2:00 MW
I attend 2:00

Excellent!

99

TEST NUMBER ONE FOR MTH221 SPRING007

AYMAN BADAWI

Name Marwan Mubaied, Id. Num. 20725, Score $\overline{100}$

QUESTION 1. (**20 points**) *Given A is 4×4 matrix such that* $A \xrightarrow{3R_2 \;(\frac{1}{3}R_2)} B \xrightarrow{R_2 \leftrightarrow R_3} C \xrightarrow{-2R_4 + R1 \to R_1} D = \begin{bmatrix} -2 & 0 & 4 & -1 \\ 0 & 4 & 2 & 2 \\ 0 & 0 & -2 & 1 \\ 0 & 0 & 2 & 5 \end{bmatrix} \xleftrightarrow{R_3+R_4 \to R_4} F = \begin{bmatrix} -2 & 0 & 4 & -1 \\ 0 & 4 & 2 & 2 \\ 0 & 0 & -2 & 1 \\ 0 & 0 & 0 & 6 \end{bmatrix}$

$\xleftarrow{2R_4+R_1 \to R_1}$

de

$\mathrm{Det}(F) = 96$

(1) *Find $\det(A)$*

$\mathrm{Det}(F) = 96 \Rightarrow \det(D) = 96 \Rightarrow \det(C) = 96$

$\det(B) = -96 \Rightarrow \boxed{\det(A) = -32}$

(2) *Find the Matrix A.*

$\begin{bmatrix} -2 & 0 & 4 & -1 \\ 0 & 4 & 2 & 2 \\ 0 & 0 & -2 & 1 \\ 0 & 0 & 2 & 5 \end{bmatrix}$ $2R_4 + R_1 \Longleftrightarrow R_1$

$\begin{bmatrix} -2 & 0 & 8 & 9 \\ 0 & 4 & 2 & 2 \\ 0 & 0 & -2 & 1 \\ 0 & 0 & 2 & 5 \end{bmatrix}$ $R_2 \longleftrightarrow R_3$ $\begin{bmatrix} -2 & 0 & 8 & 9 \\ 0 & 0 & -2 & 1 \\ 0 & 4 & 2 & 2 \\ 0 & 0 & 2 & 5 \end{bmatrix}$ $\frac{1}{3} R_2$

$A = \begin{bmatrix} -2 & 0 & 8 & 9 \\ 0 & 0 & -\frac{2}{3} & \frac{1}{3} \\ 0 & 4 & 2 & 2 \\ 0 & 0 & 2 & 5 \end{bmatrix}$

$A\ E_1\ E_2\ E_3 =$

$A = E_1^{-1}\ E_2^{-1}\ E_3^{-1}\ D$

$_2E_1\ E_2\ E_3\ \ A = D$

QUESTION NUMBER ONE CONTINUES:

(3) *Find Elementary matrices* E_1, E_2, E_3 *such that* $A = E_1E_2E_3D$.

$$D = \begin{bmatrix} -2 & 0 & 4 & -1 \\ 0 & 4 & 2 & 2 \\ 0 & 0 & -2 & 1 \\ 0 & 0 & 2 & 5 \end{bmatrix} \quad \begin{matrix} 4 & 2 \\ 0 & -2 \\ 0 & 2 \end{matrix}$$

$-40 - 8 = -48 * -2$

$$E_3^{-1} = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad E_2^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \ \& \ E_1^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1/3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1/3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} D$$

(4) *Find the (3, 4)-entry of* D^{-1}.

$$\frac{A_{43}}{\det(D)} \qquad A_{43} = (-1)^7 \det \begin{bmatrix} -2 & 0 & -1 \\ 0 & 4 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= (-1)^7 * -8 = 8$$

$\det(D) = 96$

$$\boxed{(3,4) \text{ entry } D^{-1} = \frac{8}{96}}$$

QUESTION 2. (**20 points**) *Let* $A = \begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix}$

(1) *Find* A^{-1}.

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & -2 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 & 1 \end{array}\right] \underset{\sim}{\tfrac{1}{2}R_3} \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & -2 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & \tfrac{1}{2} & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 & 1 \end{array}\right] R_3+R_4 \leftrightarrow R_4$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & -2 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & \tfrac{1}{2} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & \tfrac{1}{2} & 1 \end{array}\right] 2R_4+R_1 \leftrightarrow R_1 \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 0 & 1 & 2 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & \tfrac{1}{2} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & \tfrac{1}{2} & 1 \end{array}\right] \Rightarrow A^{-1} = \begin{bmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \tfrac{1}{2} & 0 \\ 0 & 0 & \tfrac{1}{2} & 1 \end{bmatrix}$$

A^{-1}

(2) *Find* $(A^T)^{-1}$

$$(A^T)^{-1} = (A^{-1})^T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & \tfrac{1}{2} & \cancel{\tfrac{1}{2}} \\ 2 & 0 & 0 & 1 \end{bmatrix}$$

(3) *Find the numbers in the third column of* A^2.

$$0\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + 0\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} + 2\begin{bmatrix} 0 \\ 0 \\ 2 \\ -1 \end{bmatrix} + -1\begin{bmatrix} -2 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 4 \\ -3 \end{bmatrix}$$

(4) *Write* A *as product of elementary matrices.*

$$2R_3 \sim -R_3+R_4 \leftrightarrow R_4 \sim -2R_4+R_1 \leftrightarrow R_1$$

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\cancel{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -2 \end{bmatrix}}$$

QUESTION 3. *(***20 points***) Let* $A = \begin{bmatrix} -2 & -2 & -3 & 2 & 2 \\ 1 & 1 & 2 & -3 & 1 \\ 3 & 3 & 6 & -9 & 4 \end{bmatrix}$.

(1) *Solve the system* $AX = \begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix}$ *YOU MUST USE GAUSS JORDAN ELIMINATION. GIVE ME ONE NUMERICAL SOLUTION.*

$$\left[\begin{array}{ccccc|c} -2 & -2 & -3 & 2 & 2 & -2 \\ 1 & 1 & 2 & -3 & 1 & 1 \\ 3 & 3 & 6 & -9 & 4 & 3 \end{array}\right] \overset{-\frac{1}{2}R_1}{\sim} \left[\begin{array}{ccccc|c} 1 & 1 & 3/2 & -1 & -1 & 1 \\ 1 & 1 & 2 & -3 & 1 & 1 \\ 3 & 3 & 6 & -9 & 4 & 3 \end{array}\right]$$

$-R_1+R_2 \to R_2$

$-3R_1+R_2 \to R_3$

$$\left[\begin{array}{ccccc|c} 1 & 1 & 3/2 & -1 & -1 & 1 \\ 0 & 0 & 1/2 & -2 & 2 & 0 \\ 0 & 0 & 3/2 & -6 & 7 & 0 \end{array}\right] \overset{2R_2}{\sim} \left[\begin{array}{ccccc|c} 1 & 1 & 3/2 & -1 & -1 & 1 \\ 0 & 0 & 1 & -4 & 4 & 0 \\ 0 & 0 & 3/2 & -6 & 7 & 0 \end{array}\right]$$

$-\frac{3}{2}R_2+R_1$

$-\frac{3}{2}R_2+R_3 \to$

$$\left[\begin{array}{ccccc|c} 1 & 1 & 0 & 5 & -7 & 1 \\ 0 & 0 & 1 & -4 & 4 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{array}\right]$$

$-4R_3+R_2 \to R_2$

$7R_3+R_1 \to R_1$

$$\left[\begin{array}{ccccc|c} 1 & 1 & 0 & 5 & 0 & 1 \\ 0 & 0 & 1 & -4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{array}\right]$$

$x_1 + x_2 + 5x_4 = 1$

$x_3 - 4x_4 = 0$

$x_5 = 0$

x_1, x_3, x_5 leading variables

x_2, x_4 free variables $\in R$.

$x_1 = -x_2 - 5x_4 + 1$

$x_3 = 4x_4$

$x_5 = 0$

(2) *USE the solution for (1) above to solve the homogeneous system* $AX = 0$

$x_1 = -x_2 - 5x_4$

$x_3 = 4x_4$

$x_5 = 0$

x_1, x_3, x_5 leading Variables

x_2, x_4 free variable $\in R$

QUESTION 4. *(***12 points***)*

Find the matrix 2×2 *matrix* A *such that* $\left(A^T\begin{bmatrix}0&1\\-1&3\end{bmatrix}-3A^T\right)^{-1}=\begin{bmatrix}3&7\\2&5\end{bmatrix}$

$$\left[\begin{array}{cc|cc}3&7&1&0\\2&5&0&1\end{array}\right]\xrightarrow{\frac{1}{3}R_1}\left[\begin{array}{cc|cc}1&7/3&1/3&0\\2&5&0&1\end{array}\right]\xrightarrow{-2R_1+R_2\to R_2}\left[\begin{array}{cc|cc}1&7/3&1/3&0\\0&1/3&-2/3&1\end{array}\right]\xrightarrow{3R_2}\left[\begin{array}{cc|cc}1&7/3&1/3&0\\0&1&-2&3\end{array}\right]\xrightarrow{-\frac{7}{3}R_2+R_1\to}$$

$$\left[\begin{array}{cc|cc}1&0&5&-7\\0&1&-2&3\end{array}\right]\qquad\left(\left(A^T\begin{bmatrix}0&1\\-1&3\end{bmatrix}-3A^T\right)^{-1}\right)^{-1}=\begin{bmatrix}3&7\\2&5\end{bmatrix}^{-1}$$

$$A^T\begin{bmatrix}0&1\\-1&3\end{bmatrix}-3A^T=\begin{bmatrix}5&-7\\-2&3\end{bmatrix}$$

$$A^T\left(\begin{bmatrix}0&1\\-1&3\end{bmatrix}-\begin{bmatrix}3&0\\0&3\end{bmatrix}\right)=\begin{bmatrix}5&-7\\-2&3\end{bmatrix}$$

$$A^T\left(\begin{bmatrix}-3&1\\-1&0\end{bmatrix}\right)=\begin{bmatrix}5&-7\\-2&3\end{bmatrix}$$

$$A^T\begin{bmatrix}-3&1\\-1&0\end{bmatrix}\begin{bmatrix}0&-1\\1&-3\end{bmatrix}=\begin{bmatrix}5&-7\\-2&3\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix}$$

$$(A^T)^T=\begin{bmatrix}-7&16\\3&-7\end{bmatrix}^T$$

$$A=\begin{bmatrix}-7&3\\16&-7\end{bmatrix}$$

QUESTION 5. *(***8 points***) Let* A *be a* 4×6 *matrix and* B *be the fourth column of* A*. Show that the system* $AX=B$ *is consistent by giving numerical values for* $x_1,x_2,\ldots,x_6$*. Explain why the system* $AX=B$ *has infinitely many solutions.*

~~$\left[\begin{array}{cccccc}1&2&3&4&5&6\\7&8&9&10&11&12\\13&14&15&16&17&18\\1&2&3&4&5&6\end{array}\right]\left[\begin{array}{c}4\\10\\16\\4\end{array}\right]$ A 4×6 B 4×1~~

$$\left[\begin{array}{cccccc|c}1&0&0&0&0&0&0\\0&1&0&0&0&0&0\\0&0&0&1&0&0&0\\0&0&0&0&0&1&0\end{array}\right]$$

It is consistent and has infinately many solutions

$x_1=0$
$x_2=0$
$x_4=\phi$
$x_6=0$
→ leading Variable

x_3,x_5 free variables $\in R$

The system has infinately many Solutions because All the leaders have the number "1" so it will never have no Solution and it has infinately many Solutions because the ~~#of~~ Variable is >greater than the number of equations

we have 6 Variable and 4 equations

~~adj(A) = det(A) A^{-1}~~ ~~det(A A^{-1} det(a) - 3I_3)~~

$$\cancel{\left(\begin{bmatrix}2&0&0\\0&2&0\\0&0&2\end{bmatrix} - 3\begin{bmatrix}3&0&0\\0&3&0\\0&0&3\end{bmatrix}\right)}$$

$(\det A)^2 = 2^2 = 4$

QUESTION 6. (**9 points**) *Let A, B be 3×3 matrices, $C = adj(A)$ such that $det(A) = 2$ and $det(B) = -3$.*

a. Find $det(2C)$ *b. Find $det(AC - 3I_3)$* *c. Find $det(2B^T A^{-1})$*

a. $\det(2C) = (2)^3 \det(C) = (2)^3 (\det A)^2 = 8 \cdot 4 = 32$

$= \det(A A^{-1} \det(a) - 3I_3) = \det\left(\begin{bmatrix}2&0&0\\0&2&0\\0&0&2\end{bmatrix} - \begin{bmatrix}3&0&0\\0&3&0\\0&0&3\end{bmatrix}\right) = -1$

$(2)^3 \det(B^{T}) \det(A^{-1}) = (2)^3 (-3) \frac{1}{2} = -12$

QUESTION 7. (**6 points**) *Let* $A = \begin{bmatrix} a & b & c \\ -2 & 3 & 0 \\ 4 & 3 & 3 \end{bmatrix}$. *Given* $det(A) = 4$. *Find the det* $\left(\begin{bmatrix} a & b+5 & c \\ -2 & 3 & 0 \\ 4 & 3 & 3 \end{bmatrix}\right)$ *(HINT: YOU MUST USE THE FIRST ROW TO CALCULATE THE DETERMINANT)*

$\begin{bmatrix} a & b & c \\ -2 & 3 & 0 \\ 4 & 3 & 3 \end{bmatrix}$ $a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13} = 4$

$a(9) + b(3a - 4c) + c(3a + 2b) = 4$

$a^2 + 3ab - 4bc + 3ca + 2bc = 4$

$a^2 + 3ab - 2bc + 3ca = 4$

In the Back of the page

QUESTION 8. (**5 points**) *Let* $A = \begin{bmatrix} 2 & 123 & -1 \\ 1 & 456 & 1 \\ 2 & 789 & 1 \end{bmatrix}$ *If you know that* $det(A) = -420$, *then find the value of x_2 in the solution of the linear system* $A\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

$x_2 = \dfrac{\det\begin{bmatrix} 2 & 1 & -1 \\ 1 & 0 & 1 \\ 2 & 1 & 1 \end{bmatrix}}{-420}$

$\det \begin{bmatrix} 2 & 1 & -1 \\ 1 & 0 & 1 \\ 2 & 1 & 1 \end{bmatrix} \begin{matrix} 2 & 1 \\ 1 & 0 \\ 2 & 1 \end{matrix}$

$2 - 1 - (2 + 1)$

-2

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$x_2 = \dfrac{\det\begin{bmatrix} 2 & 1 & -1 \\ 1 & 0 & 1 \\ 2 & 1 & 1 \end{bmatrix}}{-420} = \dfrac{-2}{-420} = \boxed{\dfrac{1}{210}}$

$$\cancel{\det(A A^{-1} \det(a) - 3I_3)}$$

$$\cancel{\det\left(\begin{bmatrix}2&0&0\\0&2&0\\0&0&2\end{bmatrix} - \begin{bmatrix}3&0&0\\0&3&0\\0&0&3\end{bmatrix}\right) = -1}$$