# MATH 221, REVIEW FOR THE FIRST EXAM, FALL 2005, THIS IS NOT THE TEST BUT TO TEST A TEST

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**QUESTION 1.** *1. MAKE SURE that you know how to handle GRAPH-Questions (see the book page 61, questions 32 and 33.*

*2. Make SURE that you know how to handle Questions on Circuits (see Page 29, Question number 20).*

*3. Make Sure that you know how to handle Questions on page 27, number 8, 9, 10.*

**QUESTION 2.** *Write down true or false. If false, then give a counter example:*

(1) *If $A$ is an $n \times n$ matrix and singular, then the reduced echelon form of $A$ has at least one row consists of zeros.*
(2) *If $A$ is a $3 \times 3$ matrix and $det(A^{-1}) = 2$, then $det(A^T) = 1/2$*
(3) *If $A, B$ are a $4 \times 4$ matrices and $A$ is row-equivalent to $B$, then $det(A) = det(B)$.*
(4) *If a homogeneous system has infinitely many solutions, then the system has more variables than equations.*
(5) *If $A$ is $3 \times 3$ and $AX = \begin{bmatrix} 2 \\ 3 \\ -2 \end{bmatrix}$ has no solution, then det(A) = 0*

**QUESTION 3.** *Given $A, B$ are $5 \times 5$ matrices such that $det(adj(A)) = 16$ and $det(B) = -2$*

*a) Find $det(3A^{-1}B)$*
*b) Find $det(2A^T(B^{-1})^T)$*
*c) Find $det(I_5 + Aadj(A))$.*

**QUESTION 4.** *Consider the following system*

$2x_1 - 2x_2 + 4x_3 - 2x_4 = -2$
$-x_1 + 2x_2 + x_3 + 2x_4 = 2$
$x_1 + x_2 + 4x_3 + 3x_4 = 3$

*a) Write the above system in the form $AX = B$.*

*b) Find the general solution for $AX = B$.*

*C) USE part (b) to Find the general solution for $AX = 0$*

**QUESTION 5.** *a) Given* $A \quad \widetilde{2R_2 + R_3 \to R_3} \quad A_1 \quad \widetilde{3R_3} \quad A_2 \quad \widetilde{R_2 \leftrightarrow R_1} \quad B = \begin{bmatrix} 2 & 1 & 1 \\ -2 & -2 & 0 \\ -3 & 5 & 6 \end{bmatrix}$.

*Find* $det(A)$.
*Find Elementary matrices* $E_1, E_2, E_3$ *such that* $E_1E_2E_3A = B$. *Then FIND the matrix* $A$

**QUESTION 6.** *Let* $A = \begin{bmatrix} 2 & 3 & -1 & 0 \\ 1 & -3 & -2 & 3 \\ -1 & 0 & -1 & -1 \\ -1 & 0 & 0 & 4 \end{bmatrix}$ *Find the third column of* $A^{-1}$ *without finding* $A^{-1}$.

b) *Consider the system* $AX = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$ *Use Cramer's rule to find the value of* $x_3$.

**QUESTION 7.** *Let* $A = \begin{bmatrix} 2 & 3 & -1 & 0 \\ 1 & -3 & -2 & 3 \\ -1 & 0 & -1 & -1 \\ -1 & 0 & 0 & 4 \end{bmatrix}$

*Find* $A^{-1}$
*Write* $A$ *as a product of elementary matrices.*
*Find* $(A^T)^{-1}$ *and* $(A^2)^{-1}$.

**QUESTION 8.** *Let* $A = \begin{bmatrix} 2 & a & b \\ 0 & 0 & 3 \\ 0 & x & -2 \end{bmatrix}$. *Find the values of* $a, b, x$ *that will make* $A$ *nonsingular.*

**QUESTION 9.** *Given that* $(2A^T - 3I_2)^{-1} = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$. *Find the matrix* $A$.

**QUESTION 10.** *Let* $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & -1 \\ 0 & 0 & 2 \end{bmatrix}$. *Find* $A^{-1}$ *using the adjoint method.*

**QUESTION 11.** *Let* $A, B$ *be nonzero* $n \times n$ *matrices such* $AB = 0$. *Prove that neither* $A$ *nor* $B$ *is nonsingular.*

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