MATH 221, REVIEW SHEET FOR TEST #2, SPRING 005

QUESTION 1. *T OR F*

(1) If T is a linear transformation from R^4 into P_3 such that $Ker(T) = Span(1,-1,-1,1)$, then $Range(T) = P_3$
(2) It is possible to construct a nonzero linear transformation from R^6 into R such that $dim(Ker(T)) = 4$
(3) It is possible to construct a linear transformation T from R^6 into $R_{2\times 3}$ such that $Ker(T) = (0,0,0,0,0,0)$ and $Range(T) = R_{2\times 3}$.
(4) $Span\{3+x, -6+4x\} = P_2$
(5) If A is a nonzero matrix 3×6, then $Nullity(A) \leq 5$.
(6) $dim(Span\{1+x^2, -2+x^2, -5+4x^2\}) = 2$
(7) It is possible to have 4 independent elements in R^3.
(8) It is possible that the span of 6 element in R^5 is equal to R^5.
(9) $(-2 \quad \infty)$ is a subspace of R
(10) $dim(span\{(2,0,1),(-2,0,1),(0,0,2)\}) = 2$.
(11) if A is 7×7 and $AX = 0$ has a nontrivial solution, then the columns of A are dependent
(12) If A is 10×7 and $AX = 0$ has only the trivial solution, then the rank(A) = 7
(13) if A is a 3×5 matrix and $AX = \begin{bmatrix} 2 \\ 3 \\ -4 \end{bmatrix}$ has no solution, then $dim(row(A)) \leq 2$.
(14) If X, Y are independent, then $X, Y, X+Y$ are independent
(15) Every set of 4 elements of R^4 form a basis for R^4.
(16) R^3 has a basis of the form $\{X, X+Y, Y\}$ where X, Y are some elements in R^3.

QUESTION 2. (1) Let $A = \begin{bmatrix} 1 & 1 & 1 \\ -1 & -1 & -1 \end{bmatrix}$ and $T: R^3 \longrightarrow P_2$ be a linear transformation such that $T(v) = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} v^T + \begin{bmatrix} -1 & -1 & -1 \end{bmatrix} v^T x$. Find the standard matrix representation of T. Find Ker(T). Find Range(T). Find T(-1, 2, -1).

(2) Let $A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & 1 & -1 & 1 \\ 1 & 3 & 1 & 3 \end{bmatrix}$, and let $T: R^4 \longrightarrow R^3$ be a linear transformation such that $T(w) = Aw^T$. Find $T(-1,-1,-1,-1)$. Find basis for Ker(T), find a basis for the Range(T).

(3) Let $T: R^2 \longrightarrow R^2$ be a linear transformation such that $T(2,1) = (1,1)$ and $(-4,1) \in Ker(T)$. Find $T(-8,5)$. Find the standard matrix representation of T.

(4) Let $T: P_3 \longrightarrow R$ be a linear transformation such that $T(f(x)) = \int_0^1 f(x)dx$. Find the standard matrix representation of T, then find a basis for $Ker(T)$.

(5) *Let $T : P_4 \longrightarrow P_2$ such that $T(f(x)) = f'(-1) + f(1)x$. Show that T is a linear transformation. Find the standard matrix representation of T. Find basis for Ker(T), and Range(T).*

QUESTION 3. (1) *Let $S = \{(x, y, z) \in R^3 | 2x - 5y + 6z = 0\}$. Show that S is a subspace of R^3. Find a basis for S. What is the dimension of S.*

(2) *Let $U_1 = \{A \in R_{3\times 2} \mid a_{11} + a_{21} + a_{31} = 0\}$ and let $U_2 = \{A \in R_{3\times 2} \mid a_{11} + a_{22} = 0\}$. Given that U_1 and U_2 are subspaces of $R_{3\times 2}$, and hence $U_1 \cap U_2$ is a subspace of $R_{3\times 2}$. Find a basis for U_1, a basis for U_2, and a basis for $U_1 \cap U_2$.*

(3) *Let $S = \{f(x) \in P_4 \mid f'(-2) = 0\}$. Show that S is a subspace of P_4, then find dim(S).*

(4) *let $S = \{f(x) \in P_3 \mid f(-1) = 0$ and $f\prime(-1) = 0\}$. Show that S is a subspace of P_3, then find a basis for S.*

(5) *Let* $A = \begin{bmatrix} 1 & -2 & 1 & 1 & 2 \\ -1 & 3 & 0 & 2 & -2 \\ 0 & 1 & 1 & 3 & 4 \\ 1 & 2 & 5 & 13 & 5 \end{bmatrix}$. *Find a basis for the column space of A, Find a basis for the row space of A. Find a basis for NulL(A).*

(6) *Let $U = span\{(-2, 3, -4), (0, 2, -3), (-4, 8, -11)\}$. Find the dimension of U. Find a basis for U.*

(7) *let $S = Span\{2, cos6x, sin^2(3x)\}$. Find a basis for S. What is the dimension of S.*

(8) *Let $f_1(x) =| 2x^5 |$ and $f_2 = 3x^5$. Are f_1, f_2 independent in $C[-2 \quad 2]$? are f_1, f_2 independent in $C[0 \quad 3]$?*

(9) *Is $span\{1 + x, 3 + x, -1 + x^2, 2 + x + x^2\} = P_3$? Explain*

(10) *Given A is 3×4 and $Null(A) = \{ \begin{bmatrix} x_3 + 2x_2 \\ x_2 \\ x_3 \\ -5x_3 + 6x_2 \end{bmatrix} \mid x_3, x_2 \in R\}$. Let B be a matrix such that $AB = 0$. Prove that the columns of B "live" in the Null(A). Find a matrix B 4×4 such that Rank(B) = 2 and $AB = 0$. Find a matrix C 4×6 such that Rank(C) = 1 and $AC = 0$. Is it possible to find a matrix D of rank 3 such that AD = 0? Explain*

(11) *Find a basis, say B, for R^4 such that B contains $v_1 = (2, -3, 1, 0), v_2 = (0, 3, 6, 6)$. Given than $v_3 = (2, -6, -5, -6)$ "lives" in $span\{v_1, v_2\}$. Find α_1, α_2 such that $v_3 = \alpha_1 v_1 + \alpha_2 v_2$.*