MATH 221, FIRST EXAM, SPRING 006

AYMAN BADAWI

TIME————Name——————, Id. Num.———————— Score $\frac{}{100}$

QUESTION 1. (16 points) *Let* $A = \begin{bmatrix} 2 & 4 & 2 & 0 \\ 2 & 5 & -2 & 0 \\ -1 & -2 & 0 & -1 \\ -4 & -8 & -4 & 1 \end{bmatrix}$

a) Find A^{-1}
b) Find $(A^T)^{-1}$

QUESTION 2. (14 points) *a) Let* $A = \begin{bmatrix} 2 & 4 & -2 \\ 1 & 3 & -2 \\ -1 & 0 & 6 \end{bmatrix}$ *Find the (3, 2)-entry of* A^{-1} *without finding* A^{-1}.

b) Consider the system $AX = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$ *Use Cramer's rule to find the value of* x_3.

QUESTION 3. (20 points) *Consider the following system*

$x_1 - 2x_2 + 2x_3 + 2x_4 = -4$

$-x_1 + 3x_2 + x_3 + 2x_4 = -2$

$x_1 - 2x_2 + 2x_3 + 3x_4 = 2$

a) Write the above system in the form $AX = B$.

b) Find the solution for $AX = B$.

C) USE part (b) to Find the solution for $AX = 0$

QUESTION 4. (9 points) *Given A, B are 3×3 matrices such that $det(A) = -3$ and $det(B) = 2$*

a) Find $det(-3A^{-1}B)$

b) Find $det(A^T(B^{-1})^T)$

c) Find $det(A^{-1} + 2adj(A))$.

QUESTION 5. (9 points) *Given that $(5A^{-1} + 3I_2)^T = \begin{bmatrix} -2 & 3 \\ 0 & -4 \end{bmatrix}$. Find the matrix A.*

QUESTION 6. (20 points) *a) Given* $A \quad \widetilde{3R_2} \quad A_1 \quad \widetilde{-2R_2} \quad C \quad \widetilde{\leftrightarrow} \quad B = \begin{bmatrix} 2 & 1 & 1 \\ -2 & -2 & 0 \\ -6 & -3 & 1 \end{bmatrix}$.

a)Find $det(A)$.

b)Find Elementary matrices E_1, E_2, E_3 *such that* $A = E_1E_2E_3B$.

c) FIND the matrix A

QUESTION 7. (12 points) *Let* $A = \begin{bmatrix} 2 & 2 & 2 & a & -6 \\ -2 & -1 & 5 & b & 8 \\ -4 & -4 & -4 & 10 & c \end{bmatrix}$ *be an augmented matrix of a system of linear equations:*

a) For what values of a, b, c *will the system have UNIQUE SOLUTION?*

b) For what values of a, b, c *will the system have have INFINITELY many solutions?*

c) For what values of a, b, c *will the system have NO SOLUTION?*

Department of Mathematics & Statistics, American University Of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates

E-mail address: abadawi@aus.edu, www.ayman-badawi.com