MATH 221, SECOND EXAM, SPRING 004

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QUESTION 1. *(8 points) Write Down True Or False*

(1) *If A is 4×4 matrix and invertible, then the image of A equals to R^4. ( )*

(2) *If 0 is an eigenvalue of an $n\times$ matrix A, then it is possible that the system $AX = 0$ has only the trivial solution. ( )*

(3) *If A, B are $n{\times}n$ matrices and A is row-equivalent to B, then $Char(A) = Char(B)$.*

(4) *$S = \{(x, 3x) \mid x \in R\}$ is a subspace of R^2.*

QUESTION 2. (a), (10 points) *Given $S = \{(x_1, x_2, x_3, x_4) \in R^4 \mid x_1 + x_3 = 0\}$ is a subspace of R^4. Find a basis for S. What is the dimension of S.*

(b), (10 points). *Find a basis for R^4 that contains the two elements : (1, -1, 0, 8) and (0, -2, 1, 1). Explain your work*

QUESTION 3. *Let* $A = \begin{bmatrix} 1 & -1 & 0 & 1 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 2 \end{bmatrix}$

(a), (20 points) *It is easy to see that* $Char(A) = (x-1)(x-2)^3$ *(do not show that). Find* E_2 *(the eigenspace of* A *that corresponds to the eigenvalue 2). Find a basis for* E_2*, what is the dimension of* E_2*.*

(b), (6 points) *Is* A *diagnolizable? Explain.*

c, (10 points). *Find a basis for the image of* A*. What is the dimension of the image of* A*.*

QUESTION 4. (a), (20 points) *Let* $A = \begin{bmatrix} 0 & 4 \\ 1 & 3 \end{bmatrix}$. *Show that* A *is diagnolizable by finding an invertible matrix* P *and a diagonal matrix* D *such that* $P^{-1}AP = D$.

QUESTION 5. (10 points) *a) Let* A *be an* $n \times n$ *matrix. Prove that* A *and* A^T *must have the same eigenvalues.*

b) **(6 points)** *Let* V *be an eigenvector of an invertible* $n \times n$ *matrix* A. *Show that* V *is an eigenvector of* A^{-1}.