Review for final, MTH 320, SPRING 2009

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QUESTION 1. (i) Is $(Q^*, \times)$ a group-isomorphic to $(Q, +)$?

(ii) Let $(M, *)$ be a group such that $a \in M$ and $\mid a \mid < \infty$. If $b \in M$, show that $\mid b * a * b^{-1} \mid = \mid a \mid$

(iii) Is it true that every ideal of $(Z, +, .)$ is of the form nZ for some $n \geq 1$?

(iv) How many group isomorphism are there from $(Z_8, +_8)$ into $(Z_8, +_8)$?

(v) Show there is only one nontrivial group homomorphism from $(Z_{10}, +_{10})$ into $(Z_5^*, \times_5)$.

(vi) Let $(M, *)$ be a group and assume $(M/Z(M), \wedge)$ is cyclic. Prove that M is abelian (Hint: See my proof of the result every group of of order q^2 is abelian)

(vii) Let $I = (x^2 + 1)$ be an ideal of $(Z_7[X], +, .)$. We know that $(Z_7[X]/I, +^\wedge, .^\wedge)$ is a field. How many elements does $Z_7[X]/I$ have?

(viii) Let $(M, *)$ be an infinite cyclic group . Show that $(M, *)$ is group-isomorphic to $(Z, +)$.

(ix) What are the values of $3/4$, $2/3$ in $(Z_5^*, \times_5)$.

(x) If a, b in a group such that $\mid a \mid = \mid b \mid = 12$. Is it possible that $a^6 = b^4$?

(xi) Let H be a finite subgroup of $(C^*, \times)$. Show that H is cyclic.

(xii) Let H be a finite subgroup of $(C^*, \times)$ with 6 elements. Find the elements of H.

(xiii) Show that there is a group-homomorphism , say f, from $(Z, +)$ into $(C^*, \times)$ such that $Ker(f) = 6Z$

(xiv) What is the order of $3/7 + 5Z$ in the group $(Q/5Z, \wedge)$?

(xv) A problem in Number Theory that it is two lines proof using groups: Let $p \geq 3$ be a prime number. Show that $p \mid (p-1)! + 1$.

(xvi) Let $H = \{\pm \frac{q_1^{k_1} \times \cdots \times q_n^{k_n}}{p_1^{m_1} \times \cdots \times p_i^{m_i}} \mid k_1 + \cdots + k_n = m_1 + \cdots + m_i$ where all $q_i's$ and $p_i's$ are positive prime numbers in Z, and all $k_i's$ and $m_i's$ are nonnegative integers $\}$ be a subset of Q^*.

a. Show that $(H, \times)$ is a subgroup of $(Q^*, \times)$.

b. Define $=_R$ on $(Q^*, \times)$ such that $a =_R b$ if $b^{-1} \times a = a/b \in H$. We know that $=_R$ is an equivalent relation since H is a subgroup of Q^*. Show that $49 =_R 15$. Show that $27 \neq_R 21$. In general show that if m is a prime number in Z, then $m =_R 2$.

c. Since (Q^*/H) is a group, note that each element in Q^*/H is of the form $d \times H$ for some $d \in Q^*$. Show that $(Q^*/H, \wedge)$ is cyclic and it is generated by $3 \times H$.

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