(18000, 1)

1. (15 points) At a price of \$1 per unit of a certain product, the demand is 18 thousand units and when the price rises to \$6 per unit the demand decreases to 8 thousand units.
   a. Assuming the price-demand relationship is linear, find an equation for this relationship.
   b. If the price-supply equation is given by $p = 0.25x + 2.5$ where x is the number of items produced in thousand units, find the equilibrium point.
   c. Graph the price-demand and price-supply equations in the same coordinate system. Clearly mark the equilibrium point on the graph.

(a) (18, 1) (8, 6)

$P = mx + b$ $\quad m = \frac{\Delta y}{\Delta x} = \frac{6-1}{8-18} = -0.5$

$P = -0.5x + b \Rightarrow 1 = -0.5(18) + b \Rightarrow b = 10$

$P = -0.5x + 10$ ✓

(b) Price Demand = Price Supply ✓

$-0.5x + 10 = 0.25x + 2.5$ ✓

$-0.75x = -7.5$

$\therefore x = 10$ thousands of units (10, 5)

$P(10) = 0.25(10) + 2.5 \Rightarrow P(10) = 5$ ✓

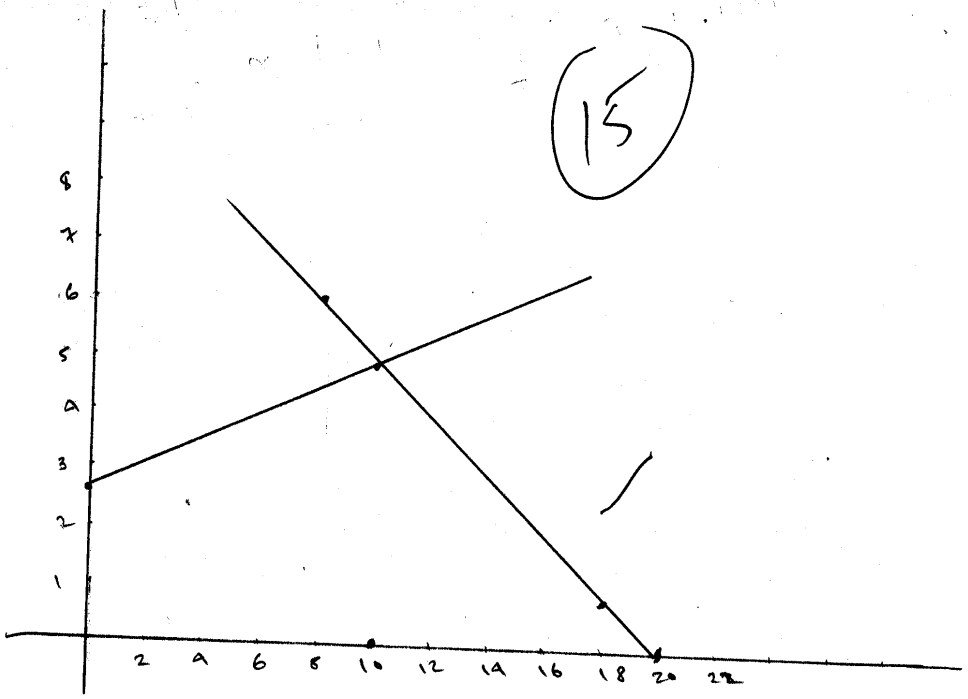
PD: y-int $\Rightarrow x = 0 \Rightarrow P = -0.5(0) + 10 \Rightarrow (0, 10)$ $y = 10$

x-int $\Rightarrow y = 0 \Rightarrow 0 = -0.5x + 10 \Rightarrow x = 20$ (20, 0)

PS: y-int $\Rightarrow x = 0 \Rightarrow P = 0.25(0) + 2.5 \Rightarrow y = 2.5 \Rightarrow (0, 2.5)$

x int $\Rightarrow y = 0 \Rightarrow 0 = 0.25x + 2.5 \Rightarrow x = 10 \Rightarrow (10, 0)$

15



Leduc

2. (15 points) Suppose that the revenue and cost functions for a small business are

$R(x) = x(70 - x)$, and

$C(x) = 20x$, where $0 \le x \le 70$.

(4) **a.** Find the break-even point(s).

(4) **b.** Find the maximum revenue.

(6) **c.** Sketch the graph of $R(x)$ and $C(x)$ on the same coordinates showing the intercepts and the vertex (for $R(x)$). Indicate the regions of loss and profit.

let $P(x) = 0$

$P(x) = R(x) - C(x)$

$= 70x - x^2 - 20x$

$= -x^2 + 50x \Rightarrow 0$

$-x^2 + 50x = 0$

$a = -1$

$b = 50$

$C = 0$

$x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} = \frac{-50 + \sqrt{50^2 - 4(-1)(0)}}{2(-1)} = 0$ ✓

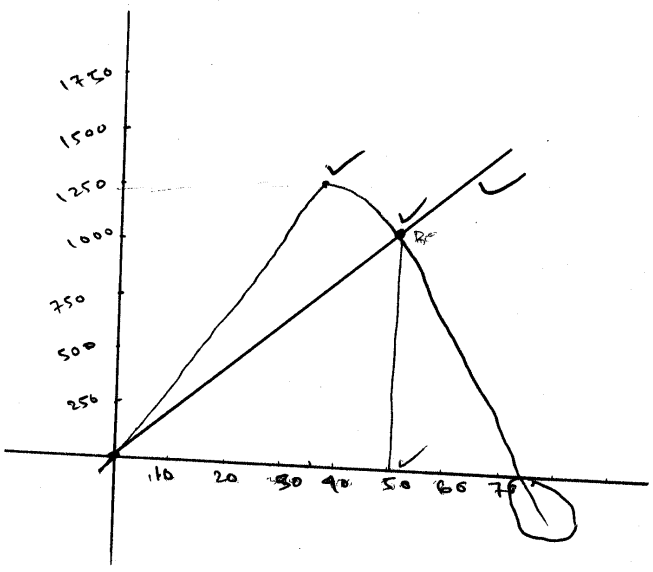
$x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a} = \frac{-50 - \sqrt{50^2 - 4(-1)(0)}}{2(-1)} = 50$ ✓

BEP (0, 0) , (50, 1000)

~~vertex~~ $x = \frac{-b}{2a} = \frac{-70}{2(-1)} = 35$ ✓

The maximum revenue

$= 70(35) - (35)^2 = 1225\$$ ✓



making money

$0 < x < 50$ ✓

lossing money

$x > 50$ ✓

3. (10 points) How long will it take 10,000 to double if it is invested at 6.5% compounded monthly? $P = 10000\$$ $r = 0.065$ $m = 12$ $i = \frac{0.065}{12} = 0.0054$

$n = mt \Rightarrow 12t$ solve for n

$A = P(1+i)^n$

$20000 = 10000(1+0.00542)^n$

$2 = (1.00542)^n$

$\log 2 = \log 1.00542 \cdot n$

$n = \frac{\log 2}{\log 1.00542} = 128.233$

$n = 12t = 128.233$

$\therefore t = \frac{128.233}{12} = 10.686$ years

10

$m = 4$ $r = 0.08$

4. (13 points) Suppose \$600 is deposited each quarter into an account paying 8% compounded quarterly. $PMT = 600\$$

   a. Find the value of the account after 4 years.
   b. Find the value of the account after 3 years.
   c. Find the interest earned during the fourth year.

$i = \frac{r}{m} = \frac{0.08}{4} = 0.02$

$n = mt = 4 \times 4 = 16$

$n = 3 \times 4 = 12$

(a) $FV_1 = PMT\left(\frac{(1+i)^n - 1}{i}\right)$

$= 600\left(\frac{(1+0.02)^{16} - 1}{0.02}\right)$

$= 11183.57\$$

(b) $FV_2 = PMT\left(\frac{(1+i)^n - 1}{i}\right)$

$= 600\left(\frac{(1+0.02)^{12} - 1}{0.02}\right)$

$= 8047.25\$$

Interest in fourth year = account after 4 − account after 3

$11183.57 - 8047.25 = 3136.32$

$\therefore$ Interest in fourth year:

$3136.32 - 2400 = 736.32\$$

(c)

13

(c) The value of the account in one year (without interest)

$PMT \times m = 600 \times 4 = 2400$

5. (10 points) Adam borrows $6000 at 7% interest rate compounded monthly and plans to amortize the loan over 3 years in equal monthly payments:
   a. What is Adam's monthly payment?
   b. How much interest will Adam pay?

$t = 3$, $m = 12$, $PV = 6000$, $n = mt = 12 \times 3 = 36$, $r = 0.07$, $i = \frac{0.07}{12} = 0.005$

(a) $PMT = PV\left(\frac{i}{1-(1+i)^{-n}}\right)$

$= 6000\left(\frac{0.00583}{1-(1+0.00583)^{-36}}\right)$

$= 185.275\ \$$

10

(b) $I = (PMT)(n) - (PV)$

$= (185.275)(36) - 6000$

$= 6669.9 - 6000$

$= 669.9\ \$$

12

6. (12 points) David and Jim agree to invest equivalent amounts in their business. David will contribute $10,000 now since he has the cash but Jim will deposit an amount each month in an account that pays 6% compounded monthly. How much should Jim deposit so that in 4 years, he will match David's contribution (Assume David's money will earn also 6% compounded monthly)?

David: $P = 10000$, $m = 12$, $r = 0.06$, $i = 0.005$, $n = 4 \times 12$

$A = P(1+i)^n$

$A = 10000(1+0.005)^{48}$

$A = 12709.89\ \$$

Jim: $t = 4$

$FV = 12709.89\ \$$

$PMT = 12709.89\left(\frac{0.005}{(1.005)^{48}-1}\right)$

$= 235.27\ \$$