PROBLEM SET 3

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Exercise 1. *Let H be a subgroup of a group S, and $a \in S \ / \ H$. Show $a * H$ is never a subgroup of S.*

Proof. We know that a subgroup H of S contains the identity of S. To form a coset of H, one has to select $a \in S \ / \ H$. Suppose this is done, then we know that $a * H \cap H$ is empty, so the identity of H (that is, the identity of S) is not an element of $a * H$. Thus $a * H$ cannot be a subgroup of S. □

Exercise 2. *Given $\alpha = (1\ 2\ 3\ 4\ 5\ 6\ 7\ 8)$, with the identity map as sketched. Find m so that α^m is the following:*

Solution. $m = 4$, with $\alpha^m = \alpha^4 = (1\ 5) \circ (2\ 6) \circ (3\ 7) \circ (4\ 8)$. □

Exercise 3. *Let $H = \{e,\ (1\ 3)\}$. This is a subgroup of S_3. Find all distinct left and right cosets of H (including $e * H$ and $H * e$).*

Solution. We know by Lagrange's Theorem that $[S_3 : H] = 3$; we begin by computing the left cosets.

$$e * H = \{e, (1\ 3)\}$$
$$(2\ 3) * H = \{(2\ 3), (1\ 2\ 3)\}$$
$$(1\ 3\ 2) * H = \{(1\ 3\ 2), (1\ 2)\}$$

and one can easily verify that $H \cup (2\ 3) * H \cup (1\ 3\ 2) * H = S_3$. We now list the right cosets,

$$H * e = \{e, (1\ 3)\}$$
$$H * (2\ 3) = \{(2\ 3), (1\ 3\ 2)\}$$
$$H * (1\ 2\ 3) = \{(1\ 2\ 3), (1\ 2)\}$$

□

Exercise 4. *Let* $(M, *)$ *be a group such that* $a^2 = e$ *for all* $a \in M$*. Prove that* M *is an abelian group.*

Proof. Take $\alpha, \beta \in M$. By the hypothesis, $(\alpha * \beta)^2 = e$. So we have $\alpha * \beta = \alpha * e * \beta = \alpha * \alpha * \beta * \alpha * \beta * \beta = e * \beta * \alpha * e = \beta * \alpha$. Thus M is an abelian group. □