 5 & \Rightarrow w = 1 \\ 1 = 8w - 7 & \Rightarrow w = 1 \end{array}\right\} \text{w is same}$$

$\therefore$ L_1 is NOT parallel to L_2, they overlap

2.2.9 Solution for EXAM II

MTH111 Exam 2 Mon-29-NOV-2021

Jude ABDULHAQ AlJundi

900091801

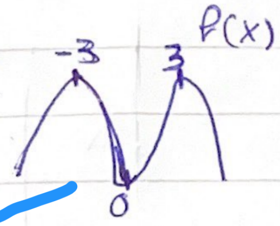
30/30

Question 1)

almost -3 and 3 but not exactly.

i) $(-\infty, -3) \cup (0, 3)$

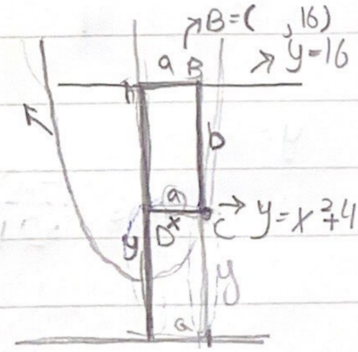
ii) $(-3, 0) \cup (3, \infty)$



iii) $f(x)$ has a local max when $x=-3$ and $x=3$,
$f(x)$ has a local min when $x=0$

6

Question 2)



$y=x^2+4$

$A=|AB|\cdot|BC|$

$y=a^2+4$

$|BC|=16-(a^2+4)$

$|AB|=a$

$A=a(16-a^2-4)=16a-4a-a^3$

$=12a-a^3$

$A'=12-3a^2=0$

$a=2$

$W=2$

$L=8$

Area=16.

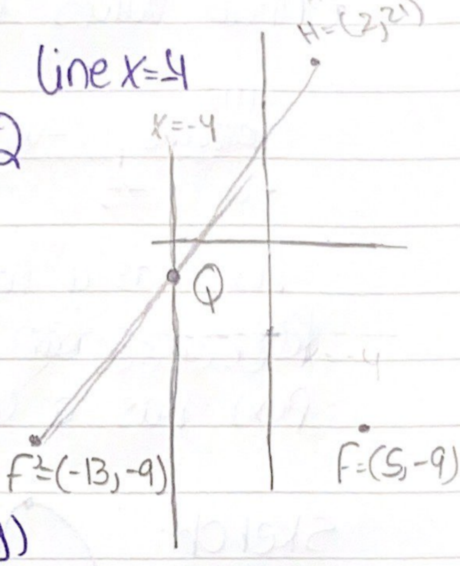
$A''=-6a$

$A''(2)=-6(2)<0$, so we have max

6

Question 3)

$H=(2,21)$ & $F=(5,-9)$ line $x=-4$



$$\text{slope} = \frac{y_1-y_2}{x_1-x_2} = \frac{-9-21}{-13-2} = 2$$

$y = mx+b$

$y = 2x+b$

$21 = 2(2)+b$

$b = 17$

$y = 2x+17$ $\quad Q=(-4, y)$

$y = 2(-4)+17 = 9$

$Q=(-4,9)$

6

Question 4)

$f(x) = (x^2+2x-7)e^x$

$f'(x) = (2x+2)(e^x) + e^x(x^2+2x-7)$

$f'(x) = 2xe^x + 2e^x + e^x x^2 + 2xe^x - 7e^x = 0$

$f'(x) = 4xe^x - 5e^x + e^x x^2 = 0$

$= e^x(4x-5+x^2) = 0$

$= e^x(x^2+4x-5) = 0$

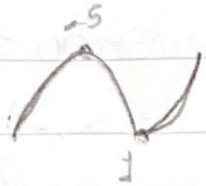
$= e^x(x-1)(x+5) = 0$

$x=1$ $\quad x=-5$ $\rightarrow \rightarrow \rightarrow \rightarrow$

Next page

$e^x(x^2+4x-5)$

Critical values: $x=1, x=-5$



+ve, -ve, +ve

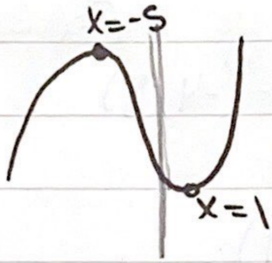
$-\infty \quad -5 \quad 1 \quad \infty$

f(x) has a local min when $x=1$

~~f(x) does not have local min~~

f(x) has a local max when $x=-5$

Sketch:



6

Roughly

No scaling

Question 5)

i) $y=e^{(2x+3)}\ln(3x+5)$

$y'=e^{(2x+3)}(2)\ln(3x+5)+\left(\frac{3}{3x+5}\right)\left(e^{(2x+3)}\right)$

2

ii) $y = \sin(3x)(2x^3+6x+1)^4$

$y' = \cos(3x)\cdot 3\cdot(2x^3+6x+1)^4 + 4(2x^3+6x+1)^3(6x^2+6)(\sin(3x))$ 3

iii) $y = \ln\left(\left(\frac{\sin x+\cos x}{3x+2}\right)^6\right)$

$y = 6\left[\ln(\sin x+\cos x) - \ln(3x+2)\right]$

$y' = 6\left[\frac{\cos x - \sin x}{\sin x+\cos x} - \frac{3}{3x+2}\right]$ 2

2.2.10 Solution for Final Exam

Name Jude Al Junai, ID g00041801

$x^2-4x-4x+16$

$x^2-8x+16$

Final Exam, MTH 111 , Fall 2021

Ayman Badawi

$$\text{Score} = \frac{64}{64}$$

$(x-4)^2-(-4)^2+10$

QUESTION 1. (5 points) Consider the parabola

$$2y = x^2 - 8x + 10$$

(i) Write it in the standard form

$2y = x^2 - 8x + 10$

$2y = [(x-4)^2 - (-4)^2 + 10]$

$2y = (x-4)^2 - 16 + 10$

$2y = (x-4)^2 - 6$

$2y + 6 = (x-4)^2$

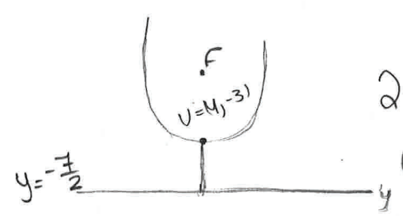
$\boxed{2(y+3) = (x-4)^2}$

(ii) Find the vertex

$V = (4, -3)$

(iii) Find the equation of the directrix line

$y = -\frac{7}{2}$



$2 = 4d$

$d = 1/2$

(iv) Find the focus

$F = (4, -\frac{5}{2})$

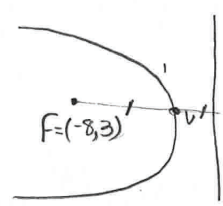
QUESTION 2. (5 points) Given $x = -2$ is the directrix of a parabola that has $F = (-8, 3)$ as its focus. Find the equation of the parabola and skëtch (roughly). Show the work.

$4d(x - x_0) = (y - y_0)^2$ $\quad d = -3$ $\quad \frac{6}{2} = 3$ $\quad \boxed{V = (-5, 3)}$

$4(-3)(x+5) = (y-3)^2$

$-12(x+5) = (y-3)^2$

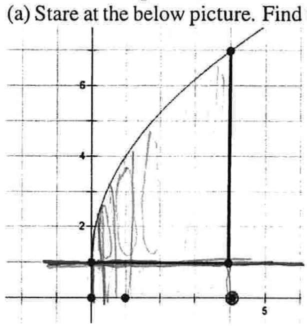
$x = -2$



$\frac{2}{3}x$

QUESTION 3. (6 points)

(a) Stare at the below picture. Find the area of the region that is bounded by $y = 3\sqrt{x}+1$, $y = 1$, $x = 0$, $and\, x = 4$



$A = \int_0^4 (3\sqrt{x}+1)dx - \int_0^4 1\, dx$

$A = 3\int_0^4 \sqrt{x}\, dx + \int_0^4 1\, dx - \int_0^4 1\, dx$

$A = 3\left(\frac{2}{3}x^{\frac{3}{2}}\Big|_0^4\right) + x\Big|_0^4 - x\Big|_0^4$

$x^{\frac{1}{2}}$

$\frac{2}{3}x^{\frac{3}{2}}$

$A = 3\left[\frac{16}{3} - 0\right] + (4-0) - (4-0)$

$\boxed{A = 16}$ ✓

(b) If the curve $y = 3\sqrt{x}+1$ is rotated 360 degrees about the line $y = 1$ what will be the volume of the outcome object?

$A = \pi r^2$

$r = 3\sqrt{x}+1-1$

$r = 3\sqrt{x}$

$V = \int \pi(3\sqrt{x})^2$

$V = \pi\int(3\sqrt{x})^2$

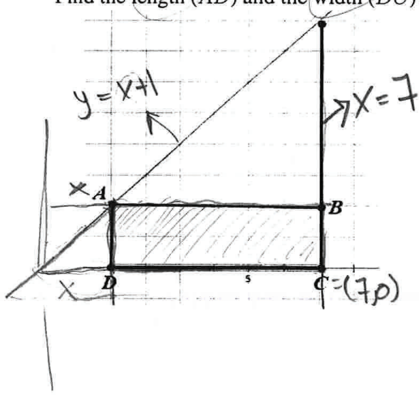
$V = \pi\int 9x$

$V = 9\pi\frac{x^2}{2}\Big|_0^4$ ✓

$\boxed{V = \frac{9}{2}\pi x^2}$

$0 \le x \le 4$

QUESTION 4. (6 points) Stare at the below pictures. We want to construct the rectangle ABCD that has maximum area. Given A lies on the line $y = x + 1$, B lies on the line $x = 7$, the point $c = (7, 0)$, and D is a point on the x-axis. Find the length (AD) and the width (DC) of such rectangle. Show the work.



$A = |AD||DC|$

$|AD| = (x+1)$

$|DC| = 7-x$

$A = (x+1)(7-x) = 7x - x^2 + 7 - x = 6x - x^2 + 7$

$A' = -2x + 6 = 0$

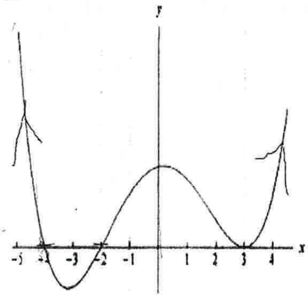
$x = 3$

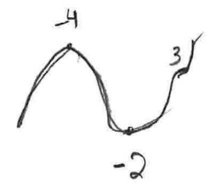
length $|AD| = 3+1 = 4$

width $|DC| = 7-3 = 4$

Area $= 4 \times 4 = 16$ ✓

QUESTION 5. (6 points) (SHOW THE WORK) Given the graph of the first derivative of $f(x)$ (i.e., $f'(x)$). Stare at it.





(i) For what values of x does $f(x)$ increase?

$(-\infty, -4) \cup (-2, \infty)$

(ii) For what values of x does $f(x)$ decrease?

$(-4, -2)$

(iii) For what values of x does $f(x)$ have local min and local max?

Local min at $x=-2$

Local max $x=-4$.

x^3

$(x+3)(x+3)$

$x^2+3x+3x+9$

$x^{-5}(x^2+6x+9) = x^{-3}+6x^{-4}+9x^{-5}$

QUESTION 6. (12 points) Evaluate the following integrals

(i) $\int \frac{(x+3)^2}{x^5}\,dx$

$\int (x^{-5}(x+3)^2)dx = \int [x^{-5}(x+6x+9)]\,dx$

$= \int (x^{-3}+6x^{-4}+9x^{-5})dx = \frac{x^{-2}}{-2} + \frac{6x^{-3}}{-3} + 9\frac{x^{-4}}{-4} + C$

$\boxed{= -\frac{1}{2x^2} - \frac{2}{x^3} - \frac{9}{4x^4} + C}$

(ii) $\int (sin(x) - cos(x))(sin(x) + cos(x) + 4)^8\,dx$

$u = \sin x + \cos x + 4$

$\frac{du}{dx} = \cos x - \sin x$

$du = \cos x - \sin x\,dx$

$du = -(\sin x - \cos x)\,dx$

$-\int u^8\,du$

$= -\left(\frac{u^9}{9}\right) + C$

$\boxed{= -\frac{(\sin x + \cos x + 4)^9}{9} + C}$

(iii) $\int \frac{x+3e^x}{x^2+6e^x+5}\,dx$

$u = x^2+6e^x+5$

$\frac{du}{dx} = 2x + 6e^x$

$du = 2(x+3e^x)\,dx$

$\frac{1}{2}\int \frac{1}{u}\,du$

$\boxed{= \frac{1}{2}\ln|x^2+6e^x+5| + C}$

$2x+6e^x$

(iv) $\int \frac{4e^{(\sqrt{x}+3)}}{\sqrt{x}}\,dx = 4\int\left(\frac{1}{\sqrt{x}}\cdot e^{\sqrt{x}+3}\right)dx$

$u=\sqrt{x}+3 \qquad = 8\int e^{u}\,du$

$\frac{du}{dx}=\frac{1}{2\sqrt{x}} \qquad = 8e^{u}+c = \boxed{8e^{\sqrt{x}+3}+C}$

QUESTION 7. (6 points) Let $f(x) = -x^3 + 3x^2 + 24x + e^{(x^3-3x^2-24x+3)}$

(i) For what values of x does $f(x)$ have local min. and local max.?

$f'(x) = -3x^2+6x+24 + e^{(x^3-3x^2-24x+3)}(3x^2-6x-24) = 0$

$f'(x) = (x-4)(x+2) + e^{(x^3-3x^2-24x+3)}(x-4)(x+2) = 0$

$x=4,\ x=-2$

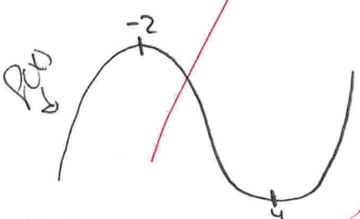
+ + + + | −ve −ve | +ve +ve

$-\infty \quad -2 \quad 4 \quad \infty$

f(x) has a local min when x=4 and a local max when x=−2

(ii) Sketch, roughly, the graph of $f(x)$.

X 6



QUESTION 8. (6 points) (a) Given $xe^{3y} + ye^{(2x+1)} + y^2 - 4x^2 + 1 = 0$. Find y'. $\frac{dy}{dx}(xe^{3y}+ye^{(2x+1)}+y^2-4x^2+1)=\frac{dy}{dx}0$

$\left(e^{3y}+e^{3y}\cdot 3x\frac{dy}{dx}\right)+\left(\frac{dy}{dx}e^{(2x+1)}+e^{(2x+1)}\cdot 2y\right)+2y\frac{dy}{dx}-8x=0$

$e^{3y}+3xe^{3y}\frac{dy}{dx}+e^{(2x+1)}\frac{dy}{dx}+2ye^{(2x+1)}+2y\frac{dy}{dx}=8x$

$3xe^{3y}\frac{dy}{dx}+e^{(2x+1)}\frac{dy}{dx}+2y\frac{dy}{dx}=8x-2ye^{(2x+1)}-e^{3y}$

final answer

Long

(b) Find the equation of the tangent line to the curve $y = x + e^{(x-2)} + ln(x-1)$ at the point $(2, 3)$.

$y=mx+b \qquad y'=1+e^{(x-2)}+\frac{1}{x-1}$

$y=3x+b \qquad y'(2)=3$

$3=3(2)+b$

$3=6+b$

$b=-3$

$\boxed{y=3x-3}$

$\frac{dy}{dx}(3xe^{3y}+e^{(2x+1)}+2y)=$

$\frac{dy}{dx}=\frac{8x-2ye^{(2x+1)}-e^{3y}}{3xe^{3y}+e^{(2x+1)}+2y}$

$y'=\frac{8x-2ye^{(2x+1)}-e^{3y}}{3xe^{3y}+e^{(2x+1)}+2y}$

use $y'=-\frac{f_x}{f_y}=-\frac{(e^{3y}+2ye^{(2x+1)}-8x)}{3x\ \ (2x+1)}$

QUESTION 9. (6 points) Given $P_1 : 2x + y - 3z = 10$ intersects the plane $P_2 : -x - y + 5z = -6$ in a line L. Find a parametric equations of L. Then find the symmetric equation of L.

$-y + \cancel{5}0 = -6$

$V_1 = \langle 2, 1, -3 \rangle \quad V_2 = \langle -1, -1, 5 \rangle$

$V_1 \times V_2 = \begin{vmatrix} i & j & k \\ 2 & 1 & -3 \\ -1 & -1 & 5 \end{vmatrix} = \left\langle \begin{vmatrix} 1 & -3 \\ -1 & 5 \end{vmatrix}, -\begin{vmatrix} 2 & -3 \\ -1 & 5 \end{vmatrix}, \begin{vmatrix} 2 & 1 \\ -1 & -1 \end{vmatrix} \right\rangle$

$= \langle 5-3, -(10-3), -2-(-1) \rangle$

$= \langle 2, -7, -1 \rangle$ ✓

let $\boxed{x=0}$

$y - 3z = 10$
$-y + 5z = -6$ } add

$2z = 4 \quad z = 2$

$y - 3(2) = 10$

$\boxed{y=16}$

$\boxed{(0,16,2)}$ ← Point ✓

Parametric equations:

$L: x = 2t + 0$

$y = -7t + 16$

$z = -1t + 2$ ✓

Symmetric equation:

$\dfrac{x}{2} = \dfrac{y-16}{-7} = \dfrac{z-2}{-1}$ ✓

$t = \dfrac{x}{2}$

$\dfrac{y-16}{-13}$

$\dfrac{z-2}{-1}$

QUESTION 10. (6 points) Given $A = (1, 1, 0)$, $B = (1, 2, 2)$, and $C = (2, 1, 2)$ are the vertices of a triangle.

(i) Find the area of the triangle ABC.

$V_1 = \langle 0, 1, 2 \rangle \quad V_2 = \langle 1, 0, 2 \rangle$

$A = \dfrac{|V_1 \times V_2|}{2}$

$V_1 \times V_2 = \begin{vmatrix} i & j & k \\ 0 & 1 & 2 \\ 1 & 0 & 2 \end{vmatrix} = \left\langle \begin{vmatrix} 1 & 2 \\ 0 & 2 \end{vmatrix}, -\begin{vmatrix} 0 & 2 \\ 1 & 2 \end{vmatrix}, \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} \right\rangle$

$= \langle 2-0, -(0-2), \cancel{0} \rangle = \langle 2, 2, \cancel{0} \rangle$ -1

$A = \dfrac{\sqrt{2^2 + 2^2 + \cancel{0}^2}}{2} = \dfrac{\cancel{2\sqrt{2}}}{2} = \boxed{\cancel{\sqrt{2}}}$ 3 $3/2$

(ii) Find the equation of the plane that passes through A, B, and C.

$2(x-1) + 2(y-2) + 0(z-2) = 0$ ✓ -1

~~$2x - 2 + 2y - 4 + 0 - 0 = 0$~~

~~$2x + 2y - 6 = 0$~~

~~$2x + 2y = 6$~~

Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: abadawi@aus.edu, www.ayman-badawi.com

2.3 QUIZZES

2.3.1 Quiz I

Name———————————————, ID ——————

Quiz I, MTH 111 , Fall 2021

Ayman Badawi

QUESTION 1. An ellipse is centralized at $(4, -2)$ Given $(4, 3)$ and $(1, -2)$ are two vertices of the ellipse.

(i) Draw such ellipse (roughly)

(ii) Find the ellipse constant

(iii) Find the foci of the ellipse.

(iv) Find the equation of the ellipse

QUESTION 2. Consider the ellipse $\dfrac{(x+3)^2}{9} + \dfrac{(y-2)^2}{4} = 1$.

(i) Draw such ellipse (roughly)

(ii) Find the ellipse constant

(iii) Find the foci of the ellipse.

(iv) Find the vertices of the minor axis.

Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: `abadawi@aus.edu, www.ayman-badawi.com`

2.3.2 Quiz II

Name————————————, ID —————

Quiz II, MTH 111 , Fall 2021

Ayman Badawi

QUESTION 1. Consider the parabola

$$y = x^2 - 6x + 10$$

(i) Write it in the standard form

(ii) Sketch, roughly

(iii) Find the focus

(iv) Find the vertex

(v) Find the equation of the directrix line

QUESTION 2. Consider the Parabola

$$-8(x - 2) = (y + 3)^2$$

(i) Sketch, roughly

(ii) Find the focus

(iii) Find the vertex

(iv) Find the equation of the directrix line

Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: abadawi@aus.edu, www.ayman-badawi.com

2.3.3 Quiz III

Name——————————————, **ID** ——————

Quiz III, MTH 111 , Fall 2021

Ayman Badawi

QUESTION 1. The following is a hyperbola. Write it in the standard form, sketch, find the vertices and the foci

$$8x^2 - 32x - y^2 + 6y = -15$$

QUESTION 2. Consider the hyperbola

$$\frac{(y-1)^2}{9} - \frac{(x+2)^2}{16} = 1$$

Sketch, find the vertices and the foci

Faculty information

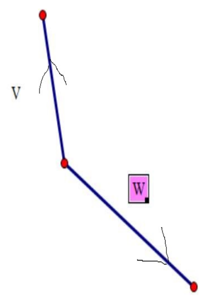
Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: `abadawi@aus.edu, www.ayman-badawi.com`

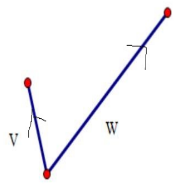
2.3.4 Quiz IV

Quiz IV, MTH 111 , Fall 2021

Ayman Badawi

Picture (2)





Picture (1)

QUESTION 1.

(i) Stare at Picture (1). Draw $Proj_W^V$ (i.e., Projection of V over W)

(ii) Stare at Picture (2). Draw $Proj_V^W$ (i.e., Projection of W over V)

QUESTION 2. Given $L_1 : x = -t + 2, y = 3t + 4, z = 2t - 3,\ (t \in R)$ and $L_2 : x = 10w - 8, y = 2w + 2, z = 2w - 5,\ (w \in R)$.

(i) If L_1 intersects L_2, find the intersection point. Show the work

(ii) Is L_1 perpendicular to L_2? Show the work

QUESTION 3. $L_1 : x = -4t + 2, y = t + 4, z = 5t - 3,\ (t \in R)$ and $L_2 : x = 8w - 6, y = -2w + 6, z = -10w + 9,\ (w \in R)$.

(i) Is L_1 parallel to L_2? Show the work

(ii) Write down the symmetric equation of L_1.

Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: `abadawi@aus.edu, www.ayman-badawi.com`

2.3.5 Quiz V

Quiz 5, MTH 111 , Fall 2021

Ayman Badawi

QUESTION 1. Let $P_1 : 2x - y + z = 4$ and $P_2 : x + y + 4z = 11$

Then P_1 intersects P_2 in a line L. Find a parametric equations of the intersection-line.

QUESTION 2. Given $P : x + y - 3z = 27$.
a) Let $L : x = t + 4, y = 2t + 6, z = t - 4$. Can we draw L entirely inside the plane P? Show the work

b) Let $V =< 2, 7, 3 >$. Can we draw V inside P? Show the work

c) Is P perpendicular to the plane $-3x + 4y - 5z = 2$? Show the work

Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: `abadawi@aus.edu, www.ayman-badawi.com`

2.3.6 Quiz VI

Name————————————, ID ———————

Quiz VI, MTH 111 , Fall 2021

Ayman Badawi

QUESTION 1. Find y' and do not simplify

(i) $y = 7x^2 + 10\sqrt{x} + sin(9x)$

(ii) $y = 2(x^3 + 7x + 3)^{11}$

(iii) $y = cos(7x)(sin(5x) + 2)$

QUESTION 2. Given $f(x) = (x^2 - 6x - 7)^3$

(i) Find all critical values

(ii) Find all local min., local max of $f(x)$.

(iii) Roughly, sketch the graph of $f(x)$.

Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: abadawi@aus.edu, www.ayman-badawi.com

2.3.7 Quiz VII

Name——————————————, **ID** ——————

Quiz VII, MTH 111 , Fall 2021

Ayman Badawi

QUESTION 1. Find y' and do not simplify

(i) $y = ln(\frac{x^2+3x}{5-2x})$

(ii) $y = e^{(x^3+6x+3)}$

(iii) $y = ln(7x^2 + 5x - 3)e^x$

QUESTION 2. Given $f(x) = ln(3x - 9 + e) + e^{(x-3)}$. Find the equation of the tangent line to the curve at the point (3, 2). (note $ln(e) = 1$)

QUESTION 3. Given $H = (3, 2)$ and $F = (-1, 4)$. Find a point, say Q, on the line $y = -3$ so that $|HQ| + |QF|$ is minimum.

Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: `abadawi@aus.edu, www.ayman-badawi.com`

2.4 Exams

2.4.1 Exam I

Exam One, MTH 111 , Fall 2021

Ayman Badawi

QUESTION 1. (8 points) Consider the equation

$$\frac{(x-2)^2}{9}+\frac{(y+1)^2}{16}=1$$

(i) Sketch

(ii) Find the foci

(iii) Find the vertices

(iv) Find the ellipse-constant k.

QUESTION 2. (8 points)

$$\frac{(x-2)^2}{4}-\frac{(y+1)^2}{5}=1$$

(i) Sketch

(ii) Find the foci

(iii) Find the vertices

(iv) Find the ellipse-constant k.

QUESTION 3. (8 points) Given $x = 4$ is the directrix line of a parabola that has $(-2, 1)$ as its focus. Find the equation of the parabola and sketch.

QUESTION 4. (8 points) Given $x + y + 2z = 3$ intersects $x + y - z = 6$ in a line L. Find a parametric equations of L.

QUESTION 5. (8 points) Given $4x + y + z = 10$ intersects the line $L : x = t + 2, y = -2t + 1, z = -t + 3$ in a point Q. Find Q.

QUESTION 6. (8 points) Given $Q_1 = (1, 2, 3)$, $Q_2 = (-2, 4, 2)$ and $Q_3 = (-4, 6, 8)$.

(i) Convince me that Q_1, Q_2, and Q_3 are not co-linear.

(ii) Find the area of the triangle $Q_1Q_2Q_3$.

(iii) Find the equation of the plane that contains Q_1, Q_2 and Q_3.

QUESTION 7. (8 points) Is $L_1 : x = 2t + 1, y = -t + 3, z = 4t + 1 \ (t \in R)$ parallel to $L_2 : x = 4w - 3, y = -2w + 5, z = 8w - 7 \ (w \in R)$?

Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: `abadawi@aus.edu, www.ayman-badawi.com`

2.4.2 Exam II

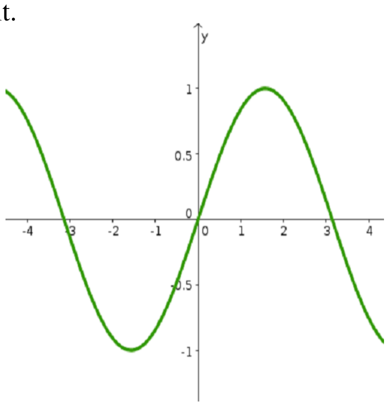
Name——————————————, ID ————————

Exam Two, MTH 111 , Fall 2021

Ayman Badawi

Score = $\frac{}{30}$

QUESTION 1. (6 points) (SHOW THE WORK) Given the graph of the first derivative of $f(x)$ (i.e., $f'(x)$). Stare at it.

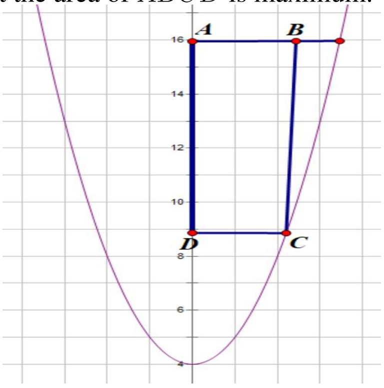


(i) For what values of x does $f(x)$ increase?

(ii) For what values of x does $f(x)$ decrease?

(iii) For what values of x does $f(x)$ have local min and local max?

QUESTION 2. (6 points) (SHOW THE WORK) We want to construct a rectangle (see picture) ABCD, where A, B are on the line $y = 16$, C is on $y = x^2 + 4$, and D is on the y-axis. Find the length and the width of $ABCD$ so that the area of $ABCD$ is maximum.



QUESTION 3. (6 points) (SHOW THE WORK) Given $H = (2, 21)$ and $F = (5, -9)$. Find a point, say Q, on the line $x = -4$ so that $|FQ| + |QH|$ is minimum.

QUESTION 4. (6 points) (SHOW THE WORK) Let $f(x) = (x^2 + 2x - 7)e^x$. For what values of x do we have local min? For what values of x do we have local max? Sketch (roughly).

QUESTION 5. (6 points) (SHOW THE WORK) Find y' and do not simplify

(i) $y = e^{(2x+3)} ln(3x + 5)$

(ii) $y = sin(3x)(2x^3 + 6x + 1)^4$

(iii) $y = ln((\frac{sin(x)+cos(x)}{3x+2})^6)$

Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: `abadawi@aus.edu, www.ayman-badawi.com`

2.4.3 Final Exam

Final Exam, MTH 111 , Fall 2021

Ayman Badawi

Score = $\overline{}$/64

QUESTION 1. (5 points) Consider the parabola

$$2y = x^2 - 8x + 10$$

(i) Write it in the standard form

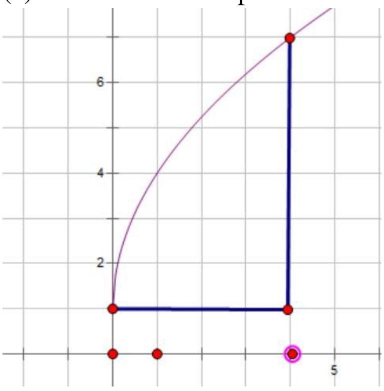
(ii) Find the vertex

(iii) Find the equation of the directrix line

(iv) Find the focus

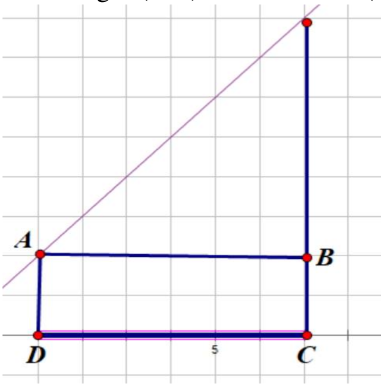
QUESTION 2. (5 points) Given $x = -2$ is the directrix of a parabola that has $F = (-8, 3)$ as its focus. Find the equation of the parabola and sketch (roughly). Show the work.

QUESTION 3. (6 points)
(a) Stare at the below picture. Find the area of the region that is bounded by $y = 3\sqrt{x}+1, y = 1, x = 0,\ and\ x = 4$

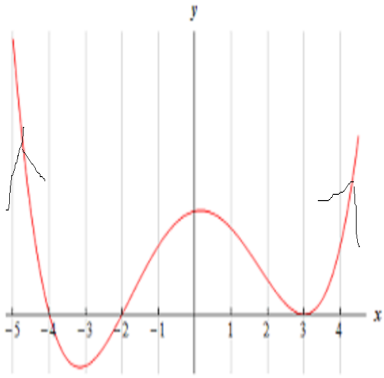


(b) If the curve $y = 3\sqrt{x} + 1$ is rotated 360 degrees about the line $y = 1$, what will be the volume of the outcome object?

QUESTION 4. (6 points) Stare at the below pictures. We want to construct the rectangle ABCD that has maximum area. Given A lies on the line $y = x + 1$, B lies on the line $x = 7$, the point $c = (7, 0)$, and D is a point on the x-axis. Find the length (AD) and the width (DC) of such rectangle. Show the work.



QUESTION 5. (**6 points**) **(SHOW THE WORK)** Given the graph of the first derivative of $f(x)$ (i.e., $f'(x)$). Stare at it.



(i) For what values of x does $f(x)$ increase?

(ii) For what values of x does $f(x)$ decrease?

(iii) For what values of x does $f(x)$ have local min and local max?

QUESTION 6. (**12 points**) Evaluate the following integrals

(i) $\int \frac{(x+3)^2}{x^5}\, dx$

(ii) $\int (sin(x) - cos(x))(sin(x) + cos(x) + 4)^8\, dx$

(iii) $\int \frac{x+3e^x}{x^2+6e^x+5}\, dx$

(iv) $\int \frac{4e^{(\sqrt{x}+3)}}{\sqrt{x}}\, dx$

QUESTION 7. (6 points) Let $f(x) = -x^3 + 3x^2 + 24x + e^{(x^3-3x^2-24x+3)}$

(i) For what values of x does $f(x)$ have local min. and local max.?

(ii) Sketch, roughly, the graph of $f(x)$.

QUESTION 8. (6 points) (a) Given $xe^{3y} + ye^{(2x+1)} + y^2 - 4x^2 + 1 = 0$. Find y'.

(b) Find the equation of the tangent line to the curve $y = x + e^{(x-2)} + ln(x-1)$ at the point $(2, 3)$.

QUESTION 9. (6 points) Given $P_1 : 2x + y - 3z = 10$ intersects the plane $P_2 : -x - y + 5z = -6$ in a line L. Find a parametric equations of L. Then find the symmetric equation of L.

QUESTION 10. (6 points) Given $A = (1, 1, 0)$, $B = (1, 2, 2)$, and $C = (2, 1, 2)$ are the vertices of a triangle.

(i) Find the area of the triangle ABC.

(ii) Find the equation of the plane that passes through A, B, and C.

Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: `abadawi@aus.edu, www.ayman-badawi.com`

Faculty information

Ayman Badawi, American University of Sharjah, UAE.
E-mail: abadawi@aus.edu