

## Exam II , MTH 213 , Fall 2011

Ayman Badawi

[YOU HAVE 10 QUESTIONS, EACH = 10 points]

**QUESTION 1.** a) Solve over  $Z_{15}$ ,  $10x \equiv 5 \pmod{15}$

b) Solve over  $Z$ ,  $10x \equiv 5 \pmod{15}$

**QUESTION 2.** Find all integers where each integer satisfies the following two conditions: I) If each integer is divided by 6, then the remainder is 3, and II) if each integer is divided by 4, then the remainder is 1.

**QUESTION 3.** Solve over  $Z$ ,  $3x \equiv 1 \pmod{4}$  and  $5x \equiv 4 \pmod{7}$  [You need to find all  $x$  values that satisfy the two congruent at the same time].

**QUESTION 4.** Find the  $\gcd(104, 38)$ . Then find two integers  $K, D$  such that  $\gcd(104, 38) = 104K + 38D$ .

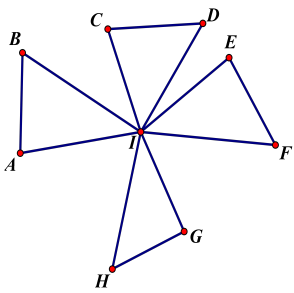
**QUESTION 5.** a) Find  $(101)_7 - (23)_7$

b) Find  $(1465)_8 + (2277)_8$

c) Convert  $(146)_{10}$  to base 9

d) Convert  $(12222)_3$  to base 27.

**QUESTION 6.** Is the following graph an Euler graph? If yes, then GIVE ME ONE EULER CIRCUIT. If NO, then tell me why?



b) In the graph above find the degree(I).

c) Is the above graph a Hamilton Graph? Explain

d) Find the distance between the two vertices  $A, G$ ; i.e. Find  $d(A, G)$

**QUESTION 7.** a) Sketch a graph with exactly 4 vertices that is a Hamilton graph but not an Euler graph.

b) Is  $K_{3,3}$  a Hamilton graph? If yes, give me a Hamilton cycle. If not, explain

c) Is  $K_{3,3}$  an Euler graph? If yes, give me an Euler Circuit. If not, explain.

c) Is it possible to construct a graph  $G(V, E)$  such that  $\text{degree}(\text{Each Vertex}) = 6$  and  $|E| = 28$  (note that your graph need not be connected)? Explain

**QUESTION 8.** Let  $m = 11^3 \times 22 \times 44 \times 5^2$ , and let  $D = \{a \in \mathbb{N} \mid 1 < a < m \text{ and } \gcd(a, m) = 220\}$  (note that  $220 = 11 \times 4 \times 5$ )

a) Find  $\phi(m)$

b) Find  $\sum_{i \in D} i$

**QUESTION 9.** Use Math Induction to Prove that  $14^n - 1$  is divisible by 13 for each integer  $n \geq 1$ .

**QUESTION 10.** Find a mathematical formula for the sequence  $a_n$ , where  $a_0 = 3$ , and  $a_1 = 4$ , and  $a_n = 4a_{n-1} - 4a_{n-2}$  for each  $n \geq 2$ . Then find the 9th term in the sequence.

**Faculty information**

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.

E-mail: [abadawi@aus.edu](mailto:abadawi@aus.edu), [www.ayman-badawi.com](http://www.ayman-badawi.com)