MATH 221, REVIEW SHEET FOR TEST #1, SPRING 005

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QUESTION 1. *T OR F, if False then give a counter example*

(1) *If* E *is an elementary matrix of type I, then* $E^T = E$.

(2) *Let* A *be* 3×3 *such that* $det(A) = 0$, *and let* R *be the reduced echelon form of* A. *Then* $RX = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$ *has no solution.*

(3) *If* $A \begin{bmatrix} 2 \\ 5 \\ 3 \end{bmatrix} = 0$, *then* $AX = 0$ *has infinitely many solutions*

(4) *If* A, B *are* $n \times n$ *and they have the same reduced echelon matrix, then* $A = B$.

(5) *If* A *is row equivalent to* B, *then the reduced echelon form of* A = *the reduced echelon form of* B.

(6) *if* A, B *are invertible, then* A *is row equivalent to* B.

(7) *If* $UA = B$ *and* U *is an invertible matrix, then* A *is row equivalent to* B.

(8) *If* A *is* 3×3 *and det(A) = -2, then* $det(3A) = -6$.

(9) *If* A, B *are* 4×4 *and* $E_1 E_2 A = B$, *where* E_1, E_2 *are elementary of type III, then det(A) = det(B).*

(10) *If* $AB = 0$, *then either* $A = 0$ *or* $B = 0$.

(11) *If* A *is* 3×3 *and* $AX = 0$ *has no nontrivial solution, then the reduced echelon form of* A *has at least one row of zeros.*

(12) *If* A *is* 6×6 *and* $AX = B$ *has a solution for every* B, 6×1, *then* A *is invertible.*

(13) *If* $AX = 0$ *has infinitely many solutions, then the system has more variables than equations.*

QUESTION 2. (1) *Let* $A = \begin{bmatrix} 0 & 1 & -3 \\ 1 & -1 & 5 \\ 3 & -2 & 8 \end{bmatrix}$. *Given that* A *is invertible. Write* A *as product of elementary Matrices. What is the (3,1)-entry of* A^{-1}. *Find the entries of the second column of* A^{-1}

(2) *Let* $A = \begin{bmatrix} 0 & 0 & -1 \\ 1 & -3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, *and* R *be the reduced echelon form of* A. *Find an invertible matrix* U *such that* $UA = R$.

QUESTION 3. *Consider the following system:*

$x_1 - 2x_2 + x_3 - x_4 = 6$
$x_2 + 2x_3 + 2x_4 = 4$
$2x_1 - 3x_2 + 4x_3 = 16$

1)Write the above system in the form $AX = B$ (where A is the coefficient MATRIX of the system, X is the Variable-Column, and B is the constant-Column)

2) Find the augmented matrix of the system

3) Find the general solution of the system
4) From (3) find the general solution to $AX = 0$.

QUESTION 4. *Let A, B be 3×3 matrices such that*
$A \underbrace{3R_2 + R_2 \to R_2} A_1 \underbrace{-6R_2} B$. *Find two elementary matrices E_1, E_2 such that $A = E_1 E_2 B$.*

QUESTION 5. *Let* $A = \begin{bmatrix} 0 & 3 & -1 & 0 \\ 0 & 0 & -2 & 3 \\ 2 & 2 & -2 & -1 \\ -4 & -7 & 5 & -6 \end{bmatrix}$ *transform A to an upper triangular matrix, then find $det(A)$.*

QUESTION 6. *Given* $A \underbrace{R_2 \leftrightarrow R_3} A_1 \underbrace{2R_3 + R_3 \to R_3} A_2 \underbrace{6R_1} A_3 = \begin{bmatrix} 3 & 0 & 0 \\ -2 & -2 & 0 \\ 3 & 5 & 6 \end{bmatrix}$. *Find $det(A)$.*

QUESTION 7. *Given A, B are 3×3 matrices such that $det(A) = -2$ and $det(B) = 3$. Find $det(2AB^T)$, $det(A^{-1} + adj(A))$.*

QUESTION 8. *Consider the following system:*
$x_1 + x_2 - x_3 = 2$
$-x_1 + 2x_2 + 3x_3 = 6$
$2x_1 + 2x_2 + x_3 = 4$

USE CRAMER's RULE TO FIND the value of x_2.

QUESTION 9. *Let* $A = \begin{bmatrix} 1 & 2 & 4 \\ -1 & -1 & 6 \\ -1 & -1 & 4 \end{bmatrix}$. *Find A^{-1} using the adjoint-method.*

QUESTION 10. *1)Let A be an $n \times n$ invertible matrix. Prove that $det(A^{-1}) = 1/det(A)$.*
2) Let A, B be $n \times n$ matrices. Prove that $det(A + B^T) = det(A^T + B)$.
3)Let A, B be NON-ZERO $n \times n$ matrices such that AB is not invertible. Prove that neither A nor B is invertible.
4) Let A be an $n \times n$ invertible matrix. Prove that $det(adj(A)) = (det(A))^{n-1}$
5) Let A, B be $n \times n$ matrices such A is invertible and $AB = BA$. Prove that $A^{-1}B = BA^{-1}$.
6) Let A be a 7×7 matrix. Prove that $A - A^T$ is singular.
7) Let A be a 8×8 matrix. Prove that $a_{31}A_{51} + a_{32}A_{52} + ... + a_{38}A_{58} = 0$.
8) If A is singular, then show that $Adj(A)$ is singular.