PROBLEM SET 5

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Exercise 1. *Let $a \in S_8$ such that $a \neq e$. Find all possibilites for $|a|$. For each order you claim, say m, give an element $a \in S_8$ such that $|a| = m$.*

Solution. The possible orders of elements living in S_8 are 2, 3, 4, 5, 6, 7, 8, 10, 12 and 15. We demonstrate this below:

$|(1\ 2\ 3) \circ (4\ 5\ 6\ 7\ 8)| = 15$

$|(1\ 2\ 3\ 4) \circ (5\ 6\ 7)| = 12$

$|(1\ 2) \circ (4\ 5\ 6\ 7\ 8)| = 10$

$|(1\ 2\ 3\ 4\ 5\ 6\ 7\ 8)| = 8$

$|(1\ 2\ 3\ 4\ 5\ 6\ 7)| = 7$

$|(1\ 2\ 3\ 4\ 5\ 6)| = 6$

$|(1\ 2\ 3\ 4\ 5)| = 5$

$|(1\ 2\ 3\ 4)| = 4$

$|(1\ 2\ 3)| = 3$

$|(1\ 2)| = 2$ □

Exercise 2. *Give me an abelian subgroup, say H, of the group $(S_5, \circ)$ such that $|H| = 6$.*

Solution. Let $h = (1\ 2\ 3) \circ (4\ 5)$. We have $|h| = \text{lcm}(3, 2) = 6$. Let $H = < h >$, thus $|H| = 6$. □

Exercise 3. *Let $(M, *) = < w >$ be a finite cyclic group of order 12 and generaterd by $w \in M$. Find all elements in M that have order 12. Also, find all elements in M that have order 4. In both cases, write your answers in terms of w.*

Solution. For the first part, the orders of w, w^5, w^7, w^{11} are 12, since 5, 7, and 11 are the only relatively prime numbers to 12. For the latter, we have w^3 and w^9, all with order 4. □