MATH 221, FINAL EXAM , SPRING 004

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QUESTION 1. (**9 points**) (**True or False**)

(1) *If A is a matrix 3×3 and $B = 2A$, then $det(B) = 2det(A)$ (*   *)*

(2) *There is a linear transformation from R^5 into R^6 such that $Range(T) = R^6$ (*   *)*

(3) *If A is 4×4 matrix and $AX = B$ has no solution for some B, 4×1, then $AX = 0$ has infinitely many solutions (*   *)*

(4) *If E is a 4×4 matrix and in reduced echelon form such that $E \neq I$, then $EX = \begin{bmatrix} 2 \\ 3 \\ 0 \\ 4 \end{bmatrix}$ has no solution (*   *)*

(5) *If T is a linear transformation from R^8 into R such that for some nonzero element $v \in R^8$ we have $T(v) = 21$, then $dim(Ker(T)) = 7$ (*   *)*

(6) $S = \{A \in R^{3\times3} \mid det(A) = 0\}$ *is a subspace of* $R^{3\times3}$ *(*   *)*

QUESTION 2. (**6 points**) *Let* $A = \begin{bmatrix} 0 & -2 & -4 & 0 \\ 3 & 6 & 3 & 6 \\ -6 & -12 & 0 & 2 \\ 0 & 0 & -6 & 10 \end{bmatrix}$. *Use row-operations to find* $det(A)$.

QUESTION 3. (**16 points**) *Let* $A = \begin{bmatrix} 1 & 0 & -1 & 0 \\ -1 & 0 & 2 & 4 \\ 2 & 0 & -2 & 0 \end{bmatrix}$, *and* $T : R^4 \to R^3$ *such that* $T(a_1, a_2, a_3, a_4) = A \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix}$.

1) Show that T *is a linear transformation*

2) Find the Range(T), basis for Range(T), and dim(Range(T)).

3) Find Ker(T), basis for Ker(T), and dim(Ker(T))

QUESTION 4. (**14 points**) *Let* $S = \{(a_1, a_2, a_3, a_4) \in R^4 \mid a_1 + 2a_2 - 4a_3 + a_4 = 0\}$ *be a subspace of* R^4.

1) Find a basis for S*, what is the dimension of* S.

2) Find an Orthogonal basis for S.

QUESTION 5. (**15 points**) *Let* $A = \begin{bmatrix} -1 & 2 & -2 & 0 \\ 0 & 3 & 1 & 2 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}$.

1) Find the characteristic polynomial of A.

2) Find E_3 *(The eigenspace of* A *that corresponds to the eigenvalue 3). Find a basis for* E_3*, what is the dimension of* E_3*?.*

4) Is A *diagnolizable? Explain*

QUESTION 6. *(***10 points***) Given* $T : R^2 \to R$ *is a linear transformation such that* $(-2, 4) \in Ker(T)$ *and* $T(4, -2) = 3$*.*

1) Find $T(12, 0)$

Is $(-6, 10) \in Ker(T)$*? Explain*

QUESTION 7. (**9 points**) *Given* A *is a* 3×3 *matrix such that* $det(A) = -2$. *Find*
1) $det(adj(A))$

2) $det(-3A^{-1}A^T)$

3)$det(2I_3 + 3adj(A)A)$

QUESTION 8. (**6 points**) *Let* $A = \begin{bmatrix} 0 & -2 & -2 \\ 2 & 1 & 4 \\ -4 & -2 & 2 \end{bmatrix}$

1) Explain why A *is invertible.*

2) find the (3,2)-entry of A^{-1}.

QUESTION 9. (**5 points**) *Let* $v, X_1, X_2, X_3 \in R^4$ *and suppose that there are UNIQUE real numbers* c_1, c_2, c_3 *such that* $v = c_1X_1 + c_2X_2 + c_3X_3$. *Prove that* X_1, X_2, X_3 *are independent.*

QUESTION 10. (**10 points**) *Let* $A = \begin{bmatrix} 1 & -2 & 3 \\ -1 & a & -3 \\ 2 & -4 & b \end{bmatrix}$, *and* $B \begin{bmatrix} 1 \\ 3 \\ c \end{bmatrix}$.

1) Find the values of a, b, c *so that the system* $AX = B$ *has a unique solution*

2)Find the values of a, b, c *so that the system* $AX = B$ *has infinitely many solutions.*

3) Find the values of a, b, c *so that the system* $AX = B$ *has no solutions*