

HW number Seven, MTH 320, SPRING 2009

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QUESTION 1. Find all subgroups of $(Z_{13}^*, \times_{13})$.

QUESTION 2. a) Let $n \geq 3$. Show that $[n-1] \in (U(Z_n), \times_n)$ is an element of order 2.

b) Show that $(U(Z_{35}, \times_{35})$ is not a cyclic group. (Hint: find elements in $U(Z_{35})$ that have order 2)

c) We know that $(Z_{47}^*, \times_{47})$ is a cyclic group. Show that there are as many elements of order 23 as there are elements of order 46.

d) Let $\alpha \in S_{99}$ such that $\mid \alpha \mid = 99$. Show that α^{66} is either a 3-cycle or the composition of disjoint 3-cycles.

QUESTION 3. a) Let $S = \{\begin{bmatrix} a & b \\ 0 & c \end{bmatrix} \mid a, b, c \in Z_{23}\}$. It is easy to see that $(S, \times_{23})$ is a monoid. Note that $\times_{23}$ is the normal multiplication of matrices but module 23. (I explain more on Sunday). Let U(S) be the set of all invertible elements of S under $\times_{23}$. Thus we know $(U(S), \times_{23})$ is a group. Find $\mid U(S) \mid$. Is U(S) an abelian group? or a non-abelian group? EXPLAIN

b) Let $a = \begin{bmatrix} 2 & 18 \\ 0 & 7 \end{bmatrix}$. Then $a \in S$. Is $a \in U(S)$? if yes then find a^{-1}. Note that S, U(S) are as in (a).

c)Let $M = \{\begin{bmatrix} a & b \\ 0 & c \end{bmatrix} \mid a, b, c \in Z\}$. It is easy to see that $(M, \times)$ is a monoid. Note that $\times$ is the normal multiplication of matrices . Let U(M) be the set of all invertible elements of M under $\times$. Thus we know $(U(M), \times)$ is a group. Is U(M) an abelian group? or a non-abelian group? EXPLAIN. If $a \in U(M)$, find a general form (describtion) of a. Let $a = \begin{bmatrix} 2 & 18 \\ 0 & 7 \end{bmatrix}$. Then $a \in M$. Is $a \in U(M)$? if yes then find a^{-1}

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