Review, MTH 320, SPRING 2009

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QUESTION 1. YOU MUST BE AWARE of all the concepts discussed in HW1 to HW8.
Few words but it says it all : "o a mathematician, real life is a special case."
"One should not aim at being possible to understand, but at being impossible to misunderstand."

QUESTION 2. Let N, H be normal subgroups of a group $(G, *)$. Prove that $NH = \{n * h : n \in N \text{ and } h \in H\}$ is a normal subgroup of G. (First prove that NH is a subgroup of G, then show it is normal in G)

QUESTION 3. Let H be a subgroup of a group G such that [G:H] = 2 (Note that [G:H] denotes the number of distinct left cosets of H) . Prove that H is a normal subgroup of G.

QUESTION 4. Let H be a normal subgroup of a group $(G, *)$ and let $a \in G$. If $\mid a \mid \neq 5$, $a * H \in G/H$ such that $\mid aH \mid = 5$, and H has exactly 7 elements, what is the order of a? prove your claim.

QUESTION 5. Let G be a group with an odd number of elements. Prove that $a^2 \neq e$ for each non identity $a \in G$.

QUESTION 6. Let G be a cyclic group with n elements. Let k be a factor of n ($k \geq 1$) Prove that the equation $x^k = e$ has exactly k distinct solutions.

QUESTION 7. Give me an example of a an abelian group G with 8 elements such that the equation $x^2 = e$ has exactly 8 distinct solutions. So you should conclude from the previous question that "cyclic" is crucial in the statement.

QUESTION 8. Let G be a group such $\mid G \mid_s$ = pq, where p and q are prime numbers. Prove that every proper subgroup of G is cyclic. (A subgroup H of G is called a proper subgroup if $H \neq G$.)

QUESTION 9. Let $a \in A_5$ such that $\mid a \mid = 2$. Show that $a = (a_1, a_2)o(a_3, a_4)$, where a_1, a_2, a_3, a_4 are distinct elements in $\{1, 2, 3, 4, 5\}$.

QUESTION 10. Let $\alpha = (1, 2, 3)(1, 2, 5, 6) \in S_6$. Find $\mid \alpha \mid$, then find α^{53}.

QUESTION 11. Let $(M, *)$ be a monoid with identity e and H be a subset of M. If (H, *) is a group, can we conclude that the identity of H is also e? Prove it or give a counter example.

QUESTION 12. Let $(M, *)$ be a group with identity e and H be a subset of M. If (H, *) is a group, can we conclude that the identity of H is also e? Prove it or give a counter example.

QUESTION 13. Give me an example of a group M such that every proper subgroup of M is cyclic but M is not cyclic.

QUESTION 14. Let (M, *) be a finite abelian group and $a \in M$ ($a \neq e$)such that $\mid a \mid = k$. Let $S = \{c \in M \mid \mid c \mid = k\}$. Can we conclude that S has exactly $\Phi(k)$ elements? Prove it or give a counter example.

QUESTION 15. Let (M, *) be a finite cyclic group and $a \in M$ ($a \neq e$)such that $\mid a \mid = k$. Let $S = \{c \in M \mid \mid c \mid = k\}$. Can we conclude that S has exactly $\Phi(k)$ elements? Prove it or give a counter example.

QUESTION 16. Let a be an element in a group G such that $a^n = e$ for some positive integer n. If m is a positive integer such that gcd(n,m) = 1, then prove that $a = b^m$ for some b in G. (Note that if gcd(m, n) = k, then there are two integers d, f such that k = dm + fn.)

QUESTION 17. If (M, *) is an infinite group, prove that M has infinitely many subgroups.

QUESTION 18. Let $(M, *)$ be an abelian group of with 45 elements. If H is a subgroup of (M, *) such that G/H is a cyclic group with 9 elements. Prove that M is a cyclic group.

QUESTION 19. Let H_1, H_2 be subgroups of a group $(M, *)$. Suppose that $H_1 \not\subseteq H_2$ and $H_2 \not\subseteq H_1$. Prove that $(H_1 \cup H_2, *)$ is never a group.

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