

HW number Six, MTH 320, SPRING 2009

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QUESTION 1. a) Let $(M, *)$ be a group and $(H, *)$ be a subgroup of M such that $H \neq M$. Define $=_R$ on M such that for every $a, b \in M$ (a, b not necessary distinct) $a =_R b$ if $b^{-1} * a \in H$. Show that $=_R$ is an equivalent relation on $(M, *)$ (you must show reflexive, symmetric, transitive)

b)Let M and H as in (a) but assume that M is an abelian group (and hence H is abelian). Let S be the set of all distinct equivalence classes of $(M, *, =_R)$. Define a binary operation $\wedge$ on S as following: Let $d, k \in S$. Then $d = [a], k = [c]$ for some $a, c \in M$. Now $[a] \wedge [c]$ means : chose $u \in [a]$ and chose $j \in [c]$ and let $[a] \wedge [c] = [u * j]$.

i) Show that $\wedge$ is a well-defined relation on S.

ii) Show that $(S, \wedge)$ is an abelian group.

QUESTION 2. Let $(M, *)$ be a group:

a) Let $a, b \in M$ such that $a * b = b * a$, $\mid a \mid = n$, $\mid b \mid = m$, and $gcd(n, m) = 1$. Show that $\mid a * b \mid = nm$.

b)Let $a, b \in M$ such that $a * b = b * a$, $\mid a \mid = n$, $\mid b \mid = m$. Show there is an element $c \in M$ such that $\mid c \mid = LCM[n, m]$. (Hint: you may want to use the conclusion of (a))

QUESTION 3. Construct the additive table for $(Z_7, +_7)$ and the multiplicative table for $(Z_7^*, \times_7)$.

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