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MTH 213, Final Exam

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Score = $\frac{64}{67}$

QUESTION 1. Consider the following permutation function

$$f : \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 7 & 3 & 2 & 6 & 1 & 8 & 5 & 4 \end{pmatrix}$$

(a) **(4 points)** Find the least positive integer m such that $f^m = f \circ f \circ \cdots \circ f = I$, the identity function

$(1\ 7\ 5)(2\ 3)(4\ 6\ 8)$

$LCM(LCM(3,2), 3) = LCM(6, 3) = \frac{6 \times 3}{gcd(6,3)} = 6$

$\boxed{m = 6}$

(b) **(2 points)** Find f^3

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 1 & 3 & 2 & 4 & 5 & 6 & 7 & 8 \end{pmatrix}$$

QUESTION 2. (6 points) Use math induction and prove that $7 | (5^{(2n+1)} + 2^{(2n+1)}), \forall n \geq 1$ [Hint: personally, I know how to do it if at some point I use the fact: if k, d are positive integers, then $k = qd + r$, where $0 \leq r < d$. If you can do it without the hint, then just ignore my hint]

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(i) Proof for $n=1$ $7 | 5^3 + 2^3 \rightarrow \frac{5^3+2^3}{7} = 19$

(ii) Assume $7 | 5^{2n+1} + 2^{2n+1}$ is true for some $n \geq 1$

(iii) Proof for $n+1$

$5^{2(n+1)+1} + 2^{2(n+1)+1}$

$5^{2n+2+1} + 2^{2n+2+1}$

$5^{2n+1} \cdot 5^2 + 2^{2n+1} \cdot 2^2$

$5^{2n+1} \cdot 5^2 + 5^2 \cdot 2^{2n+1} - 5^2 \cdot 2^{2n+1} + 2^{2n+1} \cdot 2^2$

$5^2(5^{2n+1} + 2^{2n+1}) - 5^2 \cdot 2^{2n+1} + 2^2 \cdot 2^{2n+1}$

$5^2(5^{2n+1} + 2^{2n+1}) + 2^{2n+1}(-5^2 + 2^2)$

$\underbrace{5^2(5^{2n+1} + 2^{2n+1})}_{\text{divisible by 3 using (ii)}} + \underbrace{2^{2n+1}(-21)}_{\text{divisible by 3}}$

$-21/3 = 3$

$-21 \bmod 3 = 0$

QUESTION 3. (4 points) Write down T or F

(i) If $\exists! x \in Z^+$ such that $x^2 - 2 \leq 1$, then $\exists y \in \mathbb{R}$ such that $y^2 - 2 = 0$ T

(ii) $\exists\, x \in \mathbb{Q}$ such that $x^3 = 9$ if and only if $\exists! y \in Z$ such that $y^2 = 1$. T

(iii) $\forall x \in Z, \exists y \in \mathbb{Q}$ such that $x + y = 1$ T

(iv) $\exists y \in \mathbb{Q}$ such that $\forall x \in Z$, we have $x^2 + y = 3$ F

QUESTION 4. (6 points) Consider the following code

$$For\ k = 1\ to\ (n+2)\ do$$
$$y = i^3 + i^2 + 2 * i \quad 6$$
$$For\ i = 1\ to\ (k+1)\ do$$
$$x = i^4 + 2 * i^3 + 10 \quad 8$$
$$Next\ i$$
$$Next\ k$$

a) Find the exact number of arithmetic operations that will be executed by the code.

Outer loop:
iterations: $n+2-1+1 = n+2$
operations: $6(n+2)$

Inner loop:
first time:
iterations: $2-1+1 = 2$
operations: $8(2) = 16$

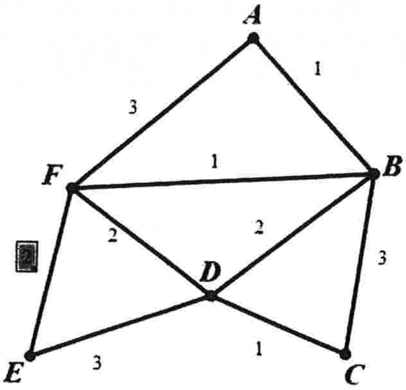
last time:
iterations: $n+2+1-1+1 = n+3$
operations: $8(n+3)$

$$\frac{8n+24+16}{2}(n+2) + 6n + 12$$

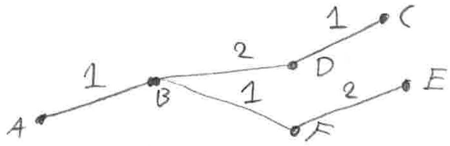
b) Find the complexity of the code, i.e. O(code)

$O(\text{code}) = O(n^2)$

QUESTION 5. (8 points)
Consider the following connected graph.



V	A	B	C	D	E	F
A	[0]	1^A	∞	∞	∞	3^A
B		$[1^A]$	4^B	3^B	∞	2^B
F			4^B	3^B	4^F	$[2]^B$
D			4^D	$[3]^B$	4^F	
C			$[4]^D$		4^F	
E					$[4^F]$	



Use Dijkstra Algorithm and find the minimum spanning tree.

QUESTION 6. (6 points) We have 5 men and 3 females. We need to form a committee of 4 persons.

(i) In how many ways can we form such committee where exactly two men are serving on the committee?

2M and 2F $\quad 5C_2 \times 3C_2 = 30$ ways

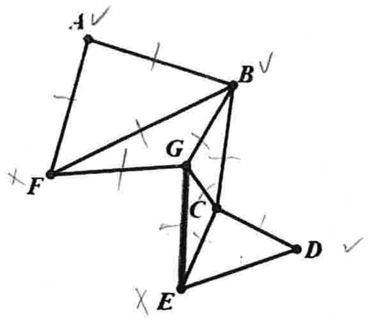
(ii) In how many ways can we form such committee where at least two females are serving on the committee?

2F and 2M or 3F and 1M | $3C_2 \times 5C_2 + 3C_3 \times 5C_1 =$
$30 + 5 = 35$ ways

(iii) Assume the names of the females are Raneem, Ideal, and Ring. In how many ways can we form such committee where Raneem, exactly two men, and one more female are serving on the committee?

$1 \times 2C_1 \times 5C_2 = 20$ ways

QUESTION 7. (9 points)(SHOW THE WORK) Consider the below graph of order 7



for it to be euleurian, each degree must be even

(i) Is the graph Eulerian? explain. If yes, construct an Euler circuit.

NO, since deg(E) = deg(F) = 3 which is odd

(ii) Is the graph an Euler trail? explain. If yes, construct an Euler trail.

yes, since there are exactly to odd degrees F – A – B – F – G – B – C – G – E – C – D – E

(iii) Is the graph Hamiltonian? explain. If yes, construct a Hamiltonian cycle (C_7)

yes it is hamiltonian

D — E — G — F — A — B — C — D

QUESTION 8. (**6 points)** Consider the linear recurrence $a_n = 4a_{n-1} - 3a_{n-2} + 6(2^n)$. Find the general formula for a_n, find a_h and a_p. No need to find c_1, c_2.

$a_n - 4a_{n-1} + 3a_{n-2} = 0$

$H(n) = 0$

$a_n - 4a_{n-1} + 3a_{n-2} = 0$

$\alpha^n - 4\alpha^{n-1} + 3\alpha^{n-2} = 0$

$\alpha^2 - 4\alpha + 3\alpha = 0$

$\alpha_1 = 3 \quad \alpha_2 = 1$

$H(n) = C_1(3)^n + C_2(1)^n$

$a_n = H(n) + P(n)$

$\boxed{a_n = C_1(3)^n + C_2 - 24(2)^n}$

$P(n) = a(2^n)$

$P(n) - 4P(n-1) + 3P(n-2) = 6(2^n)$

$a(2^n) - 4(a(2^{n-1})) + 3(a(2^{n-2})) = 6(2^n)$

$\frac{a(2^n) - 4a2^{n-1} + 3a2^{n-2}}{2^{n-2}} = \frac{6(2^n)}{2^{n-2}}$

$a(2^2) - 4a(2) + 3a = 6(2^2)$

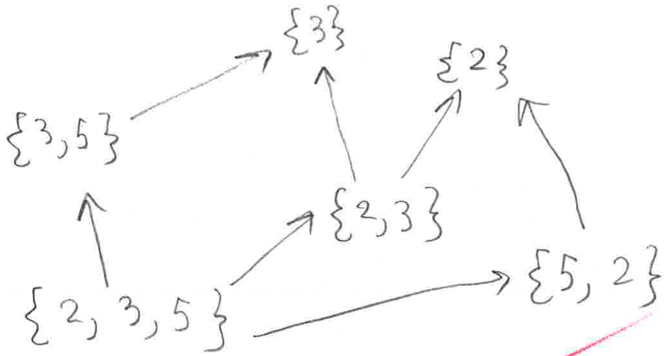
$4a - 8a + 3a = 24$

$-a = 24 \longrightarrow a = -24$

$P(n) = -24(2^n)$

QUESTION 9. (**4 points)** Let $A = \{\{3,5\},\{2\},\{3\},\{5,2\},\{2,3\},\{2,3,5\}\}$. Define " $\leq$ " on A such that for every $x, y \in A$, we have $x \leq y$ if $y \subseteq x$ We know that $\leq$ is a partial order. Draw the Hasse diagram of such relation.

y is Part of x



QUESTION 10. (6 points) Let x be a positive integer such that $0 < x < 99$, given $x\ (mod\ 11) = 2$ and $x\ (mod\ 9) = 5$. Find x

$x \pmod{11} = 2 \quad r_1 = 2 \qquad GCD(11, 9) = 1$ use CRT

$x \pmod 9 = 5 \quad r_2 = 5$

$m_1 = 11 \qquad m_2 = 9 \qquad m = 99$

$b_1 = 9 \qquad b_2 = 11$

$Z_{11} \qquad Z_9$

$9 d_1 = 1 \qquad 11 d_2 = 1$

$(9 \cdot d_1) \bmod 11 = 1 \qquad (11 \cdot d_2) \bmod 9 = 1$

$d_1 = 5 \qquad d_2 = 5$

$x = [5 \times 9 \times 2 + 5 \times 11 \times 5] \bmod 99$

$x = 365 \bmod 99$

$\boxed{x = 68}$

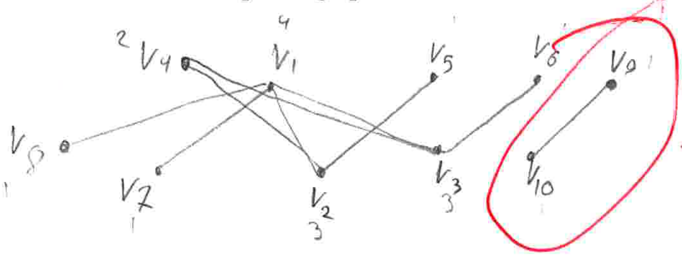
QUESTION 11. (6 points) Let G be a CONNECTED graph of order 10 with the following degrees

$v_1 \quad v_3 \quad v_5 \quad v_2 \quad v_9$

4, 3, 3, 2, 1, 1, 1, 1, 1, 1

$v_2 \quad v_4 \quad v_6 \quad v_8 \quad v_{10}$

. By drawing, convince me that G is a bipartite graph.



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It is a bipartite

A: {V_4, V_1, V_5, V_6, V_9}

B: {V_8, V_7, V_2, V_3, V_{10}}

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Is G a tree? convince me? explain briefly

No, It is not a thee since G is not connected

Faculty information

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